Elementary Algebra Textbook
Solutions Manual

Second Edition

Department of Mathematics
College of the Redwoods

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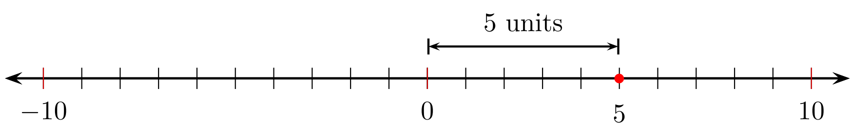
Chapter 1

The Arithmetic of Numbers

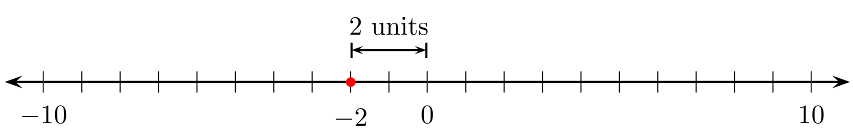
1.1 An Introduction to the Integers

1. The number 5 is 5 units from the origin.



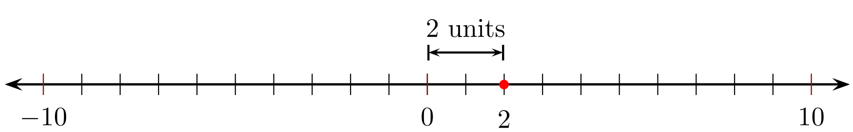
Hence, $|5| = 5$.

3. The number -2 is 2 units from the origin.



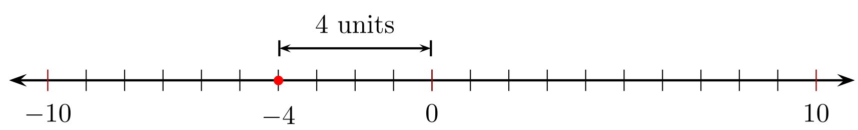
Hence, $|-2| = 2$.

5. The number 2 is 2 units from the origin.



Hence, $|2| = 2$.

7. The number -4 is 4 units from the origin.



Hence, $|-4| = 4$.

9. We have like signs. The magnitudes (absolute values) of -91 and -147 are 91 and 147, respectively. If we add the magnitudes, we get 238. If we prefix the common negative sign, we get -238. That is:

$$-91 + (-147) = -238$$

11. We have like signs. The magnitudes (absolute values) of 96 and 145 are 96 and 145, respectively. If we add the magnitudes, we get 241. If we prefix the common positive sign, we get 241. That is:

$$96 + 145 = 241$$

13. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude $(76 - 46 = 30)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$-76 + 46 = -30.$$

15. We have like signs. The magnitudes (absolute values) of -59 and -12 are 59 and 12, respectively. If we add the magnitudes, we get 71. If we prefix the common negative sign, we get -71. That is:

$$-59 + (-12) = -71$$

17. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude $(86 - 37 = 49)$, then prefix the sign of the integer with the larger magnitude. Thus,

$$37 + (-86) = -49.$$

19. To add a positive and a negative integer, subtract the smaller magnitude from the larger magnitude ($85 - 66 = 19$), then prefix the sign of the integer with the larger magnitude. Thus,

$$66 + (-85) = -19.$$

21. We have like signs. The magnitudes (absolute values) of 57 and 20 are 57 and 20, respectively. If we add the magnitudes, we get 77. If we prefix the common positive sign, we get 77. That is:

$$57 + 20 = 77$$

23. To add a negative and a positive integer, subtract the smaller magnitude from the larger magnitude ($127 - 48 = 79$), then prefix the sign of the integer with the larger magnitude. Thus,

$$-48 + 127 = 79.$$

25. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -20 - (-10) &= -20 + 10 && \text{Subtracting } -10 \text{ is the same as adding } 10. \\ &= -10 && \text{Subtract the magnitudes, and prefix with sign of larger number.} \end{aligned}$$

27. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -62 - 7 &= -62 + (-7) && \text{Subtracting } 7 \text{ is the same as adding } -7. \\ &= -69 && \text{Add the magnitudes, and prefix the common negative sign.} \end{aligned}$$

29. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -77 - 26 &= -77 + (-26) && \text{Subtracting } 26 \text{ is the same as adding } -26. \\ &= -103 && \text{Add the magnitudes, and prefix the common negative sign.} \end{aligned}$$

31. Subtraction means "add the opposite," so change the difference into a sum.

$$\begin{aligned} -7-(-16) &= -7+16 && \text{Subtracting } -16 \text{ is the same as adding 16.} \\ &= 9 && \text{Subtract the magnitudes, and prefix with sign of larger number.} \end{aligned}$$

33. In the expression $(-8)^6$, the exponent 6 tells us to write the base -8 six times as a factor. Thus,

$$(-8)^6 = (-8)(-8)(-8)(-8)(-8)(-8).$$

Now, the product of an even number of negative factors is positive.

$$(-8)^6 = 262144$$

35. In the expression $(-7)^5$, the exponent 5 tells us to write the base -7 five times as a factor. Thus,

$$(-7)^5 = (-7)(-7)(-7)(-7)(-7).$$

Now, the product of an odd number of negative factors is negative.

$$(-7)^5 = -16807$$

37. In the expression $(-9)^2$, the exponent 2 tells us to write the base -9 two times as a factor. Thus,

$$(-9)^2 = (-9)(-9).$$

Now, the product of an even number of negative factors is positive.

$$(-9)^2 = 81$$

39. In the expression $(-4)^4$, the exponent 4 tells us to write the base -4 four times as a factor. Thus,

$$(-4)^4 = (-4)(-4)(-4)(-4).$$

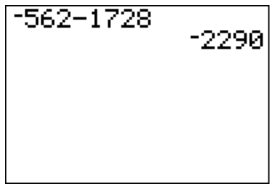
Now, the product of an even number of negative factors is positive.

$$(-4)^4 = 256$$

41. To calculate the expression $-562 - 1728$, enter the expression `-562-1728` using the following keystrokes.

(-) 5 6 2 − 1 7 2 8 ENTER

The result is shown in the following figure.

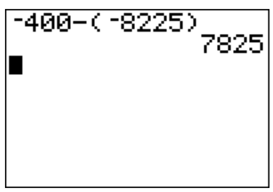


Hence, $-562 - 1728 = -2290$.

43. To calculate the expression $-400-(-8225)$, enter the expression `-400-(-8225)` using the following keystrokes.

(-) 4 0 0 − ((-) 8 2 2 5) ENTER

The result is shown in the following figure.

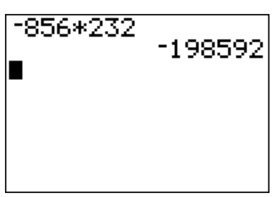


Hence, $-400 - (-8225) = 7825$.

45. To calculate the expression $(-856)(232)$, enter the expression `-856*232` using the following keystrokes.

(-) 8 5 6 × 2 3 2 ENTER

The result is shown in the following figure.

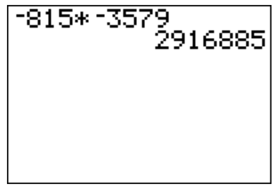


Hence, $(-856)(232) = -198592$.

47. To calculate the expression $(-815)(-3579)$, enter the expression `-815*-3579` using the following keystrokes.

(-) 8 1 5 × (-) 3 5 7 9 ENTER

The result is shown in the following figure.

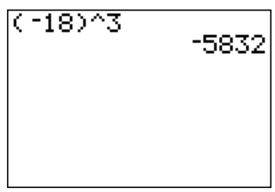


Hence, $(-815)(-3579) = 2916885$.

49. To calculate the expression $(-18)^3$, enter the expression `(-18)∧3` using the following keystrokes.

((-) 1 8) ∧ 3 ENTER

The result is shown in the following figure.

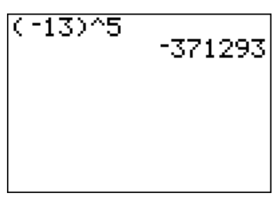


Hence, $(-18)^3 = -5832$.

51. To calculate the expression $(-13)^5$, enter the expression `(-13)∧5` using the following keystrokes.

((-) 1 3) ∧ 5 ENTER

The result is shown in the following figure.



Hence, $(-13)^5 = -371293$.

1.2 Order of Operations

1. The *Rules Guiding Order of Operations* require that multiplication is applied first, then addition.

$$\begin{aligned} -12 + 6(-4) &= -12 + (-24) && \text{Multiply first: } 6(-4) = -24. \\ &= -36 && \text{Add: } -12 + (-24) = -36. \end{aligned}$$

3. In the expression $-(-2)^5$, the exponent 5 tells us to write the base -2 five times as a factor. Thus,

$$-(-2)^5 = -(-2)(-2)(-2)(-2)(-2)$$

Multiply: $(-2)(-2)(-2)(-2)(-2) = -32$.

$$-(-2)^5 = -(-32)$$

Finally, take the opposite.

$$-(-2)^5 = 32$$

5. First take the absolute value, then negate (take the opposite) of the result.

$$\begin{aligned} -|-40| &= -(40) && \text{Absolute value: } |-40| = 40 \\ &= -40 && \text{Negate.} \end{aligned}$$

7. The *Rules Guiding Order of Operations* require that division and multiplication must be done in the order that they appear, working from left to right.

$$\begin{aligned} -24/(-6)(-1) &= 4(-1) && \text{Divide: } -24/(-6) = 4. \\ &= -4 && \text{Multiply: } 4(-1) = -4. \end{aligned}$$

9. The opposite of -50 is 50, or if you wish, the negative of -50 is 50. That is,

$$-(-50) = 50.$$

11. In the expression -3^5, the exponent 5 tells us to write the base 3 five times as a factor. Thus,

$$-3^5 = -(3)(3)(3)(3)(3).$$

Note that order of operations forces us to apply the exponent before applying the minus sign.

$$-3^5 = -243$$

13. The *Rules Guiding Order of Operations* require division and multiplication must be done in the order that they appear, working from left to right.

$$\begin{aligned} 48 \div 4(6) &= 12(6) && \text{Divide: } 48 \div 4 = 12. \\ &= 72 && \text{Multiply: } 12(6) = 72. \end{aligned}$$

15.

$$\begin{aligned} -52 - 8(-8) &= -52 - (-64) && \text{Multiply first: } 8(-8) = -64. \\ &= -52 + 64 && \text{Add the opposite.} \\ &= 12 && \text{Add: } -52 + 64 = 12. \end{aligned}$$

17. In the expression $(-2)^4$, the exponent 4 tells us to write the base -2 four times as a factor. Thus,

$$(-2)^4 = (-2)(-2)(-2)(-2).$$

Multiply. An even number of negative signs yield a positive result.

$$(-2)^4 = 16$$

19. The *Rules Guiding Order of Operations* require that we address exponents first, then multiplications, then subtractions.

$$\begin{aligned} 9 - 3(2)^2 &= 9 - 3(4) && \text{Exponent first: } (2)^2 = 4. \\ &= 9 - 12 && \text{Multiply: } 3(4) = 12. \\ &= 9 + (-12) && \text{Add the opposite.} \\ &= -3 && \text{Add: } 9 + (-12) = -3. \end{aligned}$$

21. We must first evaluate the expression inside the absolute value bars. Subtraction means "add the opposite."

$$\begin{aligned} 17 - 10|13 - 14| &= 17 - 10|13 + (-14)| && \text{Add the opposite.} \\ &= 17 - 10|-1| && \text{Add: } 13 + (-14) = -1 \\ &= 17 - 10(1) && \text{Absolute value: } |-1| = 1 \\ &= 17 - 10 && \text{Multiply: } 10(1) = 10 \\ &= 17 + (-10) && \text{Add the opposite.} \\ &= 7 && \text{Add: } 17 + (-10) = 7 \end{aligned}$$

23. The *Rules Guiding Order of Operations* require that we address exponents first, then multiplications, then subtractions.

$$\begin{aligned} -4+5(-4)^3 &= -4+5(-64) && \text{Exponent first: } (-4)^3=-64. \\ &= -4+(-320) && \text{Multiply: } 5(-64)=-320. \\ &= -324 && \text{Add: } -4+(-320)=-324. \end{aligned}$$

25. The *Rules Guiding Order of Operations* require evaluate the expression inside the parentheses first, then multiply, then subtract.

$$\begin{aligned} 8+5(-1-6) &= 8+5(-1+(-6)) && \text{Add the opposite.} \\ &= 8+5(-7) && \text{Add:} -1+(-6)=-7. \\ &= 8+(-35) && \text{Multiply: } 5(-7)=-35. \\ &= -27 && \text{Add: } 8+(-35)=-27. \end{aligned}$$

27. Following the *Rules Guiding Order of Operations*, we must first simplify the expressions inside the parentheses. Then we can apply the exponents and after that, subtract.

$$\begin{aligned} &(10-8)^2-(7-5)^2 \\ &\quad = 2^2-2^3 && \text{Subtract: } 10-8=2;\ 7-5=2. \\ &\quad = 4-8 && \text{Square: } 2^2=4. \\ & && \text{Cube: } 2^3=8. \\ &\quad = 4+(-8) && \text{Add the opposite.} \\ &\quad = -4 && \text{Add: } 4+(-8)=-4. \end{aligned}$$

29. The *Rules Guiding Order of Operations* require evaluate the expression inside the innermost parentheses first.

$$\begin{aligned} 6-9(6-4(9-7)) &= 6-9(6-4(9+(-7))); && \text{Add the opposite.} \\ &= 6-9(6-4(2)); && \text{Add: } 9+(-7)=2. \\ &= 6-9(6-8); && \text{Multiply: } 4(2)=8. \\ &= 6-9(6+(-8)); && \text{Add the opposite.} \\ &= 6-9(-2); && \text{Add: } 6+(-8)=-2. \\ &= 6-(-18); && \text{Multiply: } 9(-2)=-18. \\ &= 6+18; && \text{Add the opposite.} \\ &= 24 && \text{Add: } 6+18=24. \end{aligned}$$

31. The *Rules Guiding Order of Operations* require evaluate the expression inside the parentheses first, then multiply, then subtract.

$$\begin{aligned}
-6-5(4-6) &= -6-5(4+(-6)) && \text{Add the opposite.}\\
&= -6-5(-2) && \text{Add:} 4+(-6)=-2.\\
&= -6-(-10) && \text{Multiply: } 5(-2)=-10.\\
&= -6+10 && \text{Add the opposite.}\\
&= 4 && \text{Add: } -6+10=4.
\end{aligned}$$

33. Following the *Rules Guiding Order of Operations*, simplify the expression inside the parentheses first, then apply the exponent, then add and subtract, moving left to right.

$$\begin{aligned}
9+(9-6)^3-5 &= 9+3^3-5 && \text{Subtract: } 9-6=3.\\
&= 9+27-5 && \text{Exponent: } 3^3=27.\\
&= 9+27+(-5) && \text{Add the opposite.}\\
&= 36+(-5) && \text{Add: } 9+27=36.\\
&= 31 && \text{Add: } 36+(-5)=31.
\end{aligned}$$

35. The *Rules Guiding Order of Operations* require that we address exponents first, then multiplications, then subtractions.

$$\begin{aligned}
-5+3(4)^2 &= -5+3(16) && \text{Exponent first: } (4)^2=16.\\
&= -5+48 && \text{Multiply: } 3(16)=48.\\
&= 43 && \text{Add: } -5+48=43.
\end{aligned}$$

37. Following the *Rules Guiding Order of Operations*, simplify the expression inside the parentheses first, then apply the exponent, then add and subtract, moving left to right.

$$\begin{aligned}
8-(5-2)^3+6 &= 8-3^3+6 && \text{Subtract: } 5-2=3.\\
&= 8-27+6 && \text{Exponent: } 3^3=27.\\
&= 8+(-27)+6 && \text{Add the opposite.}\\
&= -19+6 && \text{Add: } 8+(-27)=-19.\\
&= -13 && \text{Add: } -19+6=-13.
\end{aligned}$$

39. We must first evaluate the expression inside the absolute value bars. Start with "subtraction means add the opposite."

$$\begin{aligned}
&|6-15|-|-17-11| && \\
&\quad=|6+(-15)|-|-17+(-11)| && \text{Add the opposites.}\\
&\quad=|-9|-|-28| && \text{Add: } 6+(-15)=-9 \\
& && \text{and } -17+(-11)=-28\\
&\quad=9-28 && \text{Absolute value: } |-9|=9\\
& && \text{and } |-28|=28\\
&\quad=9+(-28) && \text{Add the opposite.}\\
&\quad=-19 && \text{Add: } 9+(-28)=-19
\end{aligned}$$

41. The *Rules Guiding Order of Operations* require evaluate the expression inside the innermost parentheses first.

$$\begin{aligned}
5-5(5-6(6-4)) &= 5-5(5-6(6+(-4))); && \text{Add the opposite.}\\
&= 5-5(5-6(2)); && \text{Add: } 6+(-4)=2.\\
&= 5-5(5-12); && \text{Multiply: } 6(2)=12.\\
&= 5-5(5+(-12)); && \text{Add the opposite.}\\
&= 5-5(-7); && \text{Add: } 5+(-12)=-7.\\
&= 5-(-35); && \text{Multiply: } 5(-7)=-35.\\
&= 5+35; && \text{Add the opposite.}\\
&= 40 && \text{Add: } 5+35=40.
\end{aligned}$$

43. First replace all occurrences of the variables in the expression with open parentheses:

$$4x^2+3xy+4y^2=4(\quad)^2+3(\quad)(\quad)+4(\quad)^2$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned}
4x^2+3xy+4y^2 &= 4(-3)^2+3(-3)(0)+4(0)^2 && \text{Substitute } -3 \text{ for } x \text{ and } 0 \text{ for } y.\\
&= 4(9)+3(-3)(0)+4(0) && \text{Evaluate exponents first.}\\
&= 36 && \text{Perform multiplications, left to right.}\\
&= 36 && \text{Perform additions and subtractions, left to right.}
\end{aligned}$$

45. First replace all occurrences of the variables in the expression with open parentheses:

$$-8x+9=-8(\quad)+9$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} -8x + 9 &= -8(-9) + 9 && \text{Substitute } -9 \text{ for } x. \\ &= 72 + 9 && \text{Multiply first: } -8(-9) = 72 \\ &= 81 && \text{Add.} \end{aligned}$$

47. First replace all occurrences of the variables in the expression with open parentheses:

$$-5x^2 + 2xy - 4y^2 = -5(\quad)^2 + 2(\quad)(\quad) - 4(\quad)^2$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} -5x^2 + 2xy - 4y^2 &= -5(5)^2 + 2(5)(0) - 4(0)^2 && \text{Substitute 5 for } x \text{ and 0 for } y. \\ &= -5(25) + 2(5)(0) - 4(0) && \text{Evaluate exponents first.} \\ &= -125 && \text{Perform multiplications, left to right.} \\ &= -125 && \text{Perform additions and subtractions, left to right.} \end{aligned}$$

49. First replace all occurrences of the variables in the expression with open parentheses:

$$3x^2 + 3x - 4 = 3(\quad)^2 + 3(\quad) - 4$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} 3x^2 + 3x - 4 &= 3(5)^2 + 3(5) - 4 && \text{Substitute 5 for } x. \\ &= 3(25) + 3(5) - 4 && \text{Evaluate exponents first.} \\ &= 75 + 15 - 4 && \text{Perform multiplications, left to right.} \\ &= 86 && \text{Perform additions and subtractions, left to right.} \end{aligned}$$

51. First replace all occurrences of the variables in the expression with open parentheses:

$$-2x^2 + 2y^2 = -2(\quad)^2 + 2(\quad)^2$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned} -2x^2 + 2y^2 &= -2(1)^2 + 2(-2)^2 && \text{Substitute 1 for } x \text{ and } -2 \text{ for } y. \\ &= -2(1) + 2(4) && \text{Evaluate exponents first.} \\ &= -2 + 8 && \text{Perform multiplications, left to right.} \\ &= 6 && \text{Perform additions and subtractions, left to right.} \end{aligned}$$

53. First replace all occurrences of the variables in the expression with open parentheses:

$$-3x^2 - 6x + 3 = -3(\quad)^2 - 6(\quad) + 3$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned}
-3x^2 - 6x + 3 &= -3(2)^2 - 6(2) + 3 && \text{Substitute 2 for } x. \\
&= -3(4) - 6(2) + 3 && \text{Evaluate exponents first.} \\
&= -12 - 12 + 3 && \text{Perform multiplications, left to right.} \\
&= -21 && \text{Perform additions and subtractions, left to right.}
\end{aligned}$$

55. First replace all occurrences of the variables in the expression with open parentheses:

$$-6x - 1 = -6(\quad) - 1$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned}
-6x - 1 &= -6(1) - 1 && \text{Substitute 1 for } x. \\
&= -6 - 1 && \text{Multiply first: } -6(1) = -6 \\
&= -7 && \text{Subtract.}
\end{aligned}$$

57. First replace all occurrences of the variables in the expression with open parentheses:

$$3x^2 - 2y^2 = 3(\quad)^2 - 2(\quad)^2$$

Then replace each variable with its given value, and evaluate the expression:

$$\begin{aligned}
3x^2 - 2y^2 &= 3(-3)^2 - 2(-2)^2 && \text{Substitute } -3 \text{ for } x \text{ and } -2 \text{ for } y. \\
&= 3(9) - 2(4) && \text{Evaluate exponents first.} \\
&= 27 - 8 && \text{Perform multiplications, left to right.} \\
&= 19 && \text{Perform additions and subtractions, left to right.}
\end{aligned}$$

59. Replace each occurrence of the variables a and b with open parentheses, then substitute 27 for a and -30 for b.

$$\begin{aligned}
\frac{a^2 + b^2}{a + b} &= \frac{(\quad)^2 + (\quad)^2}{(\quad) + (\quad)} && \text{Replace variables with open parentheses.} \\
&= \frac{(27)^2 + (-30)^2}{(27) + (-30)} && \text{Substitute: 27 for } a \text{ and } -30 \text{ for } b.
\end{aligned}$$

In the numerator, exponents first, then add. In the denominator, add. Finally, divide numerator by denominator.

$$\begin{aligned}
&= \frac{729+900}{-3} && \text{Numerator, exponents first.} \\
& && \text{Denominator, add.} \\
&= \frac{1629}{-3} && \text{Numerator: } 729+900=1629 \\
&= -543 && \text{Divide: } \frac{1629}{-3} = -543
\end{aligned}$$

61. Replace each occurrence of the variables a, b, c, and d with open parentheses, then substitute -42 for a, 25 for b, 26 for c, and 43 for d.

$$\begin{aligned}
\frac{a+b}{c-d} &= \frac{(\quad)+(\quad)}{(\quad)-(\quad)} && \text{Replace variables with open parentheses.} \\
&= \frac{(-42)+(25)}{(26)-(43)} && \text{Substitute: } -42 \text{ for } a, 25 \text{ for } b, \\
& && 26 \text{ for } c, \text{ and } 43 \text{ for } d.
\end{aligned}$$

In the denominator, change the subtraction to adding the opposite. Next, simplify numerator and denominator, then divide.

$$\begin{aligned}
&= \frac{-42+25}{26+(-43)} && \text{In the denominator, add the opposite.} \\
&= \frac{-17}{-17} && \text{Numerator: } -42+25=-17 \\
& && \text{Denominator: } 26+(-43)=-17 \\
&= 1 && \text{Divide: } \frac{-17}{-17} = 1
\end{aligned}$$

63. Replace each occurrence of the variables a, b, c, and d with open parentheses, then substitute -7 for a, 48 for b, 5 for c, and 11 for d.

$$\begin{aligned}
\frac{a-b}{cd} &= \frac{(\quad)-(\quad)}{(\quad)(\quad)} && \text{Replace variables with open parentheses.} \\
&= \frac{(-7)-(48)}{(5)(11)} && \text{Substitute: } -7 \text{ for } a, 48 \text{ for } b, \\
& && 5 \text{ for } c, \text{ and } 11 \text{ for } d.
\end{aligned}$$

In the numerator, change the subtraction to adding the opposite. Next, simplify numerator and denominator, then divide.

$$\begin{aligned}
&= \frac{-7+(-48)}{(5)(11)} && \text{In the numerator, add the opposite.} \\
&= \frac{-55}{55} && \text{Numerator: } -7+(-48)=-55 \\
& && \text{Denominator: } (5)(11)=55 \\
&= -1 && \text{Divide: } \frac{-55}{55} = -1
\end{aligned}$$

65. Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variables in the expression $a^2 + b^2$ with open parentheses, then substitute 3 for a and 4 for b and simplify.

$$\begin{aligned} a^2+b^2 &= (\quad)^2 + (\quad)^2 && \text{Replace variables with open parentheses.}\\ &= (3)^2 + (4)^2 && \text{Substitute: 3 for } a\text{, 4 for } b.\\ &= 9 + 16 && \text{Square: } (3)^2 = 9;\ (4)^2 = 16.\\ &= 25 && \text{Add: } 9 + 16 = 25. \end{aligned}$$

Now we deal with the second expression.

$$\begin{aligned} (a+b)^2 &= ((\quad) + (\quad))^2 && \text{Replace variables with open parentheses.}\\ &= ((3) + (4))^2 && \text{Substitute: 3 for } a\text{, 4 for } b.\\ &= (7)^2 && \text{Add: } 3 + 4 = 7.\\ &= 49 && \text{Square: } (7)^2 = 49. \end{aligned}$$

Thus, if $a = 3$ and $b = 4$, we found that $a^2 + b^2 = 25$, but $(a + b)^2 = 49$. Hence the expressions $a^2 + b^2$ and $(a + b)^2$ did **not** produce the same results.

67. Following *Tips for Evaluating Algebraic Expressions*, first replace all occurrences of variables in the expression $|a||b|$ with open parentheses, then substitute -3 for a and 5 for b and simplify.

$$\begin{aligned} |a||b| &= |(\quad)||(\quad)| && \text{Replace variables with open parentheses.}\\ &= |(-3)||(5)| && \text{Substitute: } -3 \text{ for } a\text{, 5 for } b.\\ &= (3)(5) && \text{Simplify: } |-3| = 3;\ |5| = 5.\\ &= 15 && \text{Multiply: } (3)(5) = 15. \end{aligned}$$

Now we deal with the second expression.

$$\begin{aligned} |ab| &= |(\quad)(\quad)| && \text{Replace variables with open parentheses.}\\ &= |(-3)(5)| && \text{Substitute: } -3 \text{ for } a\text{, 5 for } b.\\ &= |-15| && \text{Multiply: } (-3)(5) = -15.\\ &= 15 && \text{Simplify: } |-15| = 15. \end{aligned}$$

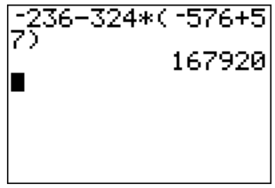
Thus, if $a = -3$ and $b = 5$, we found that $|a||b| = 15$ and $|ab| = 15$. Hence the expressions $|a||b|$ and $|ab|$ did produce the same results.

69. To evaluate the expression $-236 - 324(-576 + 57)$, enter the expression `-236-324*(-576+57)` using the following keystrokes.

(-) 2 3 6 − 3 2 4 × ((-) 5 7 6

+ 5 7) ENTER

The result is shown in the following figure.



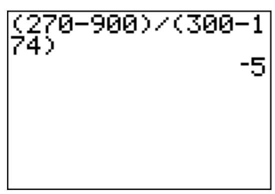
Hence, $-236 - 324(-576 + 57) = 167920$.

71. To evaluate the expression $\dfrac{270 - 900}{300 - 174}$, enter the expression `(270-900)/(300-174)` using the following keystrokes.

(2 7 0 − 9 0 0) ÷ (3 0 0

− 1 7 4) ENTER

The result is shown in the following figure.



Hence, $\dfrac{270 - 900}{300 - 174} = -5$.

73. First, store -93 in `A` with the following keystrokes. The variable `A` is located above the MATH button. Press the ALPHA key to access `A`.

(-) 9 3 STO▷ ALPHA A

Second, store 84 in `B` with the following keystrokes. The variable `B` is located above the APPS button. Press the ALPHA key to access `B`.

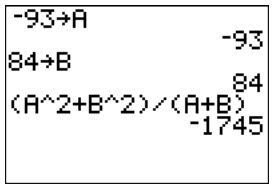
8 4 STO▷ ALPHA B

Finally, enter the expression `(A^2+B^2)/(A+B)` with the following keystrokes.

(ALPHA A ∧ 2 + ALPHA B ∧ 2) ÷

(ALPHA A + ALPHA B) ENTER

The results are shown in the following figure



Hence, $\dfrac{a^2+b^2}{a+b} = -1745$.

75. Start with the formula $F = (9/5)C + 32$, replace the variables C with open parentheses, then substitute 60 for C and simplify.

$$\begin{aligned}
F &= \frac{9}{5}C + 32 && \text{Fahrenheit formula.}\\
&= \frac{9}{5}(\quad) + 32 && \text{Replace } C \text{ with open parentheses.}\\
&= \frac{9}{5}(60) + 32 && \text{Substitute: 60 for } C.\\
&= 108 + 32 && \text{Multiply: } (9/5)60 = 108.\\
&= 140 && \text{Add: } 108 + 32 = 140.
\end{aligned}$$

Hence, the Fahrenheit temperature is 140° F.

77. Start with the kinetic energy formula $K = (1/2)mv^2$, replace the variables m and v with open parentheses, then substitute 7 for m and 50 for v and simplify.

$$\begin{aligned}
K &= \frac{1}{2}mv^2 && \text{Kinetic energy formula.}\\
&= \frac{1}{2}(\quad)(\quad)^2 && \text{Replace } m \text{ and } v \text{ with open parentheses.}\\
&= \frac{1}{2}(7)(50)^2 && \text{Substiute: 7 for } m,\ 50 \text{ for } v.\\
&= \frac{1}{2}(7)(2500) && \text{Square: } (50)^2 = 2500.\\
&= 8750 && \text{Multiply: } (1/2)(7)(2500) = 8750.
\end{aligned}$$

Hence, the kinetic energy of the object is $8,750$ joules.

1.3 The Rational Numbers

1. The greatest common divisor of the numerator and denominator is $\gcd(20, 50) = 10$. Divide numerator and denominator by 10.

$$\begin{aligned} \frac{20}{50} &= \frac{20 \div 10}{20 \div 10} && \text{Divide numerator and denominator by the greatest common divisor.} \\ &= \frac{2}{5} && \text{Simplify.} \end{aligned}$$

3. The greatest common divisor of the numerator and denominator is $\gcd(10, 48) = 2$. Divide numerator and denominator by 2.

$$\begin{aligned} \frac{10}{48} &= \frac{10 \div 2}{10 \div 2} && \text{Divide numerator and denominator by the greatest common divisor.} \\ &= \frac{5}{24} && \text{Simplify.} \end{aligned}$$

5. The greatest common divisor of the numerator and denominator is $\gcd(24, 45) = 3$. Divide numerator and denominator by 3.

$$\begin{aligned} \frac{24}{45} &= \frac{24 \div 3}{24 \div 3} && \text{Divide numerator and denominator by the greatest common divisor.} \\ &= \frac{8}{15} && \text{Simplify.} \end{aligned}$$

7. Prime factor both numerator and denominator, then cancel common factors.

$$\begin{aligned} \frac{153}{170} &= \frac{3 \cdot 3 \cdot 17}{2 \cdot 5 \cdot 17} && \text{Prime factorization.} \\ &= \frac{3 \cdot 3 \cdot \cancel{17}}{2 \cdot 5 \cdot \cancel{17}} && \text{Cancel common factors.} \\ &= \frac{3 \cdot 3}{2 \cdot 5} \\ &= \frac{9}{10} && \text{Simplify numerator and denominator.} \end{aligned}$$

9. Prime factor both numerator and denominator, then cancel common factors.

$$\begin{aligned} \frac{188}{141} &= \frac{2 \cdot 2 \cdot 47}{3 \cdot 47} && \text{Prime factorization.} \\ &= \frac{2 \cdot 2 \cdot \cancel{47}}{3 \cdot \cancel{47}} && \text{Cancel common factors.} \\ &= \frac{2 \cdot 2}{3} \\ &= \frac{4}{3} && \text{Simplify numerator and denominator.} \end{aligned}$$

11. Prime factor both numerator and denominator, then cancel common factors.

$$\begin{aligned}
\frac{159}{106} &= \frac{3 \cdot 53}{2 \cdot 53} && \text{Prime factorization.} \\
&= \frac{3 \cdot \cancel{53}}{2 \cdot \cancel{53}} && \text{Cancel common factors.} \\
&= \frac{3}{2}
\end{aligned}$$

13. First, multiply numerators and denominators. Then prime factor the resulting numerator and denominator and cancel like factors. Simplify your final answer.

$$\begin{aligned}
\frac{20}{8} \cdot \left(-\frac{18}{13}\right) &= -\frac{360}{104} && \text{Unlike signs yields negative answer.} \\
&= -\frac{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5}{2 \cdot 2 \cdot 2 \cdot 13} && \text{Prime factor numerator and denominator.} \\
&= -\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot 3 \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 13} && \text{Cancel common factors.} \\
&= -\frac{3 \cdot 3 \cdot 5}{13} \\
&= -\frac{45}{13} && \text{Simplify numerator and denominator.}
\end{aligned}$$

15. First, multiply numerators and denominators. Then prime factor the resulting numerator and denominator and cancel like factors. Simplify your final answer.

$$\begin{aligned}
-\frac{19}{4} \cdot \left(-\frac{18}{13}\right) &= \frac{342}{52} && \text{Like signs yields positive answer.} \\
&= \frac{2 \cdot 3 \cdot 3 \cdot 19}{2 \cdot 2 \cdot 13} && \text{Prime factor numerator and denominator.} \\
&= \frac{\cancel{2} \cdot 3 \cdot 3 \cdot 19}{\cancel{2} \cdot 2 \cdot 13} && \text{Cancel common factors.} \\
&= \frac{3 \cdot 3 \cdot 19}{2 \cdot 13} \\
&= \frac{171}{26} && \text{Simplify numerator and denominator.}
\end{aligned}$$

17. First, multiply numerators and denominators. Then prime factor the resulting numerator and denominator and cancel like factors. Simplify your

final answer.

$$\begin{aligned}
-\frac{16}{8}\cdot\frac{19}{6} &= -\frac{304}{48} && \text{Unlike signs yields negative answer.}\\
&= -\frac{2\cdot 2\cdot 2\cdot 2\cdot 19}{2\cdot 2\cdot 2\cdot 2\cdot 3} && \text{Prime factor numerator and denominator.}\\
&= -\frac{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot 19}{\cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot \cancel{2}\cdot 3} && \text{Cancel common factors.}\\
&= -\frac{19}{3}
\end{aligned}$$

19. Factor the individual numerators and denominators first, cancel common factors, then multiply numerators and denominators. Like signs gives a positive answer.

$$\begin{aligned}
-\frac{5}{6}\cdot\left(-\frac{12}{49}\right) &= -\frac{5}{2\cdot 3}\cdot\left(-\frac{2\cdot 2\cdot 3}{7\cdot 7}\right)\\
&= -\frac{5}{\cancel{2}\cdot \cancel{3}}\cdot\left(-\frac{\cancel{2}\cdot 2\cdot \cancel{3}}{7\cdot 7}\right)\\
&= \frac{5\cdot 2}{7\cdot 7}\\
&= \frac{10}{49}
\end{aligned}$$

21. Factor the individual numerators and denominators first, cancel common factors, then multiply numerators and denominators. Unlike signs gives a negative answer.

$$\begin{aligned}
-\frac{21}{10}\cdot\frac{12}{55} &= -\frac{3\cdot 7}{2\cdot 5}\cdot\frac{2\cdot 2\cdot 3}{5\cdot 11}\\
&= -\frac{3\cdot 7}{\cancel{2}\cdot 5}\cdot\frac{\cancel{2}\cdot 2\cdot 3}{5\cdot 11}\\
&= -\frac{3\cdot 7\cdot 2\cdot 3}{5\cdot 5\cdot 11}\\
&= -\frac{126}{275}
\end{aligned}$$

23. Factor the individual numerators and denominators first, cancel common factors, then multiply numerators and denominators. Unlike signs gives a neg-

ative answer.

$$\begin{aligned}
\frac{55}{29} \cdot \left(-\frac{54}{11}\right) &= \frac{5 \cdot 11}{29} \cdot \left(-\frac{2 \cdot 3 \cdot 3 \cdot 3}{11}\right) \\
&= \frac{5 \cdot \cancel{11}}{29} \cdot \left(-\frac{2 \cdot 3 \cdot 3 \cdot 3}{\cancel{11}}\right) \\
&= -\frac{5 \cdot 2 \cdot 3 \cdot 3 \cdot 3}{29} \\
&= -\frac{270}{29}
\end{aligned}$$

25. First, invert and multiply.

$$\frac{50}{39} \div \left(-\frac{5}{58}\right) = \frac{50}{39} \cdot \left(-\frac{58}{5}\right)$$

Factor the individual numerators and denominators first, cancel common factors, then multiply numerators and denominators. Unlike signs gives a negative answer.

$$\begin{aligned}
&= \frac{2 \cdot 5 \cdot 5}{3 \cdot 13} \cdot \left(-\frac{2 \cdot 29}{5}\right) \\
&= \frac{2 \cdot \cancel{5} \cdot 5}{3 \cdot 13} \cdot \left(-\frac{2 \cdot 29}{\cancel{5}}\right) \\
&= -\frac{2 \cdot 5 \cdot 2 \cdot 29}{3 \cdot 13} \\
&= -\frac{580}{39}
\end{aligned}$$

27. First, invert and multiply.

$$-\frac{60}{17} \div \frac{34}{31} = -\frac{60}{17} \cdot \frac{31}{34}$$

Next, factor the individual numerators and denominators first, cancel common factors, then multiply numerators and denominators. Unlike signs gives a negative answer.

$$\begin{aligned}
&= -\frac{2 \cdot 2 \cdot 3 \cdot 5}{17} \cdot \frac{31}{2 \cdot 17} \\
&= -\frac{\cancel{2} \cdot 2 \cdot 3 \cdot 5}{17} \cdot \frac{31}{\cancel{2} \cdot 17} \\
&= -\frac{2 \cdot 3 \cdot 5 \cdot 31}{17 \cdot 17} \\
&= -\frac{930}{289}
\end{aligned}$$

29. First, invert and multiply.

$$-\frac{7}{10} \div \left(-\frac{13}{28}\right) = -\frac{7}{10} \cdot \left(-\frac{28}{13}\right)$$

Factor the individual numerators and denominators first, cancel common factors, then multiply numerators and denominators. Like signs gives a positive answer.

$$\begin{aligned}
&= -\frac{7}{2 \cdot 5} \cdot \left(-\frac{2 \cdot 2 \cdot 7}{13}\right) \\
&= -\frac{7}{\cancel{2} \cdot 5} \cdot \left(-\frac{\cancel{2} \cdot 2 \cdot 7}{13}\right) \\
&= \frac{7 \cdot 2 \cdot 7}{5 \cdot 13} \\
&= \frac{98}{65}
\end{aligned}$$

31. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the common denominator and simplify.

$$\begin{aligned}
-\frac{5}{6} + \frac{1}{4} &= -\frac{5}{6} \cdot \frac{2}{2} + \frac{1}{4} \cdot \frac{3}{3} && \text{Make equivalent fractions with LCD} = 12. \\
&= -\frac{10}{12} + \frac{3}{12} && \text{Simplify numerators and denominators.} \\
&= \frac{-10 + 3}{12} && \text{Add numerators over common denominator.} \\
&= \frac{-7}{12} && \text{Simplify numerator.}
\end{aligned}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{7}{12}$$

33. Since the denominators are different, start by writing equivalent fractions using the least common denominator. Then add the numerators over the com-

mon denominator and simplify.

$$\begin{aligned}
-\frac{8}{9}+\left(-\frac{1}{3}\right) &= -\frac{8}{9}\cdot\frac{1}{1}+\left(-\frac{1}{3}\cdot\frac{3}{3}\right) && \text{Make equivalent fractions with LCD} = 9. \\
&= -\frac{8}{9}+\left(-\frac{3}{9}\right) && \text{Simplify numerators and denominators.} \\
&= \frac{-8+(-3)}{9} && \text{Add numerators over common denominator.} \\
&= \frac{-11}{9} && \text{Simplify numerator.}
\end{aligned}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{11}{9}$$

35. First simplify by rewriting as an addition problem. Then, since the denominators are different, write equivalent fractions using the least common denominator. Finally, add the numerators over the common denominator and simplify.

$$\begin{aligned}
-\frac{1}{4}-\left(-\frac{2}{9}\right) &= -\frac{1}{4}+\frac{2}{9} && \text{Rewrite as an addition problem.} \\
&= -\frac{1}{4}\cdot\frac{9}{9}+\frac{2}{9}\cdot\frac{4}{4} && \text{Equivalent fractions with LCD} = 36. \\
&= -\frac{9}{36}+\frac{8}{36} && \text{Simplify numerators and denominators.} \\
&= \frac{-9+8}{36} && \text{Add numerators over common denominator.} \\
&= \frac{-1}{36} && \text{Simplify numerator.}
\end{aligned}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$= -\frac{1}{36}$$

37. Start by changing subtraction into addition.

$$-\frac{8}{9}-\frac{4}{5}=-\frac{8}{9}+\left(-\frac{4}{5}\right) \qquad \text{Add the opposite.}$$

Make equivalent fractions over a common denominator. Then add the numerators over the common denominator and simplify.

$$\begin{aligned}
&=-\frac{8}{9}\cdot\frac{5}{5}+\left(-\frac{4}{5}\cdot\frac{9}{9}\right) && \text{Make equivalent fractions with LCD} = 45.\\
&=-\frac{40}{45}+\left(-\frac{36}{45}\right) && \text{Simplify numerators and denominators.}\\
&=\frac{-40+(-36)}{45} && \text{Add numerators over common denominator.}\\
&=\frac{-76}{45} && \text{Simplify numerator.}
\end{aligned}$$

Although this answer is perfectly acceptable, negative divided by positive gives a negative answer, so we could also write

$$=-\frac{76}{45}$$

39. Evaluate the expression inside the absolute value bars first.

$$\begin{aligned}
\frac{8}{9}-\left|\frac{5}{2}-\frac{2}{5}\right| &=\frac{8}{9}-\left|\frac{5}{2}+\left(-\frac{2}{5}\right)\right| && \text{Add the opposite.}\\
&=\frac{8}{9}-\left|\frac{5}{2}\cdot\frac{5}{5}+\left(-\frac{2}{5}\cdot\frac{2}{2}\right)\right| && \text{Make equivalent fractions}\\
&=\frac{8}{9}-\left|\frac{25}{10}+\left(-\frac{4}{10}\right)\right| && \text{with LCD} = 10.\\
&=\frac{8}{9}-\left|\frac{21}{10}\right| && \text{Add numerators over Common denominator.}
\end{aligned}$$

Next, take the absolute value, then make equivalent fractions with a common denominator and simplify.

$$\begin{aligned}
&= \frac{8}{9} - \frac{21}{10} && \text{Take absolute value.}\\
&= \frac{8}{9} + \left(-\frac{21}{10}\right) && \text{Add the opposite.}\\
&= \frac{8}{9}\cdot\frac{10}{10} + \left(-\frac{21}{10}\cdot\frac{9}{9}\right) && \text{Make equivalent fractions}\\
&= \frac{80}{90} + \left(-\frac{189}{90}\right) && \text{with LCD} = 90.\\
&= \frac{-109}{90} && \text{Add.}
\end{aligned}$$

41. First, evaluate the exponent, then multiply.

$$\begin{aligned}
&\left(-\frac{7}{6}\right)^2 + \left(-\frac{1}{2}\right)\left(-\frac{5}{3}\right)\\
&\quad = \frac{49}{36} + \left(-\frac{1}{2}\right)\left(-\frac{5}{3}\right) && \text{Evaluate exponent.}\\
&\quad = \frac{49}{36} + \frac{5}{6} && \text{Multiply.}
\end{aligned}$$

Make equivalent fractions with a common denominator, add numerators over a common denominator and simplify.

$$\begin{aligned}
&= \frac{49}{36}\cdot\frac{1}{1} + \frac{5}{6}\cdot\frac{6}{6} && \text{Make equivalent fractions}\\
&= \frac{49}{36} + \frac{30}{36} && \text{with LCD} = 36.\\
&= \frac{79}{36} && \text{Add over common denominator.}
\end{aligned}$$

43. Multiply first, then add or subtract as needed.

$$\begin{aligned}
&\left(-\frac{9}{5}\right)\left(-\frac{9}{7}\right) + \left(\frac{8}{5}\right)\left(-\frac{1}{2}\right)\\
&\quad = \frac{81}{35} + \left(-\frac{4}{5}\right) && \text{Multiply.}\\
&\quad = \frac{81}{35}\cdot\frac{1}{1} + \left(-\frac{4}{5}\cdot\frac{7}{7}\right) && \text{Make equivalent fractions}\\
&\quad = \frac{81}{35} + \left(-\frac{28}{35}\right) && \text{with LCD} = 35.\\
&\quad = \frac{53}{35} && \text{Add over common denominator.}
\end{aligned}$$

45. Multiply first, then add or subtract as needed.

$$\begin{aligned} -\frac{5}{8}+\frac{7}{2}\left(-\frac{9}{2}\right) &= -\frac{5}{8}+\left(-\frac{63}{4}\right) && \text{Multiply.} \\ &= -\frac{5}{8}\cdot\frac{1}{1}+\left(-\frac{63}{4}\cdot\frac{2}{2}\right) && \text{Make equivalent fractions} \\ &= -\frac{5}{8}+\left(-\frac{126}{8}\right) && \text{with LCD} = 8. \\ &= -\frac{131}{8} && \text{Add over common denominator.} \end{aligned}$$

47. First evaluate exponent, then multiply.

$$\begin{aligned} &\left(-\frac{7}{5}\right)\left(\frac{9}{2}\right)-\left(-\frac{2}{5}\right)^2 \\ &\quad= \left(-\frac{7}{5}\right)\left(\frac{9}{2}\right)-\frac{4}{25} && \text{Exponent first.} \\ &\quad= -\frac{63}{10}-\frac{4}{25} && \text{Multiply.} \\ &\quad= -\frac{63}{10}+\left(-\frac{4}{25}\right) && \text{Add the opposite.} \end{aligned}$$

Make equivalent fractions with a common denominator, then add numerators over a common denominator and simplify.

$$\begin{aligned} &= -\frac{63}{10}\cdot\frac{5}{5}+\left(-\frac{4}{25}\cdot\frac{2}{2}\right) && \text{Make equivalent fractions} \\ &= -\frac{315}{50}+\left(-\frac{8}{50}\right) && \text{with LCD} = 50. \\ &= -\frac{323}{50} && \text{Add over common denominator.} \end{aligned}$$

49. Multiply first, then add or subtract as needed.

$$\begin{aligned} \frac{6}{5}-\frac{2}{5}\left(-\frac{4}{9}\right) &= \frac{6}{5}-\left(-\frac{8}{45}\right) && \text{Multiply.} \\ &= \frac{6}{5}+\frac{8}{45} && \text{Add the opposite.} \\ &= \frac{6}{5}\cdot\frac{9}{9}+\frac{8}{45}\cdot\frac{1}{1} && \text{Make equivalent fractions} \\ &= \frac{54}{45}+\frac{8}{45} && \text{with LCD} = 45. \\ &= \frac{62}{45} && \text{Add over common denominator.} \end{aligned}$$

51. Multiply first.

$$\begin{aligned}
&\left(\frac{2}{3}\right)\left(-\frac{8}{7}\right)-\left(\frac{4}{7}\right)\left(-\frac{9}{8}\right)\\
&\quad=-\frac{16}{21}-\left(-\frac{9}{14}\right) && \text{Multiply.}\\
&\quad=-\frac{16}{21}+\frac{9}{14} && \text{Add the opposite.}
\end{aligned}$$

Next, make equivalent fractions with a common denominator, add numerators over a common denominator and simplify.

$$\begin{aligned}
&=-\frac{16}{21}\cdot\frac{2}{2}+\frac{9}{14}\cdot\frac{3}{3} && \text{Make equivalent fractions}\\
&=-\frac{32}{42}+\frac{27}{42} && \text{with LCD} = 42.\\
&=-\frac{5}{42} && \text{Add over common denominator.}
\end{aligned}$$

53. Replace each variable with open parentheses, then substitute $-1/2$ for x, $-1/3$ for y, and $5/2$ for z.

$$\begin{aligned}
xy-z^2&=(\quad)(\quad)-(\quad)^2 && \text{Replace variable with parentheses.}\\
&=\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)-\left(\frac{5}{2}\right)^2 && \text{Substitute } -1/2 \text{ for } x,\ -1/3\\
& && \text{for } y, \text{ and } 5/2 \text{ for } z.
\end{aligned}$$

First evaluate exponent, then multiply.

$$\begin{aligned}
&=\left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)-\frac{25}{4} && \text{Exponent first.}\\
&=\frac{1}{6}-\frac{25}{4} && \text{Multiply.}\\
&=\frac{1}{6}+\left(-\frac{25}{4}\right) && \text{Add the opposite.}
\end{aligned}$$

Make equivalent fractions with a common denominator, then add numerators over a common denominator and simplify.

$$\begin{aligned}
&=\frac{1}{6}\cdot\frac{2}{2}+\left(-\frac{25}{4}\cdot\frac{3}{3}\right) && \text{Make equivalent fractions}\\
&=\frac{2}{12}+\left(-\frac{75}{12}\right) && \text{with LCD} = 12.\\
&=-\frac{73}{12} && \text{Add over common denominator.}
\end{aligned}$$

55. Replace each variable with open parentheses, then substitute $3/4$ for x and $-1/2$ for y.

$$\begin{aligned}-5x^2+2y^2 &= -5\left(\quad\right)^2+2\left(\quad\right)^2\\ &= -5\left(\frac{3}{4}\right)^2+2\left(-\frac{1}{2}\right)^2\end{aligned}$$

Exponents first: $(3/4)^2 = 9/16$ and $(-1/2)^2 = 1/4$.

$$= -5\left(\frac{9}{16}\right)+2\left(\frac{1}{4}\right)$$

Multiply: $-5(9/16) = -45/16$ and $2(1/4) = 1/2$

$$= -\frac{45}{16}+\frac{1}{2}$$

Make equivalent fractions with LCD $= 16$, then add the numerators over the common denominator and simplify.

$$\begin{aligned}&= -\frac{45}{16}\cdot\frac{1}{1}+\frac{1}{2}\cdot\frac{8}{8}\\ &= -\frac{45}{16}+\frac{8}{16}\\ &= \frac{-37}{16}\end{aligned}$$

57. Replace each variable with open parentheses, then substitute $3/2$ for x and $-3/4$ for y.

$$\begin{aligned}2x^2-2xy-3y^2 &= 2\left(\quad\right)^2-2\left(\quad\right)\left(\quad\right)-3\left(\quad\right)^2\\ &= 2\left(\frac{3}{2}\right)^2-2\left(\frac{3}{2}\right)\left(-\frac{3}{4}\right)-3\left(-\frac{3}{4}\right)^2\end{aligned}$$

Exponents first: $(3/2)^2 = 9/4$ and $(-3/4)^2 = 9/16$.

$$= 2\left(\frac{9}{4}\right)-2\left(\frac{3}{2}\right)\left(-\frac{3}{4}\right)-3\left(\frac{9}{16}\right)$$

Multiply: $2(9/4) = 9/2$, $2(3/2)(-3/4) = -9/4$, and $3(9/16) = 27/16$.

$$= \frac{9}{2}-\left(-\frac{9}{4}\right)-\frac{27}{16}$$

Add the opposite.

$$= \frac{9}{2} + \frac{9}{4} + \left(-\frac{27}{16}\right)$$

Make equivalent fractions with LCD = 16, then add the numerators over the common denominator and simplify.

$$\begin{aligned} &= \frac{9}{2} \cdot \frac{8}{8} + \frac{9}{4} \cdot \frac{4}{4} + \left(-\frac{27}{16} \cdot \frac{1}{1}\right) \\ &= \frac{72}{16} + \frac{36}{16} + \left(-\frac{27}{16}\right) \\ &= \frac{81}{16} \end{aligned}$$

59. Replace all variables with open parentheses, then substitute $-1/3$ for x, $1/6$ for y, and $2/5$ for z.

$$\begin{aligned} x + yz &= (\quad) + (\quad)(\quad) && \text{Replace variables with parentheses.} \\ &= \left(-\frac{1}{3}\right) - \left(\frac{1}{6}\right)\left(\frac{2}{5}\right) && \text{Substitute } -1/3 \text{ for } x,\ 1/6 \text{ for } y, \text{ and } 2/5 \text{ for } z. \end{aligned}$$

Multiply first, then subtract.

$$\begin{aligned} &= -\frac{1}{3} - \frac{1}{15} && \text{Multiply.} \\ &= -\frac{1}{3} + \left(-\frac{1}{15}\right) && \text{Add the opposite.} \end{aligned}$$

Make equivalent fractions with a common denominator, then add numerators over the common denominator and simplify.

$$\begin{aligned} &= -\frac{1}{3} \cdot \frac{5}{5} + \left(-\frac{1}{15} \cdot \frac{1}{1}\right) && \text{Make equivalent fractions} \\ &= -\frac{5}{15} + \left(-\frac{1}{15}\right) && \text{with LCD} = 15. \\ &= -\frac{6}{15} && \text{Add over common denominator.} \\ &= -\frac{2}{5} && \text{Simplify.} \end{aligned}$$

61. Replace each variable with open parentheses, then substitute 4/7 for a, 7/5 for b, and 5/2 for c.

$$\begin{aligned} ab + bc &= (\quad)(\quad) + (\quad)(\quad) \\ &= \left(-\frac{4}{7}\right)\left(\frac{7}{5}\right) + \left(\frac{7}{5}\right)\left(-\frac{5}{2}\right) \end{aligned}$$

Multiply and reduce.

$$\begin{aligned} &= -\frac{28}{35} + \left(-\frac{35}{10}\right) \\ &= -\frac{4}{5} + \left(-\frac{7}{2}\right) \end{aligned}$$

Make equivalent fractions with $\text{LCD}(5, 2) = 10$, then add numerators over a common denominator and simplify.

$$\begin{aligned} &= -\frac{4}{5} \cdot \frac{2}{2} + \left(-\frac{7}{2} \cdot \frac{5}{5}\right) \\ &= -\frac{8}{10} + \left(-\frac{35}{10}\right) \\ &= \frac{-8 + (-35)}{10} \\ &= -\frac{43}{10} \end{aligned}$$

63. Replace each occurrence of the variable x with open parentheses, then substitute $-1/2$ for x.

$$x^3 = (\quad)^3$$ Replace variable with open parentheses.

$$= \left(-\frac{1}{2}\right)^3$$ Substitute $-1/2$ for x.

In the expression $(-1/2)^3$, the exponent 3 tells us to write the base $-1/2$ three times as a factor.

$$= \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$ Write $-1/2$ as a factor three times.

$$= -\frac{1}{8}$$ The product of three negative fractions is negative.

65. Replace all variables with open parentheses, then substitute $-8/5$ for x, $1/3$ for y, and $-8/5$ for z.

$$\begin{aligned} x - yz &= (\quad) - (\quad)(\quad) && \text{Replace variables with parentheses.} \\ &= \left(-\frac{8}{5}\right) - \left(\frac{1}{3}\right)\left(-\frac{8}{5}\right) && \text{Substitute } -8/5 \text{ for } x,\ 1/3 \text{ for } y, \text{ and } -8/5 \text{ for } z. \end{aligned}$$

Multiply first, then add or subtract as needed.

$$\begin{aligned} &= -\frac{8}{5} - \left(-\frac{8}{15}\right) && \text{Multiply.} \\ &= -\frac{8}{5} + \frac{8}{15} && \text{Add the opposite.} \end{aligned}$$

Make equivalent fractions with a common denominator, then add numerators over the common denominator and simplify.

$$\begin{aligned} &= -\frac{8}{5} \cdot \frac{3}{3} + \frac{8}{15} \cdot \frac{1}{1} && \text{Make equivalent fractions} \\ &= -\frac{24}{15} + \frac{8}{15} && \text{with LCD} = 15. \\ &= -\frac{16}{15} && \text{Add over common denominator.} \end{aligned}$$

67. Replace each occurrence of the variable x with open parentheses, then substitute $-8/3$ for x. Note that we must deal with the exponent first, then negate our answer in the final step.

$$\begin{aligned} -x^2 &= -(\quad)^2 && \text{Replace variable with open parentheses.} \\ &= -\left(-\frac{8}{3}\right)^2 && \text{Substitute } -8/3 \text{ for } x. \end{aligned}$$

The exponent 2 tells us to write the base $-8/3$ two times as a factor.

$$\begin{aligned} &= -\left(-\frac{8}{3}\right)\left(-\frac{8}{3}\right) && \text{Write } -8/3 \text{ as a factor two times.} \\ &= -\left(\frac{64}{9}\right) && \text{The product of two negative fractions is positive.} \\ &= -\frac{64}{9} && \text{Negate.} \end{aligned}$$

69. First, replace each variable with open parentheses, then substitute $7/2$ for x, $-5/4$ for y, and $-5/3$ for z.

$$\begin{aligned}
x^2 + yz &= (\quad)^2 + (\quad)(\quad) && \text{Replace variables with parentheses.} \\
&= \left(\frac{7}{2}\right)^2 + \left(-\frac{5}{4}\right)\left(-\frac{5}{3}\right) && \text{Substitute: } 7/2 \text{ for } x,\ -5/4 \text{ for } y, \text{ and } -5/3 \text{ for } z
\end{aligned}$$

Next, evaluate the exponent, then multiply.

$$\begin{aligned}
&= \frac{49}{4} + \left(-\frac{5}{4}\right)\left(-\frac{5}{3}\right) && \text{Evaluate exponent.} \\
&= \frac{49}{4} + \frac{25}{12} && \text{Multiply.}
\end{aligned}$$

Make equivalent fractions with a common denominator, add numerators over a common denominator and simplify.

$$\begin{aligned}
&= \frac{49}{4} \cdot \frac{3}{3} + \frac{25}{12} \cdot \frac{1}{1} && \text{Make equivalent fractions} \\
&= \frac{147}{12} + \frac{25}{12} && \text{with LCD} = 12. \\
&= \frac{172}{12} && \text{Add over common denominator.} \\
&= \frac{43}{3} && \text{Simplify.}
\end{aligned}$$

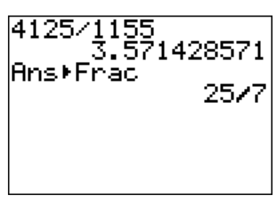
71. The *Rules Guiding Order of Operations* requires that we first perform the division. Hence:

$$a + b/c + d = a + \frac{b}{c} + d$$

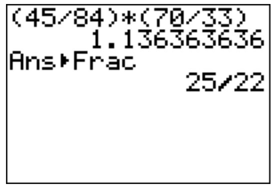
73. The *Rules Guiding Order of Operations* requires that we attend the parentheses first, then the division, then the addition. Hence:

$$a + b/(c + d) = a + \frac{b}{c + d}$$

75. Enter the expression 4125/1155, then press the ENTER key. Select **1:▶Frac** from the MATH menu, then press ENTER again. The result is shown in the following figure.

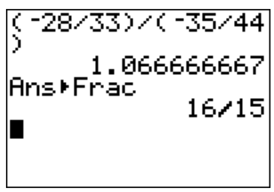


77. Enter the expression $(45/84) * (70/33)$, then press the ENTER key. Select **1:▶Frac** from the MATH menu, then press ENTER again. The result is shown in the following figure.



Note: The parentheses are not required here, as was explained in part (b) of Example **??***, but they do help emphasize the proper order of operations in this case.*

79. Enter the expression $(-28/33)/(-35/44)$, then press the ENTER key. Select **1:▶Frac** from the MATH menu, then press ENTER again. The result is shown in the following figure.



Note: The parentheses are required here, as was explained in part (c) of Example **??**.

1.4 Decimal Notation

1. The two decimals are both negative. First add the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 2.835 \\ \underline{+\,8.759} \\ 11.594 \end{array}$$

Finish by prefixing the common negative sign. Hence,

$$-2.835 + (-8.759) = -11.594$$

3. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$19.5 - (-1.6) = 19.5 + 1.6$$

Then compute the sum. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 19.5 \\ \underline{+\,1.6} \\ 21.1 \end{array}$$

Thus,

$$\begin{aligned} 19.5 - (-1.6) &= 19.5 + 1.6 \\ &= 21.1 \end{aligned}$$

5. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-2 - 0.49 = -2 + (-0.49)$$

In this addition problem, the decimals have like signs. Therefore, start by adding the magnitudes. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 2.00 \\ \underline{+\,0.49} \\ 2.49 \end{array}$$

Finish by prefixing the common negative sign. Thus,

$$\begin{aligned} -2 - 0.49 &= -2 + (-0.49) \\ &= -2.49 \end{aligned}$$

7. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$\begin{array}{r} 1.2 \\ \underline{\times\;\;0.05} \\ 0.060 \end{array}$$

Like signs give a positive result. Therefore,

$$(-1.2)(-0.05) = 0.06$$

9. The decimals have unlike signs. First subtract the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 23.49 \\ -\ 0.13 \\ \hline 23.36 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$-0.13 + 23.49 = 23.36$$

11. The decimals have unlike signs. First subtract the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 41.205 \\ -\ 16.400 \\ \hline 24.805 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Hence,

$$16.4 + (-41.205) = -24.805$$

13. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$0.49\,\overline{)0.45\,08}$$

Then, by long division,

$$\begin{array}{r} 0.92 \\ 49\overline{)45.08} \\ \underline{44\ 1} \\ 98 \\ \underline{98} \\ 0 \end{array}$$

Unlike signs give a negative quotient, so $-0.4508 \div 0.49 = -0.92$.

15. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$\begin{array}{r} 1.42 \\ \times\ \ 3.6 \\ \hline 852 \\ 4\,26\ \ \\ \hline 5.112 \end{array}$$

Like signs give a positive result. Therefore,

$$(-1.42)(-3.6) = 5.112$$

17. First divide the magnitudes. Move the decimal point in the divisor and dividend two places to the right:

$$0.24\,\overline{)2.18\,4}$$

Then, by long division,

$$\begin{array}{r} 9.1 \\ 24\overline{)218.4} \\ 216\ \ \\ \hline 2\,4 \\ 2\,4 \\ \hline 0 \end{array}$$

Unlike signs give a negative quotient, so $2.184 \div (-0.24) = -9.1$.

19. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$\begin{array}{r} 7.1 \\ \times\ \ 4.9 \\ \hline 6\,39 \\ 28\,4\ \ \\ \hline 34.79 \end{array}$$

Like signs give a positive result. Therefore,

$$(-7.1)(-4.9) = 34.79$$

21. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$9.5\,\overline{)7.4\,10}$$

Then, by long division,

$$\begin{array}{r} 0.78 \\ 95\overline{)74.10} \\ \underline{66\,5} \\ 7\,60 \\ \underline{7\,60} \\ 0 \end{array}$$

Unlike signs give a negative quotient, so $7.41 \div (-9.5) = -0.78$.

23. First divide the magnitudes. Move the decimal point in the divisor and dividend one place to the right:

$$2.8\,\overline{)24.0\,8}$$

Then, by long division,

$$\begin{array}{r} 8.6 \\ 28\overline{)240.8} \\ \underline{224} \\ 16\,8 \\ \underline{16\,8} \\ 0 \end{array}$$

Unlike signs give a negative quotient, so $-24.08 \div 2.8 = -8.6$.

25. Use vertical format with the unsigned numbers. Since there are a total of 3 digits to the right of the decimal point in the original numbers, the answer also has 3 digits to the right of the decimal point.

$$\begin{array}{r} 4.04 \\ \times\ \ 0.6 \\ \hline 2.424 \end{array}$$

Like signs give a positive result. Therefore,

$$(-4.04)(-0.6) = 2.424$$

27. First rewrite the problem as an addition problem by adding the opposite of the second number:

$$-7.2 - (-7) = -7.2 + 7$$

In this addition problem, the decimals have unlike signs. Therefore, start by subtracting the smaller magnitude from the larger magnitude. Include trailing zeros if necessary to align the decimal points.

$$\begin{array}{r} 7.200 \\ -\ 7.000 \\ \hline 0.200 \end{array}$$

Finish by prefixing the sign of the decimal with the larger magnitude. Thus,

$$\begin{aligned} -7.2 - (-7) &= -7.2 + 7 \\ &= -0.2 \end{aligned}$$

29. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$\begin{array}{r} 46.9 \\ \times\ \ 0.1 \\ \hline 4.69 \end{array}$$

Unlike signs give a negative result. Therefore,

$$(46.9)(-0.1) = -4.69$$

31. Use vertical format with the unsigned numbers. Since there are a total of 2 digits to the right of the decimal point in the original numbers, the answer also has 2 digits to the right of the decimal point.

$$\begin{array}{r} 86.6 \\ \times\ \ 1.9 \\ \hline 77\ 94 \\ 86\ 6 \\ \hline 164.54 \end{array}$$

Unlike signs give a negative result. Therefore,

$$(86.6)(-1.9) = -164.54$$

33.

35. Simplify the expression inside the absolute value bars first.

$$
\begin{aligned}
&-3.49+|-6.9-(-15.7)| \\
&\quad=-3.49+|-6.9+15.7| && \text{Subtract: Add the opposite.} \\
&\quad=-3.49+|8.8| && \text{Add: } -6.9+15.7=8.8. \\
&\quad=-3.49+8.8 && \text{Take absolute value: } |8.8|=8.8. \\
&\quad=5.31 && \text{Add: } -3.49+8.8=5.31.
\end{aligned}
$$

37. Simplify the expression inside the absolute value bars. First, add the opposite.

$$
\begin{aligned}
&|18.9-1.55|-|-16.1-(-17.04)| \\
&\quad=|18.9+(-1.55)|-|-16.1+17.04| && \text{Add the opposite.} \\
&\quad=|17.35|-|0.939999999999998| && \text{Add.} \\
&\quad=17.35-0.939999999999998 && \text{Take absolute value.} \\
&\quad=16.41 && \text{Subtract.}
\end{aligned}
$$

39. Cube first, then subtract.

$$
\begin{aligned}
&8.2-(-3.1)^3 \\
&\quad=8.2-(-29.791) && \text{Cube: } (-3.1)^3=-29.791 \\
&\quad=8.2+29.791 && \text{Add the opposite.} \\
&\quad=37.991 && \text{Add: } 8.2+29.791=37.991.
\end{aligned}
$$

41. First evaluate exponents, then multiply, and then subtract.

$$
\begin{aligned}
&5.7-(-8.6)(1.1)^2 \\
&\quad=5.7-(-8.6)(1.21) && \text{Exponents first: } 1.1^2=1.21. \\
&\quad=5.7-(-10.406) && \text{Multiply: } (-8.6)(1.21)=-10.406. \\
&\quad=5.7+10.406 && \text{Subtract: Add the opposite.} \\
&\quad=16.106 && \text{Add: } 5.7+10.406=16.106.
\end{aligned}
$$

43. First evaluate exponents, then multiply, and then subtract.

$$\begin{aligned}
&(5.67)(6.8) - (1.8)^2 \\
&\quad = (5.67)(6.8) - 3.24 && \text{Exponents first: } (1.8)^2 = 3.24. \\
&\quad = 38.556 - 3.24 && \text{Multiply: } (5.67)(6.8) = 38.556. \\
&\quad = 38.556 + (-3.24) && \text{Subtract: Add the opposite.} \\
&\quad = 35.316 && \text{Add: } 38.556 + (-3.24) = 35.316.
\end{aligned}$$

45. Simplify the expression inside the parentheses first.

$$\begin{aligned}
&9.6 + (-10.05 - 13.16) \\
&\quad = 9.6 + (-10.05 + (-13.16)) && \text{Subtract: Add the opposite.} \\
&\quad = 9.6 + (-23.21) && \text{Add: } -10.05 + (-13.16) = -23.21. \\
&\quad = -13.61 && \text{Add: } 9.6 + (-23.21) = -13.61.
\end{aligned}$$

47. Multiply first, then add.

$$\begin{aligned}
&8.1 + 3.7(5.77) \\
&\quad = 8.1 + 21.349 && \text{Multiply: } 3.7(5.77) = 21.349 \\
&\quad = 29.449 && \text{Add: } 8.1 + 21.349 = 29.449.
\end{aligned}$$

49. The *Rules Guiding Order of Operations* tell us that we should first evaluate the expression inside the parentheses, then divide, then add.

$$\begin{aligned}
7.5 + 34.5/(-1.6 + 8.5) &= 7.5 + 34.5/6.9 && \text{Add: } -1.6 + 8.5 = 6.9 \\
&= 7.5 + 5 && \text{Divide: } 34.5/6.9 = 5 \\
&= 12.5 && \text{Add: } 7.5 + 5 = 12.5
\end{aligned}$$

51. The *Rules Guiding Order of Operations* tell us that we should first evaluate the expression inside the parentheses, then divide, then add.

$$\begin{aligned}
(8.0 + 2.2)/5.1 - 4.6 &= 10.2/5.1 - 4.6 && \text{Add: } 8.0 + 2.2 = 10.2 \\
&= 2 - 4.6 && \text{Divide: } 10.2/5.1 = 2 \\
&= -2.6 && \text{Subtract: } 2 - 4.6 = -2.6
\end{aligned}$$

53. Simplify the expression inside the absolute value bars first.

$$\begin{aligned} &-18.24 - |-18.5 - 19.7| \\ &\quad = -18.24 - |-18.5 + (-19.7)| && \text{Subtract: Add the opposite.} \\ &\quad = -18.24 - |-38.2| && \text{Add: } -18.5 + (-19.7) = -38.2. \\ &\quad = -18.24 - 38.2 && \text{Take absolute value: } |-38.2| = 38.2. \\ &\quad = -18.24 + (-38.2) && \text{Subtract: Add the opposite.} \\ &\quad = -56.44 && \text{Add: } -18.24 + (-38.2) = -56.44. \end{aligned}$$

55. First take the absolute value, then subtract.

$$\begin{aligned} &-4.37 - |-8.97| \\ &\quad = -4.37 - 8.97 && \text{Absolute value: } |-8.97| = 8.97 \\ &\quad = -4.37 + (-8.97) && \text{Add the opposite.} \\ &\quad = -13.34 && \text{Add: } -4.37 + (-8.97) = -13.34. \end{aligned}$$

57. Simplify the expression inside the parentheses first.

$$\begin{aligned} &7.06 - (-1.1 - 4.41) \\ &\quad = 7.06 - (-1.1 + (-4.41)) && \text{Subtract: Add the opposite.} \\ &\quad = 7.06 - (-5.51) && \text{Add: } -1.1 + (-4.41) = -5.51. \\ &\quad = 7.06 + 5.51 && \text{Subtract: Add the opposite.} \\ &\quad = 12.57 && \text{Add: } 7.06 + 5.51 = 12.57. \end{aligned}$$

59. Square first, then subtract.

$$\begin{aligned} &-2.2 - (-4.5)^2 \\ &\quad = -2.2 - 20.25 && \text{Square: } (-4.5)^2 = 20.25 \\ &\quad = -2.2 + (-20.25) && \text{Add the opposite.} \\ &\quad = -22.45 && \text{Add: } -2.2 + (-20.25) = -22.45. \end{aligned}$$

61. Replace each variable with open parentheses, then substitute -2.9 for a and -5.4 for b.

$$\begin{aligned} a - b^2 &= (\quad) - (\quad)^2 \\ &= (-2.9) - (-5.4)^2 \end{aligned}$$

Square first, then subtract.

$$\begin{aligned}
&-2.9-(-5.4)^2 \\
&\quad=-2.9-29.16 && \text{Square: } (-5.4)^2=29.16 \\
&\quad=-2.9+(-29.16) && \text{Add the opposite.} \\
&\quad=-32.06 && \text{Add: } -2.9+(-29.16)=-32.06.
\end{aligned}$$

63. Replace each variable with open parentheses, then substitute -19.55 for a, 5.62 for b, and -5.21 for c.

$$\begin{aligned}
a+|b-c| &= (\quad)+|\,(\quad)-(\quad)\,| \\
&= (-19.55)+|\,(5.62)-(-5.21)\,|
\end{aligned}$$

Simplify the expression inside the absolute value bars first.

$$\begin{aligned}
&-19.55+|5.62-(-5.21)| \\
&\quad=-19.55+|5.62+5.21| && \text{Subtract: Add the opposite.} \\
&\quad=-19.55+|10.83| && \text{Add: } 5.62+5.21=10.83. \\
&\quad=-19.55+10.83 && \text{Take absolute value: } |10.83|=10.83. \\
&\quad=-8.72 && \text{Add: } -19.55+10.83=-8.72.
\end{aligned}$$

65. Replace each variable with open parentheses, then substitute 4.3 for a, 8.5 for b, and 1.73 for c.

$$\begin{aligned}
a-bc &= (\quad)-(\quad)(\quad) \\
&= (4.3)-(8.5)\,(1.73)
\end{aligned}$$

Multiply first, then subtract.

$$\begin{aligned}
&4.3-8.5(1.73) \\
&\quad=4.3-14.705 && \text{Multiply: } 8.5(1.73)=14.705 \\
&\quad=4.3+(-14.705) && \text{Add the opposite.} \\
&\quad=-10.405 && \text{Add: } 4.3+(-14.705)=-10.405.
\end{aligned}$$

67. Replace each variable with open parentheses, then substitute -7.36 for a, -17.6 for b, and -19.07 for c.

$$\begin{aligned}
a-(b-c) &= (\quad)-((\quad)-(\quad)) \\
&= (-7.36)-((-17.6)-(-19.07))
\end{aligned}$$

Simplify the expression inside the parentheses first.

$$\begin{aligned}
&-7.36-(-17.6-(-19.07))\\
&\quad=-7.36-(-17.6+19.07) && \text{Subtract: Add the opposite.}\\
&\quad=-7.36-1.47 && \text{Add: } -17.6+19.07=1.47.\\
&\quad=-7.36+(-1.47) && \text{Subtract: Add the opposite.}\\
&\quad=-8.83 && \text{Add: } -7.36+(-1.47)=-8.83.
\end{aligned}$$

69. Replace each variable with open parentheses, then substitute 4.7 for a, 54.4 for b, 1.7 for c, and 5.1 for d..

$$\begin{aligned}
a+b/(c+d) &= (\quad)+(\quad)/((\quad)+(\quad))\\
&= (4.7)+(54.4)/((1.7)+(5.1))
\end{aligned}$$

The *Rules Guiding Order of Operations* tell us that we should first evaluate the expression inside the parentheses, then divide, then add.

$$\begin{aligned}
4.7+54.4/(1.7+5.1) &= 4.7+54.4/6.8 && \text{Add: } 1.7+5.1=6.8\\
&= 4.7+8 && \text{Divide: } 54.4/6.8=8\\
&= 12.7 && \text{Add: } 4.7+8=12.7
\end{aligned}$$

71. Replace each variable with open parentheses, then substitute -2.45 for a, 5.6 for b, and -3.2 for c.

$$\begin{aligned}
ab-c^2 &= (\quad)(\quad)-(\quad)^2\\
&= (-2.45)(5.6)-(-3.2)^2
\end{aligned}$$

First evaluate exponents, then multiply, and then subtract.

$$\begin{aligned}
&(-2.45)(5.6)-(-3.2)^2\\
&\quad=(-2.45)(5.6)-10.24 && \text{Exponents first: } (-3.2)^2=10.24.\\
&\quad=-13.72-10.24 && \text{Multiply: } (-2.45)(5.6)=-13.72.\\
&\quad=-13.72+(-10.24) && \text{Subtract: Add the opposite.}\\
&\quad=-23.96 && \text{Add: } -13.72+(-10.24)=-23.96.
\end{aligned}$$

73. Replace each variable with open parentheses, then substitute -4.9 for a and -2.67 for b.

$$\begin{aligned}
a-|b| &= (\quad)-|\,(\quad)\,|\\
&= (-4.9)-|\,(-2.67)\,|
\end{aligned}$$

First take the absolute value, then subtract.

$$\begin{aligned} &-4.9 - |-2.67| \\ &= -4.9 - 2.67 && \text{Absolute value: } |-2.67| = 2.67 \\ &= -4.9 + (-2.67) && \text{Add the opposite.} \\ &= -7.57 && \text{Add: } -4.9 + (-2.67) = -7.57. \end{aligned}$$

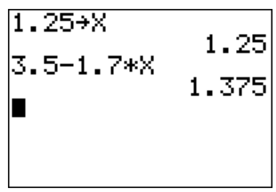
75. First, store 1.25 in the variable **X** with the following keystrokes.

$\boxed{1}\ \boxed{\cdot}\ \boxed{2}\ \boxed{5}\ \boxed{\text{STO}\triangleright}\ \boxed{\text{X,T},\theta,n}\qquad \boxed{\text{ENTER}}$

Next, enter the expression $3.5 - 1.7x$ with the following keystrokes.

$\boxed{3}\ \boxed{\cdot}\ \boxed{5}\ \boxed{-}\ \boxed{1}\ \boxed{\cdot}\ \boxed{7}\ \boxed{\times}\ \boxed{\text{X,T},\theta,n}\ \boxed{\text{ENTER}}$

The results are shown below.



Thus, the answer is approximately 1.375. We now need to round this answer to the nearest tenth. Mark the rounding digit in the tenths place and the test digit to its immediate right.

Test digit

$$1.\boxed{3}\,\boxed{7}\,5$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then replace all digits to the right of the rounding digit with zeros.

$$1.375 \approx 1.400$$

Delete the trailing zeros from the end of the fractional part of a decimal. This does not change our answer's value.

$$1.375 \approx 1.4$$

Therefore, if $x = 1.25$. then to the nearest tenth:

$$3.5 - 1.7x \approx 1.4$$

77. First, store 2.86 in the variable **X** with the following keystrokes.

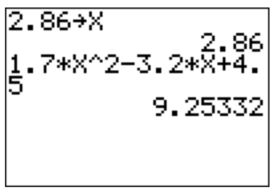
$\boxed{2}$ $\boxed{\cdot}$ $\boxed{8}$ $\boxed{6}$ $\boxed{\text{STO}\triangleright}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\text{ENTER}}$

Next, enter the expression $1.7x^2 - 3.2x + 4.5$ with the following keystrokes.

$\boxed{1}$ $\boxed{\cdot}$ $\boxed{7}$ $\boxed{\times}$ $\boxed{\text{X,T},\theta,n}$ $\boxed{\wedge}$ $\boxed{2}$ $\boxed{-}$ $\boxed{3}$ $\boxed{\cdot}$ $\boxed{2}$ $\boxed{\times}$ $\boxed{\text{X,T},\theta,n}$

$\boxed{+}$ $\boxed{4}$ $\boxed{\cdot}$ $\boxed{5}$ $\boxed{\text{ENTER}}$

The results are shown below.



Thus, the answer is approximately 9.25332. We now need to round this answer to the nearest hundredth. Mark the rounding digit in the hundredths place and the test digit to its immediate right.

Test digit

$9.2\boxed{5}\,\boxed{3}\,32$

Rounding digit

Because the test digit is less than 5, leave the rounding digit alone, then replace all digits to the right of the rounding digit with zeros.

$$9.25332 \approx 9.25000$$

Delete the trailing zeros from the end of the fractional part of a decimal. This does not change our answer's value.

$$9.25332 \approx 9.25$$

Therefore, if $x = 2.86$. then to the nearest hundredth:

$$1.7x^2 - 3.2x + 4.5 \approx 9.25$$

79. First, store -1.27 in the variable **X** with the following keystrokes.

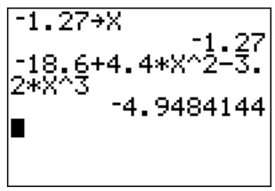
(-) 1 · 2 7 STO▷ X,T,θ,n ENTER

Next, enter the expression $-18.6 + 4.4x^2 - 3.2x^3$ with the following keystrokes.

(-) 1 8 · 6 + 4 · 4 × X,T,θ,n ∧ 2 −

3 · 2 × X,T,θ,n ∧ 3 ENTER

The results are shown below.



Thus, the answer is approximately -4.9484144. We now need to round this answer to the nearest thousandth. Mark the rounding digit in the thousandths place and the test digit to its immediate right.

Test digit

$$-4.94\boxed{8}\,\boxed{4}\,144$$

Rounding digit

Because the test digit is less than 5, keep the rounding digit the same, then replace all digits to the right of the rounding digit with zeros.

$$-4.9484144 \approx -4.9480000$$

Delete the trailing zeros from the end of the fractional part of a decimal. This does not change our answer's value.

$$-4.9484144 \approx -4.948$$

Therefore, if $x = -1.27$. then to the nearest thousandth:

$$-18.6 + 4.4x^2 - 3.2x^3 \approx -4.948$$

1.5 Algebraic Expressions

1. Use the associative property to regroup, then simplify.

$$\begin{aligned} -3(6a) &= ((-3)\cdot 6)a && \text{Apply the associative property.} \\ &= -18a && \text{Simplify.} \end{aligned}$$

3. Use the associative property to regroup, then simplify.

$$\begin{aligned} -9(6ab) &= ((-9)\cdot 6)ab && \text{Apply the associative property.} \\ &= -54ab && \text{Simplify.} \end{aligned}$$

5. Use the associative property to regroup, then simplify.

$$\begin{aligned} -7(3x^2) &= ((-7)\cdot 3)x^2 && \text{Apply the associative property.} \\ &= -21x^2 && \text{Simplify.} \end{aligned}$$

7. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$\begin{aligned} 4(3x-7y) &= 4(3x)-4(7y) && \text{Apply the distributive property.} \\ &= 12x-28y && \text{Simplify.} \end{aligned}$$

9. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$\begin{aligned} -6(-y+9) &= -6(-y)+(-6)(9) && \text{Apply the distributive property.} \\ &= 6y-54 && \text{Simplify.} \end{aligned}$$

11. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$\begin{aligned} -9(s+9) &= -9(s)+(-9)(9) && \text{Apply the distributive property.} \\ &= -9s-81 && \text{Simplify.} \end{aligned}$$

13. To negate a sum, simply negate each term of the sum:

$$-(-3u-6v+8) = 3u+6v-8$$

15. Use the distributive property to expand the expression, and then use order of operations to simplify.

$$\begin{aligned} -8(4u^2 - 6v^2) &= -8(4u^2) - (-8)(6v^2) && \text{Apply the distributive property.} \\ &= -32u^2 + 48v^2 && \text{Simplify.} \end{aligned}$$

17. To negate a sum, simply negate each term of the sum:

$$-(7u + 10v + 8) = -7u - 10v - 8$$

19. First use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} -19x + 17x - 17x &= (-19 + 17 - 17)x && \text{Distributive property.} \\ &= -19x && \text{Simplify.} \end{aligned}$$

21. First use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} 14x^3 - 10x^3 &= (14 - 10)x^3 && \text{Distributive property.} \\ &= 4x^3 && \text{Simplify.} \end{aligned}$$

23. First use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} 9y^2x + 13y^2x - 3y^2x &= (9 + 13 - 3)y^2x && \text{Distributive property.} \\ &= 19y^2x && \text{Simplify.} \end{aligned}$$

25. First use the distributive property to factor out the common variable part. Then simplify.

$$\begin{aligned} 15m + 14m &= (15 + 14)m && \text{Distributive property.} \\ &= 29m && \text{Simplify.} \end{aligned}$$

27. Combine like terms and write down the answer.

$$9 - 17m - m + 7 = 16 - 18m$$

Hint: $9 + 7 = 16$ *and* $-17m - m = -18m$.

29. Combine like terms and write down the answer.

$$-6y^2 - 3x^3 + 4y^2 + 3x^3 = -2y^2$$

Hint: $-6y^2 + 4y^2 = -2y^2$ *and* $-3x^3 + 3x^3 = 0$.

31. Combine like terms and write down the answer.

$$-5m - 16 + 5 - 20m = -25m - 11$$

Hint: $-5m - 20m = -25m$ *and* $-16 + 5 = -11$.

33. Combine like terms and write down the answer.

$$-16x^2y + 7y^3 - 12y^3 - 12x^2y = -28x^2y - 5y^3$$

Hint: $-16x^2y - 12x^2y = -28x^2y$ *and* $7y^3 - 12y^3 = -5y^3$.

35. Combine like terms and write down the answer.

$$-14r + 16 - 7r - 17 = -21r - 1$$

Hint: $-14r - 7r = -21r$ *and* $16 - 17 = -1$.

37. Combine like terms and write down the answer.

$$14 - 16y - 10 - 13y = 4 - 29y$$

Hint: $14 - 10 = 4$ *and* $-16y - 13y = -29y$.

39. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} 3 - (-5y + 1) &= 3 + 5y - 1 && \text{Distribute (negate the sum).} \\ &= 2 + 5y && \text{Combine like terms.} \end{aligned}$$

41. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} &-(9y^2 + 2x^2) - 8(5y^2 - 6x^2) \\ &\quad = -9y^2 - 2x^2 - 40y^2 + 48x^2 && \text{Distribute.} \\ &\quad = -49y^2 + 46x^2 && \text{Combine like terms.} \end{aligned}$$

43. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} 2(10-6p)+10(-2p+5) &= 20-12p-20p+50 && \text{Distribute.} \\ &= 70-32p && \text{Combine like terms.} \end{aligned}$$

45. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} 4(-10n+5)-7(7n-9) &= -40n+20-49n+63 && \text{Distribute.} \\ &= -89n+83 && \text{Combine like terms.} \end{aligned}$$

47. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} -4x-4-(10x-5) &= -4x-4-10x+5 && \text{Distribute (negate the sum).} \\ &= -14x+1 && \text{Combine like terms.} \end{aligned}$$

49. First use the distributive property to expand the expression. Then rearrange the terms and combine like terms.

$$\begin{aligned} -7-(5+3x) &= -7-5-3x && \text{Distribute (negate the sum).} \\ &= -12-3x && \text{Combine like terms.} \end{aligned}$$

51. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} -8(-5y-8)-7(-2+9y) &= 40y+64+14-63y && \text{Distribute.} \\ &= -23y+78 && \text{Combine like terms.} \end{aligned}$$

53. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} &4(-7y^2-9x^2y)-6(-5x^2y-5y^2) \\ &\quad = -28y^2-36x^2y+30x^2y+30y^2 && \text{Distribute.} \\ &\quad = 2y^2-6x^2y && \text{Combine like terms.} \end{aligned}$$

55. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} 6s-7-(2-4s) &= 6s-7-2+4s && \text{Distribute (negate the sum).} \\ &= 10s-9 && \text{Combine like terms.} \end{aligned}$$

57. Use the distributive property to expand the expression. Then combine like terms mentally.

$$\begin{aligned} 9(9-10r)+(-8-2r) &= 81-90r-8-2r && \text{Distribute.} \\ &= 73-92r && \text{Combine like terms.} \end{aligned}$$

59. We analyze the expression inside the parentheses first. Multiply first, distributing the -5.

$$\begin{aligned} -7x+7[2x-5[8x+5]] &= -7x+7(2x-40x-25) \\ &= -7x+7(-38x-25) \end{aligned}$$

Next, distribute 7 and simplify.

$$\begin{aligned} &= -7x-266x-175 \\ &= -273x-175 \end{aligned}$$

61. We analyze the expression inside the parentheses first. Multiply first, distributing the 2.

$$\begin{aligned} 6x-4[-3x+2[5x-7]] &= 6x-4(-3x+10x-14) \\ &= 6x-4(7x-14) \end{aligned}$$

Next, distribute -4 and simplify.

$$\begin{aligned} &= 6x-28x+56 \\ &= -22x+56 \end{aligned}$$

63. We analyze the expression inside the parentheses first. Multiply first, distributing the -3.

$$\begin{aligned} -8x-5[2x-3[-4x+9]] &= -8x-5(2x+12x-27) \\ &= -8x-5(14x-27) \end{aligned}$$

Next, distribute -5 and simplify.

$$\begin{aligned} &= -8x-70x+135 \\ &= -78x+135 \end{aligned}$$

Chapter 2

Solving Linear Equations

2.1 Solving Equations: One Step

1. Substitute 2 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x+2&=4 && \text{Original equation.}\\ 2+2&=4 && \text{Substitute 2 for } x.\\ 4&=4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 2 is a solution of the equation $x + 2 = 4$. For contrast, substitute 3 for x in the equation and simplify.

$$\begin{aligned} x+2&=4 && \text{Original equation.}\\ 3+2&=4 && \text{Substitute 3 for } x.\\ 5&=4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 3 is **not** a solution of $x + 2 = 4$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

3. Substitute 13 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x-9&=4 && \text{Original equation.}\\ 13-9&=4 && \text{Substitute 13 for } x.\\ 4&=4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 13 is a solution of the equation $x - 9 = 4$. For contrast, substitute 14 for x in the equation and simplify.

$$\begin{aligned} x - 9 &= 4 && \text{Original equation.} \\ 14 - 9 &= 4 && \text{Substitute 14 for } x. \\ 5 &= 4 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 14 is **not** a solution of $x - 9 = 4$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

5. Substitute 9 for x in the equation, then simplify both sides of the resulting equation.

$$\begin{aligned} x - 3 &= 6 && \text{Original equation.} \\ 9 - 3 &= 6 && \text{Substitute 9 for } x. \\ 6 &= 6 && \text{Simplify both sides.} \end{aligned}$$

This last equation is a true statement. Hence, 9 is a solution of the equation $x - 3 = 6$. For contrast, substitute 10 for x in the equation and simplify.

$$\begin{aligned} x - 3 &= 6 && \text{Original equation.} \\ 10 - 3 &= 6 && \text{Substitute 10 for } x. \\ 7 &= 6 && \text{Simplify both sides.} \end{aligned}$$

This last equation is **not** a true statement. Hence, 10 is **not** a solution of $x - 3 = 6$. Readers should check that the remaining two given numbers are not solutions by substituting them into the equation and showing that a false statement results.

7. The number -6 is the only solution of the equation $x - 1 = -7$. Similarly, -8 is the only solution of the equation $x = -8$. Therefore $x - 1 = -7$ and $x = -8$ do not have the same solution sets and are **not** equivalent.

9. The number 0 is the only solution of the equation $x - 5 = -5$. Similarly, 0 is the only solution of the equation $x = 0$. Therefore $x - 5 = -5$ and $x = 0$ have the same solution sets and are equivalent.

11. By inspection, the equation $x^2 = 1$ has two solutions, -1 and 1. On the other hand, the equation $x = 1$ has a single solution, namely 1. Hence the equations $x^2 = 1$ and $x = 1$ do not have the same solution sets and are **not** equivalent.

13. To undo the effect of subtracting 20, add 20 to both sides of the equation.

$x - 20 = 9$	Original equation.
$x - 20 + 20 = 9 + 20$	Add 20 to both sides.
$x = 29$	On the left, adding 20 "undoes" the effect of subtracting 20 and returns x. On the right, $9 + 20 = 29$.

Hence, 29 is a solution of $x - 20 = 9$.

15. To undo the effect of subtracting 3, add 3 to both sides of the equation.

$16 = x - 3$	Original equation.
$16 + 3 = x - 3 + 3$	Add 3 to both sides.
$19 = x$	On the right, adding 3 "undoes" the effect of subtracting 3 and returns x. On the left, $16 + 3 = 19$.

Hence, 19 is a solution of $16 = x - 3$.

17. To undo the effect of adding 11, subtract 11 from both sides of the equation.

$x + 11 = 20$	Original equation.
$x + 11 - 11 = 20 - 11$	Subtract 11 from both sides.
$x = 9$	On the left, subtracting 11 "undoes" the effect of adding 11 and returns x. On the right, $20 - 11 = 9$.

Hence, 9 is a solution of $x + 11 = 20$.

19. To undo the effect of subtracting 19, add 19 to both sides of the equation.

$9 = x - 19$	Original equation.
$9 + 19 = x - 19 + 19$	Add 19 to both sides.
$28 = x$	On the right, adding 19 "undoes" the effect of subtracting 19 and returns x. On the left, $9 + 19 = 28$.

Hence, 28 is a solution of $9 = x - 19$.

21. To undo the effect of adding 9, subtract 9 from both sides of the equation.

$20 = 9 + x$	Original equation.
$20 - 9 = 9 + x - 9$	Subtract 9 from both sides.
$11 = x$	On the right, subtracting 9 "undoes" the effect of adding 9 and returns x. On the left, $20 - 9 = 11$.

Hence, 11 is a solution of $20 = 9 + x$.

23. To undo the effect of adding 17, subtract 17 from both sides of the equation.

$18 = 17 + x$	Original equation.
$18 - 17 = 17 + x - 17$	Subtract 17 from both sides.
$1 = x$	On the right, subtracting 17 "undoes" the effect of adding 17 and returns x. On the left, $18 - 17 = 1$.

Hence, 1 is a solution of $18 = 17 + x$.

25. To undo the effect of adding 7, subtract 7 from both sides of the equation.

$7 + x = 19$	Original equation.
$7 + x - 7 = 19 - 7$	Subtract 7 from both sides.
$x = 12$	On the left, subtracting 7 "undoes" the effect of adding 7 and returns x. On the right, $19 - 7 = 12$.

Hence, 12 is a solution of $7 + x = 19$.

27. To undo the effect of subtracting 9, add 9 to both sides of the equation.

$x - 9 = 7$	Original equation.
$x - 9 + 9 = 7 + 9$	Add 9 to both sides.
$x = 16$	On the left, adding 9 "undoes" the effect of subtracting 9 and returns x. On the right, $7 + 9 = 16$.

Hence, 16 is a solution of $x - 9 = 7$.

29. To undo the effect of adding 15, subtract 15 from both sides of the equation.

$x + 15 = 19$	Original equation.
$x + 15 - 15 = 19 - 15$	Subtract 15 from both sides.
$x = 4$	On the left, subtracting 15 “undoes” the effect of adding 15 and returns x. On the right, $19 - 15 = 4$.

Hence, 4 is a solution of $x + 15 = 19$.

31. To undo the effect of adding 10, subtract 10 from both sides of the equation.

$10 + x = 15$	Original equation.
$10 + x - 10 = 15 - 10$	Subtract 10 from both sides.
$x = 5$	On the left, subtracting 10 “undoes” the effect of adding 10 and returns x. On the right, $15 - 10 = 5$.

Hence, 5 is a solution of $10 + x = 15$.

33. To undo the effects of subtracting 4/9, first add 4/9 to both sides of the equation. Then make equivalent fractions with a common denominator and simplify.

$x - \frac{4}{9} = \frac{2}{7}$	Original equation.
$x - \frac{4}{9} + \frac{4}{9} = \frac{2}{7} + \frac{4}{9}$	Add 4/9 to both sides.
$x = \frac{18}{63} + \frac{28}{63}$	On the left, adding 4/9 “undoes” the effect of subtracting 4/9 and returns x. On the right, make equivalent fractions with a common denominator.
$x = \frac{46}{63}$	Simplify.

35. To undo the effects of adding 7/4, first subtract 7/4 from both sides of the equation. Then make equivalent fractions with a common denominator and

simplify.

$$\begin{aligned}
x + \frac{7}{4} &= -\frac{4}{9} && \text{Original equation.}\\
x + \frac{7}{4} - \frac{7}{4} &= -\frac{4}{9} - \frac{7}{4} && \text{Subtract } 7/4 \text{ from both sides.}\\
x &= -\frac{16}{36} - \frac{63}{36} && \text{On the left, subtracting } 7/4 \text{ "undoes" the effect of adding } 7/4 \text{ and returns } x. \text{ On the right, make equivalent fractions with a common denominator.}\\
x &= \frac{-79}{36} && \text{Simplify.}
\end{aligned}$$

37. To undo the effects of adding 5/9, first subtract 5/9 from both sides of the equation. Then make equivalent fractions with a common denominator and simplify.

$$\begin{aligned}
x + \frac{5}{9} &= \frac{7}{2} && \text{Original equation.}\\
x + \frac{5}{9} - \frac{5}{9} &= \frac{7}{2} - \frac{5}{9} && \text{Subtract } 5/9 \text{ from both sides.}\\
x &= \frac{63}{18} - \frac{10}{18} && \text{On the left, subtracting } 5/9 \text{ "undoes" the effect of adding } 5/9 \text{ and returns } x. \text{ On the right, make equivalent fractions with a common denominator.}\\
x &= \frac{53}{18} && \text{Simplify.}
\end{aligned}$$

39. To undo the effects of subtracting 9/8, first add 9/8 to both sides of the equation. Then make equivalent fractions with a common denominator and simplify.

$$\begin{aligned}
x - \frac{9}{8} &= -\frac{1}{2} && \text{Original equation.}\\
x - \frac{9}{8} + \frac{9}{8} &= -\frac{1}{2} + \frac{9}{8} && \text{Add } 9/8 \text{ to both sides.}\\
x &= -\frac{4}{8} + \frac{9}{8} && \text{On the left, adding } 9/8 \text{ "undoes" the effect of subtracting } 9/8 \text{ and returns } x. \text{ On the right, make equivalent fractions with a common denominator.}\\
x &= \frac{5}{8} && \text{Simplify.}
\end{aligned}$$

41. To undo multiplying by -5.1, divide both sides of the equation by -5.1.

$$
\begin{aligned}
-5.1x &= -12.75 && \text{Original equation.}\\
\frac{-5.1x}{-5.1} &= \frac{-12.75}{-5.1} && \text{Divide both sides by } -5.1.\\
x &= 2.5 && \text{Simplify: } -12.75/(-5.1) = 2.5.
\end{aligned}
$$

43. To undo multiplying by -6.9, divide both sides of the equation by -6.9.

$$
\begin{aligned}
-6.9x &= -58.65 && \text{Original equation.}\\
\frac{-6.9x}{-6.9} &= \frac{-58.65}{-6.9} && \text{Divide both sides by } -6.9.\\
x &= 8.5 && \text{Simplify: } -58.65/(-6.9) = 8.5.
\end{aligned}
$$

45. To undo multiplying by -3.6, divide both sides of the equation by -3.6.

$$
\begin{aligned}
-3.6x &= -24.12 && \text{Original equation.}\\
\frac{-3.6x}{-3.6} &= \frac{-24.12}{-3.6} && \text{Divide both sides by } -3.6.\\
x &= 6.7 && \text{Simplify: } -24.12/(-3.6) = 6.7.
\end{aligned}
$$

47. To undo the effect of dividing by 2, multiply both sides of the equation by 2.

$$
\begin{aligned}
\frac{x}{2} &= -11 && \text{Original equation.}\\
2\left(\frac{x}{2}\right) &= 2\,(-11) && \text{Multiply both sides by 2.}\\
x &= -22 && \text{On the left, simplify. On the right, multiply: } 2(-11) = -22.
\end{aligned}
$$

49. To undo the effect of dividing by 8, multiply both sides of the equation by 8.

$$
\begin{aligned}
\frac{x}{8} &= -18 && \text{Original equation.}\\
8\left(\frac{x}{8}\right) &= 8\,(-18) && \text{Multiply both sides by 8.}\\
x &= -144 && \text{On the left, simplify. On the right, multiply: } 8(-18) = -144.
\end{aligned}
$$

51. To undo the effect of dividing by −7, multiply both sides of the equation by −7.

$$\frac{x}{-7} = 15 \qquad \text{Original equation.}$$

$$-7\left(\frac{x}{-7}\right) = -7\,(15) \qquad \text{Multiply both sides by } -7.$$

$$x = -105 \qquad \text{On the left, simplify. On the right, multiply: } -7(15) = -105.$$

2.2 Solving Equations: Multiple Steps

1. On the left, order of operations demands that we first multiply x by 2, then subtract 20. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 20 to both sides of the equation, then divide both sides of the resulting equation by 2.

$$2x - 20 = -12 \qquad \text{Original equation.}$$

$$2x - 20 + 20 = -12 + 20 \qquad \text{To "undo" subtracting 20, add 20 to both sides of the equation.}$$

$$2x = 8 \qquad \text{Simplify both sides.}$$

$$\frac{2x}{2} = \frac{8}{2} \qquad \text{To "undo" multiplying by 2, divide both sides of the equation by 2.}$$

$$x = 4 \qquad \text{Simplify both sides.}$$

3. On the left, order of operations demands that we first multiply x by 3, then add −11. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 11 to both sides of the equation, then divide both sides of the resulting equation by 3.

$$-11 + 3x = -44 \qquad \text{Original equation.}$$

$$-11 + 3x + 11 = -44 + 11 \qquad \text{To "undo" adding } -11\text{, add 11 to both sides of the equation.}$$

$$3x = -33 \qquad \text{Simplify both sides.}$$

$$\frac{3x}{3} = \frac{-33}{3} \qquad \text{To "undo" multiplying by 3, divide both sides of the equation by 3.}$$

$$x = -11 \qquad \text{Simplify both sides.}$$

5. On the left, order of operations demands that we first multiply x by -5, then add 17. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 17 from both sides of the equation, then divide both sides of the resulting equation by -5.

$-5x + 17 = 112$	Original equation.
$-5x + 17 - 17 = 112 - 17$	To "undo" adding 17, subtract 17 from both sides of the equation.
$-5x = 95$	Simplify both sides.
$\dfrac{-5x}{-5} = \dfrac{95}{-5}$	To "undo" multiplying by -5, divide both sides of the equation by -5.
$x = -19$	Simplify both sides.

7. On the left, order of operations demands that we first multiply x by -16, then subtract 14. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 14 to both sides of the equation, then divide both sides of the resulting equation by -16.

$-16x - 14 = 2$	Original equation.
$-16x - 14 + 14 = 2 + 14$	To "undo" subtracting 14, add 14 to both sides of the equation.
$-16x = 16$	Simplify both sides.
$\dfrac{-16x}{-16} = \dfrac{16}{-16}$	To "undo" multiplying by -16, divide both sides of the equation by -16.
$x = -1$	Simplify both sides.

9. On the left, order of operations demands that we first multiply x by -13, then add 5. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 5 from both sides of the equation, then divide both sides of the resulting equation by -13.

$5 - 13x = 70$	Original equation.
$5 - 13x - 5 = 70 - 5$	To "undo" adding 5, subtract 5 from both sides of the equation.
$-13x = 65$	Simplify both sides.
$\dfrac{-13x}{-13} = \dfrac{65}{-13}$	To "undo" multiplying by -13, divide both sides of the equation by -13.
$x = -5$	Simplify both sides.

11. On the left, order of operations demands that we first multiply x by -9, then add -2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 2 to both sides of the equation, then divide both sides of the resulting equation by -9.

$$-2 - 9x = -74 \quad \text{Original equation.}$$
$$-2 - 9x + 2 = -74 + 2 \quad \text{To "undo" adding } -2\text{, add 2 to both sides of the equation.}$$
$$-9x = -72 \quad \text{Simplify both sides.}$$
$$\frac{-9x}{-9} = \frac{-72}{-9} \quad \text{To "undo" multiplying by } -9\text{, divide both sides of the equation by } -9.$$
$$x = 8 \quad \text{Simplify both sides.}$$

13. On the left, order of operations demands that we first multiply x by -1, then add 7. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 7 from both sides of the equation, then divide both sides of the resulting equation by -1.

$$7 - x = -7 \quad \text{Original equation.}$$
$$7 - x - 7 = -7 - 7 \quad \text{To "undo" adding 7, subtract 7 from both sides of the equation.}$$
$$-1x = -14 \quad \text{Simplify both sides.}$$
$$\frac{-1x}{-1} = \frac{-14}{-1} \quad \text{To "undo" multiplying by } -1\text{, divide both sides of the equation by } -1.$$
$$x = 14 \quad \text{Simplify both sides.}$$

15. On the left, order of operations demands that we first multiply x by -4, then add 14. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 14 from both sides of the equation, then divide both sides of the resulting equation by -4.

$$-4x + 14 = 74 \quad \text{Original equation.}$$
$$-4x + 14 - 14 = 74 - 14 \quad \text{To "undo" adding 14, subtract 14 from both sides of the equation.}$$
$$-4x = 60 \quad \text{Simplify both sides.}$$
$$\frac{-4x}{-4} = \frac{60}{-4} \quad \text{To "undo" multiplying by } -4\text{, divide both sides of the equation by } -4.$$
$$x = -15 \quad \text{Simplify both sides.}$$

17. On the left, order of operations demands that we first divide x by 7, then subtract $1/3$. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add $1/3$ to both sides of the equation, then multiply both sides of the resulting equation by 7.

$$\frac{x}{7} - \frac{1}{3} = -\frac{9}{8}$$ Original equation.

$$\frac{x}{7} - \frac{1}{3} + \frac{1}{3} = -\frac{9}{8} + \frac{1}{3}$$ To "undo" subtracting $1/3$, add $1/3$ to both sides of the equation.

$$\frac{x}{7} = -\frac{27}{24} + \frac{8}{24}$$ On the left, simplify. On the right, make equivalent fractions with a common denominator.

$$\frac{x}{7} = -\frac{19}{24}$$ Add: $-\frac{27}{24} + \frac{8}{24} = -\frac{19}{24}$.

$$7\left(\frac{x}{7}\right) = \left(-\frac{19}{24}\right)7$$ To "undo" dividing by 7, multiply both sides of the equation by 7.

$$x = -\frac{133}{24}$$ On the left, simplify. On the right, multiply: $\left(-\frac{19}{24}\right)7 = -\frac{133}{24}$.

19. On the left, order of operations demands that we first divide x by 7, then add $4/9$. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract $4/9$ from both sides of the equation, then multiply both sides of the resulting equation by 7.

$$\frac{x}{7} + \frac{4}{9} = \frac{3}{2}$$ Original equation.

$$\frac{x}{7} + \frac{4}{9} - \frac{4}{9} = \frac{3}{2} - \frac{4}{9}$$ To "undo" adding $4/9$, subtract $4/9$ from both sides of the equation.

$$\frac{x}{7} = \frac{27}{18} - \frac{8}{18}$$ On the left, simplify. On the right, make equivalent fractions with a common denominator.

$$\frac{x}{7} = \frac{19}{18}$$ Subtract: $\frac{27}{18} - \frac{8}{18} = \frac{19}{18}$.

$$7\left(\frac{x}{7}\right) = \left(\frac{19}{18}\right)7$$ To "undo" dividing by 7, multiply both sides of the equation by 7.

$$x = \frac{133}{18}$$ On the left, simplify. On the right, multiply: $\left(\frac{19}{18}\right)7 = \frac{133}{18}$.

21. On the left, order of operations demands that we first divide x by 2, then add 2/3. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 2/3 from both sides of the equation, then multiply both sides of the resulting equation by 2.

$$\frac{x}{2}+\frac{2}{3}=\frac{4}{7}$$ Original equation.

$$\frac{x}{2}+\frac{2}{3}-\frac{2}{3}=\frac{4}{7}-\frac{2}{3}$$ To "undo" adding 2/3, subtract 2/3 from both sides of the equation.

$$\frac{x}{2}=\frac{12}{21}-\frac{14}{21}$$ On the left, simplify. On the right, make equivalent fractions with a common denominator.

$$\frac{x}{2}=-\frac{2}{21}$$ Subtract: $\dfrac{12}{21}-\dfrac{14}{21}=-\dfrac{2}{21}$.

$$2\left(\frac{x}{2}\right)=\left(-\frac{2}{21}\right)2$$ To "undo" dividing by 2, multiply both sides of the equation by 2.

$$x=-\frac{4}{21}$$ On the left, simplify. On the right, multiply: $\left(-\dfrac{2}{21}\right)2=-\dfrac{4}{21}$.

23. On the left, order of operations demands that we first divide x by 5, then subtract 9/2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 9/2 to both sides of the equation, then multiply both sides of the resulting equation by 5.

$$\frac{x}{5}-\frac{9}{2}=-\frac{5}{3}$$ Original equation.

$$\frac{x}{5}-\frac{9}{2}+\frac{9}{2}=-\frac{5}{3}+\frac{9}{2}$$ To "undo" subtracting 9/2, add 9/2 to both sides of the equation.

$$\frac{x}{5}=-\frac{10}{6}+\frac{27}{6}$$ On the left, simplify. On the right, make equivalent fractions with a common denominator.

$$\frac{x}{5}=\frac{17}{6}$$ Add: $-\dfrac{10}{6}+\dfrac{27}{6}=\dfrac{17}{6}$.

$$5\left(\frac{x}{5}\right)=\left(\frac{17}{6}\right)5$$ To "undo" dividing by 5, multiply both sides of the equation by 5.

$$x=\frac{85}{6}$$ On the left, simplify. On the right, multiply: $\left(\dfrac{17}{6}\right)5=\dfrac{85}{6}$.

25. On the left, order of operations demands that we first multiply x by 0.3, then add 1.7. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 1.7 from both sides of the equation, then divide both sides by 0.3.

$0.3x + 1.7 = 3.05$	Original equation.
$0.3x + 1.7 - 1.7 = 3.05 - 1.7$	To "undo" adding 1.7, subtract 1.7 from both sides.
$0.3x = 1.35$	On the left, simplify. On the right, subtract: $3.05 - 1.7 = 1.35$.
$\dfrac{0.3x}{0.3} = \dfrac{1.35}{0.3}$	To "undo" multiplying by 0.3, divide both sides by 0.3.
$x = 4.5$	On the left, simplify. On the right, divide: $1.35/0.3 = 4.5$.

27. On the left, order of operations demands that we first multiply x by 1.2, then add 5.2. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first subtract 5.2 from both sides of the equation, then divide both sides by 1.2.

$1.2x + 5.2 = 14.92$	Original equation.
$1.2x + 5.2 - 5.2 = 14.92 - 5.2$	To "undo" adding 5.2, subtract 5.2 from both sides.
$1.2x = 9.72$	On the left, simplify. On the right, subtract: $14.92 - 5.2 = 9.72$.
$\dfrac{1.2x}{1.2} = \dfrac{9.72}{1.2}$	To "undo" multiplying by 1.2, divide both sides by 1.2.
$x = 8.1$	On the left, simplify. On the right, divide: $9.72/1.2 = 8.1$.

29. On the left, order of operations demands that we first multiply x by 3.5, then subtract 3.7. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 3.7 to both sides of the

equation, then divide both sides by 3.5.

$$\begin{aligned}
3.5x - 3.7 &= -26.10 && \text{Original equation.}\\
3.5x - 3.7 + 3.7 &= -26.10 + 3.7 && \text{To "undo" subtracting 3.7, add 3.7 to both sides.}\\
3.5x &= -22.4 && \text{On the left, simplify. On the right, add: } -26.10 + 3.7 = -22.4.\\
\frac{3.5x}{3.5} &= \frac{-22.4}{3.5} && \text{To "undo" multiplying by 3.5, divide both sides by 3.5.}\\
x &= -6.4 && \text{On the left, simplify. On the right, divide: } -22.4/3.5 = -6.4.
\end{aligned}$$

31. On the left, order of operations demands that we first multiply x by -4.7, then subtract 7.4. To solve this equation for x, we must "undo" each of these operations in inverse order. Thus, we will first add 7.4 to both sides of the equation, then divide both sides by -4.7.

$$\begin{aligned}
-4.7x - 7.4 &= -48.29 && \text{Original equation.}\\
-4.7x - 7.4 + 7.4 &= -48.29 + 7.4 && \text{To "undo" subtracting 7.4, add 7.4 to both sides.}\\
-4.7x &= -40.89 && \text{On the left, simplify. On the right, add: } -48.29 + 7.4 = -40.89.\\
\frac{-4.7x}{-4.7} &= \frac{-40.89}{-4.7} && \text{To "undo" multiplying by } -4.7, \text{ divide both sides by } -4.7.\\
x &= 8.7 && \text{On the left, simplify. On the right, divide: } -40.89/-4.7 = 8.7.
\end{aligned}$$

33. We need to isolate all terms containing x on one side of the equation. We can eliminate $-5x$ from the right-hand side of $13 - 9x = 11 - 5x$ by adding $5x$ to both sides of the equation.

$$\begin{aligned}
13 - 9x &= 11 - 5x && \text{Original equation.}\\
13 - 9x + 5x &= 11 - 5x + 5x && \text{Add } 5x \text{ to both sides.}\\
-4x + 13 &= 11 && \text{Simplify both sides.}
\end{aligned}$$

Next, eliminate 13 from the left-hand side of the last equation by subtracting 13 from both sides of the equation.

$$\begin{aligned}
-4x + 13 - 13 &= 11 - 13 && \text{Subtract 13 both sides.}\\
-4x &= -2 && \text{Simplify both sides.}
\end{aligned}$$

Note how we have isolated all terms containing x on one side of the equation. Finally, to "undo" multiplying by -4, divide both sides of the equation by -4.

$$\begin{aligned} \frac{-4x}{-4} &= \frac{-2}{-4} && \text{Divide both sides by } -4. \\ x &= \frac{1}{2} && \text{Reduce to lowest terms.} \end{aligned}$$

35. We need to isolate all terms containing x on one side of the equation. We can eliminate $19x$ from the right-hand side of $11x + 10 = 19x + 20$ by subtracting $19x$ from both sides of the equation.

$$\begin{aligned} 11x + 10 &= 19x + 20 && \text{Original equation.} \\ 11x + 10 - 19x &= 19x + 20 - 19x && \text{Subtract } 19x \text{ from both sides.} \\ -8x + 10 &= 20 && \text{Simplify both sides.} \end{aligned}$$

Next, eliminate 10 from the left-hand side of the last equation by subtracting 10 from both sides of the equation.

$$\begin{aligned} -8x + 10 - 10 &= 20 - 10 && \text{Subtract 10 both sides.} \\ -8x &= 10 && \text{Simplify both sides.} \end{aligned}$$

Note how we have isolated all terms containing x on one side of the equation. Finally, to "undo" multiplying by -8, divide both sides of the equation by -8.

$$\begin{aligned} \frac{-8x}{-8} &= \frac{10}{-8} && \text{Divide both sides by } -8. \\ x &= -\frac{5}{4} && \text{Reduce to lowest terms.} \end{aligned}$$

37. We need to isolate all terms containing x on one side of the equation. We can eliminate $-19x$ from the right-hand side of $11 - 15x = 13 - 19x$ by adding $19x$ to both sides of the equation.

$$\begin{aligned} 11 - 15x &= 13 - 19x && \text{Original equation.} \\ 11 - 15x + 19x &= 13 - 19x + 19x && \text{Add } 19x \text{ to both sides.} \\ 4x + 11 &= 13 && \text{Simplify both sides.} \end{aligned}$$

Next, eliminate 11 from the left-hand side of the last equation by subtracting 11 from both sides of the equation.

$$\begin{aligned} 4x + 11 - 11 &= 13 - 11 && \text{Subtract 11 both sides.} \\ 4x &= 2 && \text{Simplify both sides.} \end{aligned}$$

Note how we have isolated all terms containing x on one side of the equation. Finally, to "undo" multiplying by 4, divide both sides of the equation by 4.

$$\begin{aligned} \frac{4x}{4} &= \frac{2}{4} && \text{Divide both sides by 4.} \\ x &= \frac{1}{2} && \text{Reduce to lowest terms.} \end{aligned}$$

39. We need to isolate all terms containing x on one side of the equation. We can eliminate $-19x$ from the right-hand side of $9x + 8 = 4 - 19x$ by adding $19x$ to both sides of the equation.

$$\begin{aligned} 9x + 8 &= 4 - 19x && \text{Original equation.} \\ 9x + 8 + 19x &= 4 - 19x + 19x && \text{Add } 19x \text{ to both sides.} \\ 28x + 8 &= 4 && \text{Simplify both sides.} \end{aligned}$$

Next, eliminate 8 from the left-hand side of the last equation by subtracting 8 from both sides of the equation.

$$\begin{aligned} 28x + 8 - 8 &= 4 - 8 && \text{Subtract 8 both sides.} \\ 28x &= -4 && \text{Simplify both sides.} \end{aligned}$$

Note how we have isolated all terms containing x on one side of the equation. Finally, to "undo" multiplying by 28, divide both sides of the equation by 28.

$$\begin{aligned} \frac{28x}{28} &= \frac{-4}{28} && \text{Divide both sides by 28.} \\ x &= -\frac{1}{7} && \text{Reduce to lowest terms.} \end{aligned}$$

41. We need to isolate all terms containing x on one side of the equation. We can eliminate $-18x$ from the right-hand side of $7x + 11 = 16 - 18x$ by adding $18x$ to both sides of the equation.

$$\begin{aligned} 7x + 11 &= 16 - 18x && \text{Original equation.} \\ 7x + 11 + 18x &= 16 - 18x + 18x && \text{Add } 18x \text{ to both sides.} \\ 25x + 11 &= 16 && \text{Simplify both sides.} \end{aligned}$$

Next, eliminate 11 from the left-hand side of the last equation by subtracting 11 from both sides of the equation.

$$\begin{aligned} 25x + 11 - 11 &= 16 - 11 && \text{Subtract 11 both sides.} \\ 25x &= 5 && \text{Simplify both sides.} \end{aligned}$$

Note how we have isolated all terms containing x on one side of the equation. Finally, to "undo" multiplying by 25, divide both sides of the equation by 25.

$$\begin{aligned} \frac{25x}{25} &= \frac{5}{25} && \text{Divide both sides by 25.} \\ x &= \frac{1}{5} && \text{Reduce to lowest terms.} \end{aligned}$$

43. We need to isolate all terms containing x on one side of the equation. We can eliminate $4x$ from the right-hand side of $12x + 9 = 4x + 7$ by subtracting $4x$ from both sides of the equation.

$$\begin{aligned} 12x + 9 &= 4x + 7 && \text{Original equation.} \\ 12x + 9 - 4x &= 4x + 7 - 4x && \text{Subtract } 4x \text{ from both sides.} \\ 8x + 9 &= 7 && \text{Simplify both sides.} \end{aligned}$$

Next, eliminate 9 from the left-hand side of the last equation by subtracting 9 from both sides of the equation.

$$\begin{aligned} 8x + 9 - 9 &= 7 - 9 && \text{Subtract 9 both sides.} \\ 8x &= -2 && \text{Simplify both sides.} \end{aligned}$$

Note how we have isolated all terms containing x on one side of the equation. Finally, to "undo" multiplying by 8, divide both sides of the equation by 8.

$$\begin{aligned} \frac{8x}{8} &= \frac{-2}{8} && \text{Divide both sides by 8.} \\ x &= -\frac{1}{4} && \text{Reduce to lowest terms.} \end{aligned}$$

45. We'll first simplify the expression on the left-hand side of the equation using the *Rules Guiding Order of Operations*.

$$\begin{aligned} 8(5x - 3) - 3(4x + 6) &= 4 && \text{Original equation.} \\ 40x - 24 - 12x - 18 &= 4 && \text{Multiply: } 8(5x - 3) = 40x - 24. \\ & && \text{Multiply: } -3(4x + 6) = -12x - 18. \\ 28x - 42 &= 4 && \text{Add: } 40x - 12x = 28x. \\ & && \text{Add: } -24 - 18 = -42. \end{aligned}$$

Next, isolate terms containing the variable x on one side of the equation. To remove the term -42 from the left-hand side, add 42 to both sides of the equation.

$$\begin{aligned} 28x - 42 + 42 &= 4 + 42 && \text{Add 42 to both sides.} \\ 28x &= 46 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying 28, divide both sides of the equation by 28.

$$\begin{aligned} \frac{28x}{28} &= \frac{46}{28} && \text{Divide both sides by 28.} \\ x &= \frac{23}{14} && \text{Reduce.} \end{aligned}$$

47. We'll first simplify the expression on the left-hand side of the equation using the *Rules Guiding Order of Operations*.

$$\begin{aligned} 2x - 4(4 - 9x) &= 4(7x + 8) && \text{Original equation.} \\ 2x - 16 + 36x &= 28x + 32 && \text{Multiply: } -4(4 - 9x) = -16 + 36x. \\ & && \text{Multiply: } 4(7x + 8) = 28x + 8. \\ 38x - 16 &= 28x + 32 && \text{Add: } 2x + 36x = 38x. \end{aligned}$$

Now we will isolate all terms containing x on one side of the equation. To remove the term $28x$ from the right-hand side, subtract $28x$ from both sides of the equation.

$$\begin{aligned} 38x - 16 - 28x &= 28x + 32 - 28x && \text{Subtract } 28x \text{ from both sides.} \\ 10x - 16 &= 32 && \text{Simplify both sides.} \end{aligned}$$

To remove the term -16 from the left-hand side, add 16 to both sides of the equation.

$$\begin{aligned} 10x - 16 + 16 &= 32 + 16 && \text{Add 16 to both sides.} \\ 10x &= 48 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 10, divide both sides of the equation by 10.

$$\begin{aligned} \frac{10x}{10} &= \frac{48}{10} && \text{Divide both sides by 10.} \\ x &= \frac{24}{5} && \text{Reduce.} \end{aligned}$$

49. We'll first simplify the expression on the left-hand side of the equation using the *Rules Guiding Order of Operations*.

$$\begin{aligned} 2(6 - 2x) - (4x - 9) &= 9 && \text{Original equation.} \\ 12 - 4x - 4x + 9 &= 9 && \text{Multiply: } 2(6 - 2x) = 12 - 4x. \\ & && \text{Distribute the minus sign:} \\ & && -(4x - 9) = -4x + 9. \\ 21 - 8x &= 9 && \text{Add: } 12 + 9 = 21. \\ & && \text{Add: } -4x - 4x = -8x. \end{aligned}$$

Next, isolate all terms containing the variable x on one side of the equation. To remove the term 21 from the left-hand side, subtract 21 from both sides of the equation.

$$\begin{aligned} 21 - 8x - 21 &= 9 - 21 && \text{Subtract 21 from both sides.} \\ -8x &= -12 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -8, divide both sides of the equation by -8.

$$\begin{aligned} \frac{-8x}{-8} &= \frac{-12}{-8} && \text{Divide both sides by } -8. \\ x &= \frac{3}{2} && \text{Reduce.} \end{aligned}$$

51. We'll first simplify the expression on the left-hand side of the equation using the *Rules Guiding Order of Operations*.

$$\begin{aligned} 3(5x-6) - 7(7x+9) &= 3 && \text{Original equation.} \\ 15x - 18 - 49x - 63 &= 3 && \text{Multiply: } 3(5x-6) = 15x - 18. \\ & && \text{Multiply: } -7(7x+9) = -49x - 63. \\ -34x - 81 &= 3 && \text{Add: } 15x - 49x = -34x. \\ & && \text{Add: } -18 - 63 = -81. \end{aligned}$$

Next, isolate terms containing the variable x on one side of the equation. To remove the term -81 from the left-hand side, add 81 to both sides of the equation.

$$\begin{aligned} -34x - 81 + 81 &= 3 + 81 && \text{Add 81 to both sides.} \\ -34x &= 84 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying -34, divide both sides of the equation by -34.

$$\begin{aligned} \frac{-34x}{-34} &= \frac{84}{-34} && \text{Divide both sides by } -34. \\ x &= -\frac{42}{17} && \text{Reduce.} \end{aligned}$$

53. We'll first simplify the expression on the left-hand side of the equation using the *Rules Guiding Order of Operations*.

$$\begin{aligned} 2x - 2(4-9x) &= 8(6x+2) && \text{Original equation.} \\ 2x - 8 + 18x &= 48x + 16 && \text{Multiply: } -2(4-9x) = -8 + 18x. \\ & && \text{Multiply: } 8(6x+2) = 48x + 2. \\ 20x - 8 &= 48x + 16 && \text{Add: } 2x + 18x = 20x. \end{aligned}$$

Now we will isolate all terms containing x on one side of the equation. To remove the term $48x$ from the right-hand side, subtract $48x$ from both sides of the equation.

$$\begin{aligned} 20x - 8 - 48x &= 48x + 16 - 48x && \text{Subtract } 48x \text{ from both sides.} \\ -28x - 8 &= 16 && \text{Simplify both sides.} \end{aligned}$$

To remove the term -8 from the left-hand side, add 8 to both sides of the equation.

$$\begin{aligned} -28x - 8 + 8 &= 16 + 8 && \text{Add 8 to both sides.} \\ -28x &= 24 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by -28, divide both sides of the equation by -28.

$$\begin{aligned} \frac{-28x}{-28} &= \frac{24}{-28} && \text{Divide both sides by } -28. \\ x &= -\frac{6}{7} && \text{Reduce.} \end{aligned}$$

55. We'll first simplify the expression on the left-hand side of the equation using the *Rules Guiding Order of Operations*.

$$\begin{aligned} 2(7 - 9x) - (2x - 8) &= 7 && \text{Original equation.} \\ 14 - 18x - 2x + 8 &= 7 && \text{Multiply: } 2(7 - 9x) = 14 - 18x. \\ & && \text{Distribute the minus sign:} \\ & && -(2x - 8) = -2x + 8. \\ 22 - 20x &= 7 && \text{Add: } 14 + 8 = 22. \\ & && \text{Add: } -18x - 2x = -20x. \end{aligned}$$

Next, isolate all terms containing the variable x on one side of the equation. To remove the term 22 from the left-hand side, subtract 22 from both sides of the equation.

$$\begin{aligned} 22 - 20x - 22 &= 7 - 22 && \text{Subtract 22 from both sides.} \\ -20x &= -15 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -20, divide both sides of the equation by -20.

$$\begin{aligned} \frac{-20x}{-20} &= \frac{-15}{-20} && \text{Divide both sides by } -20. \\ x &= \frac{3}{4} && \text{Reduce.} \end{aligned}$$

2.3 Solving Equations: Clearing Frations and Decimals

1. Multiply 16 and 9 to get 144, then divide 144 by 2 to get 72.

$$\begin{aligned} 16\left(\frac{9}{2}x\right) &= \left(16\cdot\frac{9}{2}\right)x && \text{Associative property of multiplication.} \\ &= \left(\frac{144}{2}\right)x && \text{Multiply: } 16\cdot 9 = 144. \\ &= 72x && \text{Divide: } 144/2 = 72. \end{aligned}$$

Alternate solution: Divide 2 into 16 to get 8, then multiply 8 by 9 to get 72.

$$\begin{aligned} 16\left(\frac{9}{2}x\right) &= \left(16\cdot\frac{9}{2}\right)x && \text{Associative property of multiplication.} \\ &= (8\cdot 9)\,x && \text{Divide: } 16/2 = 8. \\ &= 72x && \text{Multiply: } 8\cdot 9 = 72. \end{aligned}$$

Note that the second method is more efficient, because it involves smaller numbers, making it easier to perform the steps mentally. That is, write

$$16\left(\frac{9}{2}x\right) = 72x,$$

without writing down any steps.

3. Multiply 14 and 3 to get 42, then divide 42 by 2 to get 21.

$$\begin{aligned} 14\left(\frac{3}{2}x\right) &= \left(14\cdot\frac{3}{2}\right)x && \text{Associative property of multiplication.} \\ &= \left(\frac{42}{2}\right)x && \text{Multiply: } 14\cdot 3 = 42. \\ &= 21x && \text{Divide: } 42/2 = 21. \end{aligned}$$

Alternate solution: Divide 2 into 14 to get 7, then multiply 7 by 3 to get 21.

$$\begin{aligned} 14\left(\frac{3}{2}x\right) &= \left(14\cdot\frac{3}{2}\right)x && \text{Associative property of multiplication.} \\ &= (7\cdot 3)\,x && \text{Divide: } 14/2 = 7. \\ &= 21x && \text{Multiply: } 7\cdot 3 = 21. \end{aligned}$$

Note that the second method is more efficient, because it involves smaller numbers, making it easier to perform the steps mentally. That is, write

$$14\left(\frac{3}{2}x\right) = 21x,$$

without writing down any steps.

5. Multiply 70 and 9 to get 630, then divide 630 by 7 to get 90.

$$\begin{aligned} 70\left(\frac{9}{7}x\right) &= \left(70 \cdot \frac{9}{7}\right)x && \text{Associative property of multiplication.} \\ &= \left(\frac{630}{7}\right)x && \text{Multiply: } 70 \cdot 9 = 630. \\ &= 90x && \text{Divide: } 630/7 = 90. \end{aligned}$$

Alternate solution: Divide 7 into 70 to get 10, then multiply 10 by 9 to get 90.

$$\begin{aligned} 70\left(\frac{9}{7}x\right) &= \left(70 \cdot \frac{9}{7}\right)x && \text{Associative property of multiplication.} \\ &= (10 \cdot 9)\,x && \text{Divide: } 70/7 = 10. \\ &= 90x && \text{Multiply: } 10 \cdot 9 = 90. \end{aligned}$$

Note that the second method is more efficient, because it involves smaller numbers, making it easier to perform the steps mentally. That is, write

$$70\left(\frac{9}{7}x\right) = 90x,$$

without writing down any steps.

7. Clear fractions from the equation by multiplying both sides by the least common denominator. The least common denominator in this case is 21.

$$\begin{aligned} -\frac{9}{7}x - \frac{1}{3} &= \frac{5}{3} && \text{Original equation.} \\ 21\left(-\frac{9}{7}x - \frac{1}{3}\right) &= 21\left(\frac{5}{3}\right) && \text{Multiply both sides by 21.} \\ 21\left(-\frac{9}{7}x\right) - 21\left(\frac{1}{3}\right) &= 21\left(\frac{5}{3}\right) && \text{Distribute the 21 on each side.} \\ -27x - 7 &= 35 && \text{Multiply.} \end{aligned}$$

Note that the fractions are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. We can remove the term -7 from the left-hand side by adding 7 to both sides of the equation.

$$\begin{aligned} -27x - 7 + 7 &= 35 + 7 && \text{Add 7 to both sides.} \\ -27x &= 42 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by -27, divide both sides of the equation by -27.

$$\begin{aligned} \frac{-27x}{-27} &= \frac{42}{-27} && \text{Divide both sides by } -27. \\ x &= -\frac{14}{9} && \text{Simplify both sides.} \end{aligned}$$

9. Clear fractions from the equation by multiplying both sides by the least common denominator. The least common denominator in this case is 9.

$$\begin{aligned}
\frac{7}{3}x + \frac{5}{9} &= \frac{2}{3}x - \frac{4}{3} && \text{Original equation.}\\
9\left(\frac{7}{3}x + \frac{5}{9}\right) &= 9\left(\frac{2}{3}x - \frac{4}{3}\right) && \text{Multiply both sides by 9.}\\
9\left(\frac{7}{3}x\right) + 9\left(\frac{5}{9}\right) &= 9\left(\frac{2}{3}x\right) - 9\left(-\frac{4}{3}\right) && \text{Distribute the 9 on each side.}\\
21x + 5 &= 6x - 12 && \text{Multiply.}
\end{aligned}$$

Note that the fractions are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. We can remove the term $6x$ from the right-hand side by subtracting $6x$ from both sides of the equation.

$$\begin{aligned}
21x + 5 - 6x &= 6x - 12 - 6x && \text{Subtract } 6x \text{ from both sides.}\\
15x + 5 &= -12 && \text{Simplify both sides.}
\end{aligned}$$

Next, we can remove the term 5 from the left-hand side by subtracting 5 from both sides of the equation.

$$\begin{aligned}
15x + 5 - 5 &= -12 - 5 && \text{Subtract 5 from both sides.}\\
15x &= -17 && \text{Simplify both sides.}
\end{aligned}$$

Finally, to "undo" multiplying by 15, divide both sides of the equation by 15.

$$\begin{aligned}
\frac{15x}{15} &= \frac{-17}{15} && \text{Divide both sides by 15.}\\
x &= -\frac{17}{15} && \text{Simplify both sides.}
\end{aligned}$$

11. Clear fractions from the equation by multiplying both sides by the least common denominator. The least common denominator in this case is 28.

$$\begin{aligned}
\frac{9}{4}x - \frac{8}{7} &= \frac{3}{2} && \text{Original equation.}\\
28\left(\frac{9}{4}x - \frac{8}{7}\right) &= 28\left(\frac{3}{2}\right) && \text{Multiply both sides by 28.}\\
28\left(\frac{9}{4}x\right) - 28\left(\frac{8}{7}\right) &= 28\left(\frac{3}{2}\right) && \text{Distribute the 28 on each side.}\\
63x - 32 &= 42 && \text{Multiply.}
\end{aligned}$$

Note that the fractions are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. We can remove the

term -32 from the left-hand side by adding 32 to both sides of the equation.

$$\begin{aligned} 63x - 32 + 32 &= 42 + 32 && \text{Add 32 to both sides.} \\ 63x &= 74 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 63, divide both sides of the equation by 63.

$$\begin{aligned} \frac{63x}{63} &= \frac{74}{63} && \text{Divide both sides by 63.} \\ x &= \frac{74}{63} && \text{Simplify both sides.} \end{aligned}$$

13. Clear fractions from the equation by multiplying both sides by the least common denominator. The least common denominator in this case is 12.

$$\begin{aligned} -\frac{3}{4}x &= -\frac{8}{3} && \text{Original equation.} \\ 12\left(-\frac{3}{4}x\right) &= 12\left(-\frac{8}{3}\right) && \text{Multiply both sides by 12.} \\ -9x &= -32 && \text{Multiply.} \end{aligned}$$

To "undo" multiplying by -9, divide both sides of the equation by -9.

$$\begin{aligned} \frac{-9x}{-9} &= \frac{-32}{-9} && \text{Divide both sides by } -9. \\ x &= \frac{32}{9} && \text{Simplify.} \end{aligned}$$

15. Clear fractions from the equation by multiplying both sides by the least common denominator. The least common denominator in this case is 20.

$$\begin{aligned} x + \frac{3}{4} &= \frac{6}{5} && \text{Original equation.} \\ 20\left(x + \frac{3}{4}\right) &= 20\left(\frac{6}{5}\right) && \text{Multiply both sides by 20.} \\ 20x + 20\left(\frac{3}{4}\right) &= 20\left(\frac{6}{5}\right) && \text{On the left, distribute the 20.} \\ 20x + 15 &= 24 && \text{Multiply.} \end{aligned}$$

Next, isolate all terms containing x on one side of the equation.

$$\begin{aligned} 20x + 15 - 15 &= 24 - 15 && \text{Subtract 15 from both sides.} \\ 20x &= 9 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 20, divide both sides of the equation by 20.

$$\begin{aligned} \frac{20x}{20} &= \frac{9}{20} && \text{Divide both sides by 20.} \\ x &= \frac{9}{20} && \text{Simplify both sides.} \end{aligned}$$

17. Clear fractions from the equation by multiplying both sides by the least common denominator. The least common denominator in this case is 60.

$$\begin{aligned} -\frac{1}{3}x - \frac{4}{3} &= -\frac{3}{4}x - \frac{8}{5} && \text{Original equation.} \\ 60\left(-\frac{1}{3}x - \frac{4}{3}\right) &= 60\left(-\frac{3}{4}x - \frac{8}{5}\right) && \text{Multiply both sides by 60.} \\ 60\left(-\frac{1}{3}x\right) - 60\left(\frac{4}{3}\right) &= 60\left(-\frac{3}{4}x\right) - 60\left(-\frac{8}{5}\right) && \text{Distribute the 60 on each side.} \\ -20x - 80 &= -45x - 96 && \text{Multiply.} \end{aligned}$$

Note that the fractions are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. We can remove the term $-45x$ from the right-hand side by adding $45x$ to both sides of the equation.

$$\begin{aligned} -20x - 80 + 45x &= -45x - 96 + 45x && \text{Add } 45x \text{ to both sides.} \\ 25x - 80 &= -96 && \text{Simplify both sides.} \end{aligned}$$

Next, we can remove the term -80 from the left-hand side by adding 80 to both sides of the equation.

$$\begin{aligned} 25x - 80 + 80 &= -96 + 80 && \text{Add 80 to both sides.} \\ 25x &= -16 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 25, divide both sides of the equation by 25.

$$\begin{aligned} \frac{25x}{25} &= \frac{-16}{25} && \text{Divide both sides by 25.} \\ x &= -\frac{16}{25} && \text{Simplify both sides.} \end{aligned}$$

19. At a minimum, we need to move each decimal point two places to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 100.

$$\begin{aligned} 2.39x + 0.71 &= -1.98x + 2.29 && \text{Original Equation.} \\ 100(2.39x + 0.71) &= 100(-1.98x + 2.29) && \text{Multiply both sides by 100.} \\ 239x + 71 &= -198x + 229 && \text{Distribute the 100.} \end{aligned}$$

Note that the decimals are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. Remove the term $-198x$ from the right-hand side by adding $198x$ to both sides of the equation.

$$\begin{aligned} 239x + 71 + 198x &= -198x + 229 + 198x && \text{Add } 198x \text{ to both sides.} \\ 437x + 71 &= 229 && \text{Simplify both sides.} \end{aligned}$$

Subtract 71 from to eliminate the term 71 from the left-hand side of the equation.

$$\begin{aligned} 437x + 71 - 71 &= 229 - 71 && \text{Subtract 71 from both sides.} \\ 437x &= 158 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 437, divide both sides of the equation by 437.

$$\begin{aligned} \frac{437x}{437} &= \frac{158}{437} && \text{Divide both sides by 437.} \\ x &= \frac{158}{437} && \text{Simplify.} \end{aligned}$$

21. At a minimum, we need to move each decimal point two places to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 100.

$$\begin{aligned} 0.4x - 1.55 &= 2.14 && \text{Original Equation.} \\ 100(0.4x - 1.55) &= 100(2.14) && \text{Multiply both sides by 100.} \\ 40x - 155 &= 214 && \text{Distribute the 100.} \end{aligned}$$

Note that the decimals are now cleared from the equation. We can continue by adding 155 to both sides of the equation.

$$\begin{aligned} 40x - 155 + 155 &= 214 + 155 && \text{Add 155 to both sides.} \\ 40x &= 369 && \text{Simplify both sides.} \\ \frac{40x}{40} &= \frac{369}{40} && \text{Divide both sides by 40.} \\ x &= \frac{369}{40} && \text{Simplify.} \end{aligned}$$

23. At a minimum, we need to move each decimal point two places to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 100.

$$\begin{aligned} 2.6x - 2.54 &= -2.14x && \text{Original Equation.} \\ 100(2.6x - 2.54) &= 100(-2.14x) && \text{Multiply both sides by 100.} \\ 260x - 254 &= -214x && \text{Distribute the 100.} \end{aligned}$$

Note that the decimals are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. Remove the term $-214x$ from the right-hand side of the equation by adding $214x$ to both sides of the equation.

$$\begin{aligned} 260x - 254 + 214x &= -214x + 214x && \text{Add } 214x \text{ to both sides.} \\ 474x - 254 &= 0 && \text{Simplify both sides.} \end{aligned}$$

Add 254 to both sides to remove the term -254 from the left-hand side of the equation.

$$\begin{aligned} 474x - 254 + 254 &= 0 + 254 && \text{Add 254 to both sides.} \\ 474x &= 254 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 474, divide both sides of the equation by 474.

$$\begin{aligned} \frac{474x}{474} &= \frac{254}{474} && \text{Divide both sides by 474.} \\ x &= \frac{127}{237} && \text{Simplify.} \end{aligned}$$

25. At a minimum, we need to move each decimal point one place to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 10.

$$\begin{aligned} 0.7x &= -2.3x - 2.8 && \text{Original Equation.} \\ 10(0.7x) &= 10(-2.3x - 2.8) && \text{Multiply both sides by 10.} \\ 7x &= -23x - 28 && \text{Distribute the 10.} \end{aligned}$$

Note that the decimals are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. We can remove the term $-23x$ from the right-hand side by adding $23x$ to both sides of the equation.

$$\begin{aligned} 7x + 23x &= -23x - 28 + 23x && \text{Add } 23x \text{ to both sides.} \\ 30x &= -28 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 30, divide both sides of the equation by 30.

$$\begin{aligned} \frac{30x}{30} &= \frac{-28}{30} && \text{Divide both sides by 30.} \\ x &= -\frac{14}{15} && \text{Simplify.} \end{aligned}$$

27. At a minimum, we need to move each decimal point one place to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 10.

$$\begin{aligned} -4.8x - 2.7 &= -1.9 && \text{Original Equation.} \\ 10(-4.8x - 2.7) &= 10(-1.9) && \text{Multiply both sides by 10.} \\ -48x - 27 &= -19 && \text{Distribute the 10.} \end{aligned}$$

Note that the decimals are now cleared from the equation. We can continue by adding 27 to both sides of the equation.

$$\begin{aligned} -48x - 27 + 27 &= -19 + 27 && \text{Add 27 to both sides.} \\ -48x &= 8 && \text{Simplify both sides.} \\ \frac{-48x}{-48} &= \frac{8}{-48} && \text{Divide both sides by } -48. \\ x &= -\frac{1}{6} && \text{Simplify.} \end{aligned}$$

29. At a minimum, we need to move each decimal point one place to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 10.

$$\begin{aligned} 1.7x + 2.1 &= -1.6x + 2.5 && \text{Original Equation.} \\ 10(1.7x + 2.1) &= 10(-1.6x + 2.5) && \text{Multiply both sides by 10.} \\ 17x + 21 &= -16x + 25 && \text{Distribute the 10.} \end{aligned}$$

Note that the decimals are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. Remove $-16x$ from the right-hand side by adding $16x$ to both sides of the equation.

$$\begin{aligned} 17x + 21 + 16x &= -16x + 25 + 16x && \text{Add } 16x \text{ to both sides.} \\ 33x + 21 &= 25 && \text{Simplify both sides.} \end{aligned}$$

Subtract 21 from both sides to remove the term 21 from the left-hand side of the equation.

$$\begin{aligned} 33x + 21 - 21 &= 25 - 21 && \text{Subtract 21 from both sides.} \\ 33x &= 4 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 33, divide both sides of the equation by 33.

$$\begin{aligned} \frac{33x}{33} &= \frac{4}{33} && \text{Divide both sides by 33.} \\ x &= \frac{4}{33} && \text{Simplify.} \end{aligned}$$

31. At a minimum, we need to move each decimal point one place to the right in order to clear the decimals from the equation. Consequently, we multiply both sides of the equation by 10.

$$\begin{aligned} 2.5x + 1.9 &= 0.9x && \text{Original Equation.} \\ 10(2.5x + 1.9) &= 10(0.9x) && \text{Multiply both sides by 10.} \\ 25x + 19 &= 9x && \text{Distribute the 10.} \end{aligned}$$

Note that the decimals are now cleared from the equation. Next, isolate all terms containing the variable x on one side of the equation. Remove the term $9x$ from the right-hand side by subtracting $9x$ from both sides of the equation.

$$\begin{aligned} 25x + 19 - 9x &= 9x - 9x && \text{Subtract } 9x \text{ from both sides.} \\ 16x + 19 &= 0 && \text{Simplify both sides.} \end{aligned}$$

Subtract 19 from both sides to remove the term 19 from the left-hand side of the equation.

$$\begin{aligned} 16x + 19 - 19 &= 0 - 19 && \text{Subtract 19 from both sides.} \\ 16x &= -19 && \text{Simplify both sides.} \end{aligned}$$

Finally, to "undo" multiplying by 16, divide both sides of the equation by 16.

$$\begin{aligned} \frac{16x}{16} &= \frac{-19}{16} && \text{Divide both sides by 16.} \\ x &= -\frac{19}{16} && \text{Simplify.} \end{aligned}$$

2.4 Formulae

1. We are instructed to solve the equation $F = kx$ for x. Thus, we must isolate all terms containing the variable x on one side of the equation. We begin by dividing both sides of the equation by k.

$$\begin{aligned} F &= kx && \text{Original equation.} \\ \frac{F}{k} &= \frac{kx}{k} && \text{Divide both sides by } k. \\ \frac{F}{k} &= x && \text{Simplify.} \end{aligned}$$

Note that this last equation is identical to the following equation

$$x = \frac{F}{k}$$

Note that we have $x =$"Stuff", where "Stuff" contains no occurrences of x, the variable we are solving for.

3. We are instructed to solve the equation $E = mc^2$ for m. Thus, we must isolate all terms containing the variable m on one side of the equation. We begin by dividing both sides of the equation by c^2.

$$\begin{aligned}
E &= mc^2 && \text{Original equation.}\\
\frac{E}{c^2} &= \frac{mc^2}{c^2} && \text{Divide both sides by } c^2.\\
\frac{E}{c^2} &= m && \text{Simplify.}
\end{aligned}$$

Note that this last equation is identical to the following equation

$$m = \frac{E}{c^2}$$

We now have $m =$"Stuff", where "Stuff" contains no occurrences of m, the variable we are solving for.

5. We are instructed to solve the equation $A = \pi r_1 r_2$ for r_2. Thus, we must isolate all terms containing the variable r_2 on one side of the equation. We begin by dividing both sides of the equation by πr_1.

$$\begin{aligned}
A &= \pi r_1 r_2 && \text{Original equation.}\\
\frac{A}{\pi r_1} &= \frac{\pi r_1 r_2}{\pi r_1} && \text{Divide both sides by } \pi r_1.\\
\frac{A}{\pi r_1} &= r_2 && \text{Simplify.}
\end{aligned}$$

Note that this last equation is identical to the following equation

$$r_2 = \frac{A}{\pi r_1}$$

Note that we have $r_2 =$"Stuff", where "Stuff" contains no occurrences of r_2, the variable we are solving for.

7. We are instructed to solve the equation $F = ma$ for a. Thus, we must isolate all terms containing the variable a on one side of the equation. We begin by dividing both sides of the equation by m.

$$\begin{aligned}
F &= ma && \text{Original equation.}\\
\frac{F}{m} &= \frac{ma}{m} && \text{Divide both sides by } m.\\
\frac{F}{m} &= a && \text{Simplify.}
\end{aligned}$$

Note that this last equation is identical to the following equation

$$a = \frac{F}{m}$$

Note that we have $a =$"Stuff", where "Stuff" contains no occurrences of a, the variable we are solving for.

9. We are instructed to solve the equation $C = 2\pi r$ for r. Thus, we must isolate all terms containing the variable r on one side of the equation. We begin by dividing both sides of the equation by 2π.

$$\begin{aligned} C &= 2\pi r && \text{Original equation.} \\ \frac{C}{2\pi} &= \frac{2\pi r}{2\pi} && \text{Divide both sides by } 2\pi. \\ \frac{C}{2\pi} &= r && \text{Simplify.} \end{aligned}$$

Note that this last equation is identical to the following equation

$$r = \frac{C}{2\pi}$$

Note that we have $r =$"Stuff", where "Stuff" contains no occurrences of r, the variable we are solving for.

11. We are instructed to solve the equation $y = mx + b$ for x. Thus, we must isolate all terms containing the variable x on one side of the equation. First, subtract b from both sides of the equation.

$$\begin{aligned} y &= mx + b && \text{Original equation.} \\ y - b &= mx + b - b && \text{Subtract } b \text{ from both sides.} \\ y - b &= mx && \text{Combine like terms.} \end{aligned}$$

Note that all the terms comtaining x, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by m to complete the solution.

$$\begin{aligned} \frac{y-b}{m} &= \frac{mx}{m} && \text{Divide both sides by } m. \\ \frac{y-b}{m} &= x && \text{Simplify.} \end{aligned}$$

Note that this last equation is identical to the following equation

$$x = \frac{y-b}{m} \qquad \text{Simplify.}$$

Note that we have $x =$"Stuff", where "Stuff" contains no occurrences of x, the variable we are solving for.

13. We are instructed to solve the equation $F = qvB$ for v. Thus, we must isolate all terms containing the variable v on one side of the equation. We begin by dividing both sides of the equation by qB.

$$\begin{aligned} F &= qvB && \text{Original equation.} \\ \frac{F}{qB} &= \frac{qvB}{qB} && \text{Divide both sides by } qB. \\ \frac{F}{qB} &= v && \text{Simplify.} \end{aligned}$$

Note that this last equation is identical to the following equation

$$v = \frac{F}{qB}$$

Note that we have $v =$"Stuff", where "Stuff" contains no occurrences of v, the variable we are solving for.

15. We are instructed to solve the equation $V = \frac{1}{3}\pi r^2 h$ for h. Thus, we must isolate all terms containing the variable h on one side of the equation. First, clear the fractions by multiplying both sides of the equation by the common denominator, 3.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h && \text{Original equation.} \\ 3(V) &= 3\left(\frac{1}{3}\pi r^2 h\right) && \text{Multiply both sides by 3.} \\ 3V &= \pi r^2 h && \text{Simplify. Cancel 3's.} \end{aligned}$$

Note that all the terms comtaining h, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by πr^2 to complete the solution.

$$\begin{aligned} \frac{3V}{\pi r^2} &= \frac{\pi r^2 h}{\pi r^2} && \text{Divide both sides by } \pi r^2. \\ \frac{3V}{\pi r^2} &= h && \text{Simplify.} \end{aligned}$$

Note that this last equation is identical to the following equation

$$h = \frac{3V}{\pi r^2}$$

Note that we have $h =$"Stuff", where "Stuff" contains no occurrences of h, the variable we are solving for.

17. We are instructed to solve the equation $I = \dfrac{V}{R}$ for R. Thus, we must isolate all terms containing the variable R on one side of the equation. First, clear the fractions by multiplying both sides of the equation by the common denominator, R.

$$\begin{aligned} I &= \frac{V}{R} && \text{Original equation.} \\ R(I) &= R\left(\frac{V}{R}\right) && \text{Multiply both sides by } R. \\ RI &= V && \text{Simplify. Cancel R's.} \end{aligned}$$

Note that all the terms comtaining R, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by I to complete the solution.

$$\begin{aligned} \frac{RI}{I} &= \frac{V}{I} && \text{Divide both sides by } I. \\ R &= \frac{V}{I} \end{aligned}$$

Note that we have $R =$"Stuff", where "Stuff" contains no occurrences of R, the variable we are solving for.

19. We are instructed to solve the equation $F = \dfrac{kqQ}{r^2}$ for q. Thus, we must isolate all terms containing the variable q on one side of the equation. First, clear the fractions by multiplying both sides of the equation by the common denominator, r^2.

$$\begin{aligned} F &= \frac{kqQ}{r^2} && \text{Original equation.} \\ r^2(F) &= r^2\left(\frac{kqQ}{r^2}\right) && \text{Multiply both sides by } r^2. \\ r^2F &= kqQ && \text{Simplify. Cancel } r^2\text{'s.} \end{aligned}$$

Note that all the terms comtaining q, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by kQ to complete the solution.

$$\begin{aligned} \frac{r^2F}{kQ} &= \frac{kqQ}{kQ} && \text{Divide both sides by } kQ. \\ \frac{r^2F}{kQ} &= q \end{aligned}$$

Note that this last equation is identical to the following equation

$$q = \frac{r^2 F}{kQ}$$

Note that we have $q =$"Stuff", where "Stuff" contains no occurrences of q, the variable we are solving for.

21. We are instructed to solve the equation $P = 2W + 2L$ for W. Thus, we must isolate all terms containing the variable W on one side of the equation. First, subtract $2L$ from both sides of the equation.

$$\begin{aligned} P &= 2W + 2L && \text{Original equation.} \\ P - 2L &= 2W + 2L - 2L && \text{Subtract } 2L \text{ from both sides.} \\ P - 2L &= 2W && \text{Combine like terms.} \end{aligned}$$

Note that all the terms comtaining W, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by 2 to complete the solution.

$$\begin{aligned} \frac{P - 2L}{2} &= \frac{2W}{2} && \text{Divide both sides by 2.} \\ \frac{P - 2L}{2} &= W && \text{Simplify.} \end{aligned}$$

Note that this last equation is identical to the following equation

$$W = \frac{P - 2L}{2}$$

Note that we have $W =$"Stuff", where "Stuff" contains no occurrences of W, the variable we are solving for.

23. We are instructed to solve the equation $A = \frac{1}{2}h(b_1 + b_2)$ for h. Thus, we must isolate all terms containing the variable h on one side of the equation. First, clear the fractions by multiplying both sides of the equation by the common denominator, 2.

$$\begin{aligned} A &= \frac{1}{2}h(b_1 + b_2) && \text{Original equation.} \\ 2(A) &= 2\left(\frac{1}{2}h(b_1 + b_2)\right) && \text{Multiply both sides by 2.} \\ 2A &= h(b_1 + b_2) && \text{Simplify. Cancel 2's.} \end{aligned}$$

Note that all the terms comtaining h, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by $b_1 + b_2$ to complete the solution.

$$\frac{2A}{b_1+b_2} = \frac{h(b_1+b_2)}{b_1+b_2} \qquad \text{Divide both sides by } b_1+b_2.$$

$$\frac{2A}{b_1+b_2} = h \qquad \text{Simplify.}$$

Note that this last equation is identical to the following equation

$$h = \frac{2A}{b_1+b_2}$$

Note that we have $h =$"Stuff", where "Stuff" contains no occurrences of h, the variable we are solving for.

25. We are instructed to solve the equation $y - y_0 = m(x - x_0)$ for m. Thus, we must isolate all terms containing the variable m on one side of the equation. We begin by dividing both sides of the equation by $x - x_0$.

$$y - y_0 = m(x - x_0) \qquad \text{Original equation.}$$

$$\frac{y-y_0}{x-x_0} = \frac{m(x-x_0)}{x-x_0} \qquad \text{Divide both sides by } x - x_0.$$

$$\frac{y-y_0}{x-x_0} = m \qquad \text{Simplify.}$$

Note that this last equation is identical to the following equation

$$m = \frac{y-y_0}{x-x_0}$$

Note that we have $m =$"Stuff", where "Stuff" contains no occurrences of m, the variable we are solving for.

27. We are instructed to solve the equation $F = \dfrac{GMm}{r^2}$ for M. Thus, we must isolate all terms containing the variable M on one side of the equation. First, clear the fractions by multiplying both sides of the equation by the common denominator, r^2.

$$F = \frac{GMm}{r^2} \qquad \text{Original equation.}$$

$$r^2(F) = r^2\left(\frac{GMm}{r^2}\right) \qquad \text{Multiply both sides by } r^2.$$

$$r^2F = GMm \qquad \text{Simplify. Cancel } r^2\text{'s.}$$

Note that all the terms comtaining M, the variable we are solving for, are already isolated on one side of the equation. We need only divide both sides of the equation by Gm to complete the solution.

$$\frac{r^2F}{Gm} = \frac{GMm}{Gm} \qquad \text{Divide both sides by } Gm.$$

$$\frac{r^2F}{Gm} = M \qquad \text{Simplify.}$$

Note that this last equation is identical to the following equation:

$$M = \frac{r^2F}{Gm}$$

Note that we have $M =$"Stuff", where "Stuff" contains no occurrences of M, the variable we are solving for.

29. We are instructed to solve the equation $d = vt$ for v. Thus, we must isolate all terms containing the variable v on one side of the equation. We begin by dividing both sides of the equation by t.

$$d = vt \qquad \text{Original equation.}$$

$$\frac{d}{t} = \frac{vt}{t} \qquad \text{Divide both sides by } t.$$

$$\frac{d}{t} = v \qquad \text{Simplify.}$$

Note that this last equation is identical to the following equation

$$v = \frac{d}{t}$$

Note that we have $v =$"Stuff", where "Stuff" contains no occurrences of v, the variable we are solving for.

31. Start with the area formula and divide both sides by W.

$$A = LW$$

$$\frac{A}{W} = \frac{LW}{W}$$

$$\frac{A}{W} = L$$

Hence:

$$L = \frac{A}{W}$$

To determine the length, substitute 1073 for A and 29 for W and simplify.

$$\begin{aligned}L &= \frac{1073}{29}\\ L &= 37\end{aligned}$$

Hence, the length is $L = 37$ meters.

33. Start with the area formula, then divide both sides by h.

$$\begin{aligned}A &= bh\\ \frac{A}{h} &= \frac{bh}{h}\\ \frac{A}{h} &= b\end{aligned}$$

Hence:

$$b = \frac{A}{h}$$

Next, substitute 2418 for A, 31 for h, and simplify.

$$\begin{aligned}b &= \frac{2418}{31}\\ b &= 78\end{aligned}$$

Hence, $b = 78$ feet.

35. Start with the area formula, then clear the equation of fractions by multiplying both sides of the equation by 2.

$$\begin{aligned}A &= \frac{1}{2}\,bh\\ 2[A] &= \left[\frac{1}{2}\,bh\right]2\\ 2A &= bh\end{aligned}$$

Divide both sides by h.

$$\begin{aligned}\frac{2A}{h} &= \frac{bh}{h}\\ \frac{2A}{h} &= b\end{aligned}$$

Hence:

$$b = \frac{2A}{h}$$

Next, substitute 1332 for A, 36 for h, and simplify.

$$\begin{aligned} b &= \frac{2(1332)}{36} \\ b &= \frac{2664}{36} \\ b &= 74 \end{aligned}$$

Hence, $b = 74$ inches.

37. We need to isolate terms containing W on one side of the equation. Start with the perimeter equation and subtract $2L$ from both sides.

$$\begin{aligned} P &= 2W + 2L \\ P - 2L &= 2W + 2L - 2L \\ P - 2L &= 2W \end{aligned}$$

Divide both sides by 2 and simplify.

$$\begin{aligned} \frac{P - 2L}{2} &= \frac{2W}{2} \\ \frac{P - 2L}{2} &= W \end{aligned}$$

Hence:

$$W = \frac{P - 2L}{2}$$

Substitute 256 for P, 73 for L, then simplify.

$$\begin{aligned} W &= \frac{256 - 2(73)}{2} \\ W &= \frac{110}{2} \\ W &= 55 \end{aligned}$$

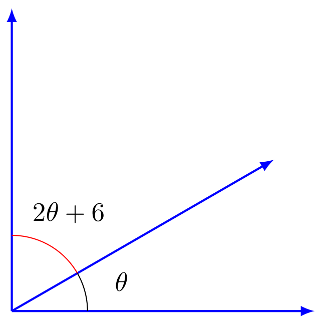
Hence, the width is $W = 55$ meters.

2.5 Applications

1. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let θ represent the measure of the first angle.

2. *Set up an equation.* A sketch will help summarize the information given in the problem. First, we sketch two angles whose sum is 90 degrees. The second angle is 6 degrees larger than 2 times the first angle, so the second angle has measure $2\theta + 6$.



The angles are complementary, so their sum is 90 degrees. Thus the equation is:

$$\theta + (2\theta + 6) = 90$$

3. *Solve the equation.* Simplify the left-hand side by combining like terms.

$$\begin{aligned} \theta + (2\theta + 6) &= 90 \\ 3\theta + 6 &= 90 \end{aligned}$$

Subtract 6 from both sides of the equation, then divide both sides of the resulting equation by 3.

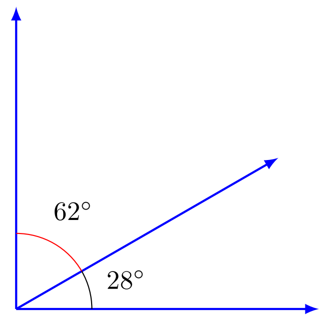
$$\begin{aligned} 3\theta + 6 - 6 &= 90 - 6 \\ 3\theta &= 84 \\ \frac{3\theta}{3} &= \frac{84}{3} \\ \theta &= 28 \end{aligned}$$

4. *Answer the question.* To find the second angle, substitute 28 for θ in $2\theta + 6$ to get:

$$\begin{aligned} 2\theta + 6 &= 2(28) + 6 \\ &= 62 \end{aligned}$$

Hence, the two angles are 28 and 62 degrees.

5. *Look back.* Let's label the angles with their numerical values.

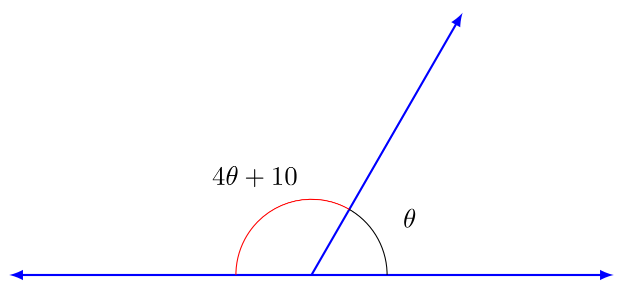


Clearly, their sum is 90°, so we have the correct answer.

3. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let θ represent the measure of the first angle.

2. *Set up an equation.* A sketch will help summarize the information given in the problem. First, we sketch two angles whose sum is 180 degrees. The second angle is 10 degrees larger than 4 times the first angle, so the second angle has measure $4\theta + 10$.



The angles are supplementary, so their sum is 180 degrees. Thus the equation is:

$$\theta + (4\theta + 10) = 180$$

3. *Solve the equation.* Simplify the left-hand side by combining like terms.

$$\begin{aligned} \theta + (4\theta + 10) &= 180 \\ 5\theta + 10 &= 180 \end{aligned}$$

Subtract 10 from both sides of the equation, then divide both sides of the resulting equation by 5.

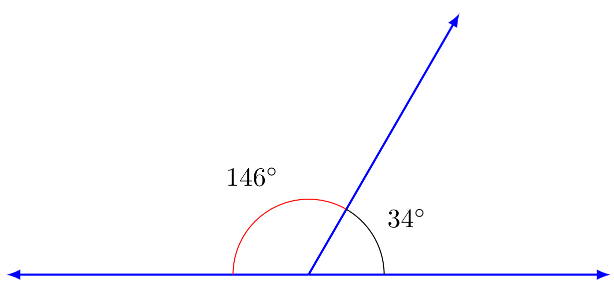
$$\begin{aligned} 5\theta + 10 - 10 &= 180 - 10 \\ 5\theta &= 170 \\ \frac{5\theta}{5} &= \frac{170}{5} \\ \theta &= 34 \end{aligned}$$

4. *Answer the question.* To find the second angle, substitute 34 for θ in $4\theta + 10$ to get:

$$\begin{aligned} 4\theta + 10 &= 4(34) + 10 \\ &= 146 \end{aligned}$$

Hence, the two angles are 34 and 146 degrees.

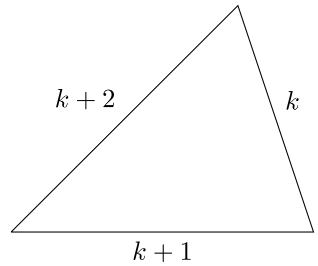
5. *Look back.* Let's label the angles with their numerical values.



Clearly, their sum is 180°, so we have the correct answer.

5. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* An example of three consecutive integers is 19, 20, and 21. These are not the integers we seek, but they do give us some sense of the meaning of three consecutive integers. Note that each consecutive integer is one larger than the preceding integer. Thus, if k is the length of the first side of the triangle, then the next two sides are $k + 1$ and $k + 2$. In this example, our variable dictionary will take the form of a well-labeled figure.



2. *Set up an Equation.* The perimeter of the triangle is the sum of the three sides. If the perimeter is 483 meters, then:

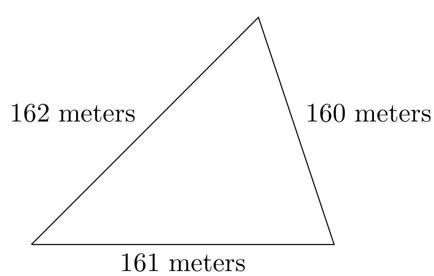
$$k + (k+1) + (k+2) = 483$$

3. *Solve the Equation.* To solve for k, first simplify the left-hand side of the equation by combining like terms.

$$\begin{aligned}
k + (k+1) + (k+2) &= 483 && \text{Original equation.}\\
3k + 3 &= 483 && \text{Combine like terms.}\\
3k + 3 - 3 &= 483 - 3 && \text{Subtract 3 from both sides.}\\
3k &= 480 && \text{Simplify.}\\
\frac{3k}{3} &= \frac{480}{3} && \text{Divide both sides by 3.}\\
k &= 160 && \text{Simplify.}
\end{aligned}$$

4. *Answer the Question.* Thus, the first side has length 160 meters. Because the next two consecutive integers are $k+1 = 161$ and $k+2 = 162$, the three sides of the triangle measure 160, 161, and 162 meters, respectively.

5. *Look Back.* An image helps our understanding. The three sides are consecutive integers.



Note that the perimeter (sum of the three sides) is:

$$160\,\text{meters} + 161\,\text{meters} + 162\,\text{meters} = 483\,\text{meters} \tag{2.1}$$

Thus, the perimeter is 483 meters, as it should be. Our solution is correct.

7. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let x represent the unknown number.

2. *Set up an equation.* The statement "four less than eight times a certain number is -660" becomes the equation:

$$8x - 4 = -660$$

3. *Solve the equation.* Add 4 to both sides, then divide the resulting equation by 8.

$$\begin{aligned} 8x - 4 &= -660 \\ 8x - 4 + 4 &= -660 + 4 \\ 8x &= -656 \\ \frac{8x}{8} &= \frac{-656}{8} \\ x &= -82 \end{aligned}$$

4. *Answer the question.* The unknown number is -82.

5. *Look back.* "Four less than eight times -82" translates as $8(-82) - 4$, which equals -660. The solution make sense.

9. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let d represent the distance left for Alan to hike. Because Alan is four times further from the beginning of the trail than the end, the distance Alan has already completed is $4d$. Let's construct a little table to help summarize the information provided in this problem.

Section of Trail	Distance (mi)
Distance to finish	d
Distance from start	$4d$
Total distance	70

2. *Set up an Equation.* As you can see in the table above, the second column shows that the sum of the two distances is 70 miles. In symbols:

$$d + 4d = 70$$

3. *Solve the Equation.* To solve for d, first simplify the left-hand side of the equation by combining like terms.

$$\begin{aligned} d + 4d &= 70 && \text{Original equation.} \\ 5d &= 70 && \text{Combine like terms.} \\ \frac{5d}{5} &= \frac{70}{5} && \text{Divide both sides by 5.} \\ d &= 14 && \text{Simplify.} \end{aligned}$$

4. *Answer the Question.* Alan still has 14 miles to hike.

5. *Look Back.* Because the amount left to hike is $d = 14$ miles, Alan's distance from the start of the trail is $4d = 4(14)$, or 56 miles. If we arrange these results in tabular form, it is evident that not only is the distance from the start of the trail four times that of the distance left to the finish, but also the sum of their lengths is equal to the total length of the trail.

Section of Trail	Distance (mi)	Distance (mi)
Distance to finish	d	14
Distance from start	$4d$	56
Total distance	70	70

Thus, we have the correct solution.

11. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let p represent the percentage of Martha 's sixth grade class that is absent.

2. *Set up the equation.* The question is "what percent of the class size equals the number of students absent?" The phrase "p percent of 36 is 2" becomes the equation:

$$p \times 36 = 2$$

Or equivalently:

$$36p = 2$$

3. *Solve the equation.* Use a calculator to help divide both sides of the equation by 36.

$$\begin{aligned} 36p &= 2 \\ \frac{36p}{36} &= \frac{2}{36} \\ p &= 0.0555555556 \end{aligned}$$

4. *Answer the question.* We need to change our answer to a percent. First, round the percentage answer p to the nearest hundredth.

Test digit

$$0.0\boxed{5}\,\boxed{5}\,5555556$$

Rounding digit

Because the test digit is greater than or equal to 5, add 1 to the rounding digit, then truncate. Hence, to the nearest hundredth, 0.0555555556 is approximately 0.06. To change this answer to a percent, multiply by 100, or equivalently, move the decimal two places to the right. Hence, 6% of Martha's sixth grade class is absent.

5. *Look back.* If we take 6% of Martha's class size, we get:

$$\begin{aligned} 6\% \times 36 &= 0.06 \times 36 \\ &= 2.16 \end{aligned}$$

Rounded to the nearest student, this means there are 2 students absent, indicating we've done the problem correctly.

13. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let x represent the length of the first piece.

2. *Set up an equation.* The second piece is 3 times as long as the first piece, so the second piece has length $3x$. The third piece is 6 centimeters longer than the first piece, so the second piece has length $x + 6$. Let's construct a table to summarize the information provided in this problem.

Piece	Length (centimeters)
First	x
Second	$3x$
Third	$x + 6$
Total length	211

As you can see in the table above, the second column shows that the sum of the three pieces is 211 centimeters. Hence, the equation is:

$$x + 3x + (x + 6) = 211$$

3. *Solve the equation.* First, simplify the left-hand side of the equation by combining like terms.

$$\begin{aligned} x + 3x + (x + 6) &= 211 \\ 5x + 6 &= 211 \end{aligned}$$

Subtract 6 from both sides of the equation, then divide both sides of the resulting equation by 5.

$$\begin{aligned} 5x + 6 - 6 &= 211 - 6 \\ 5x &= 205 \\ \frac{5x}{5} &= \frac{205}{5} \\ x &= 41 \end{aligned}$$

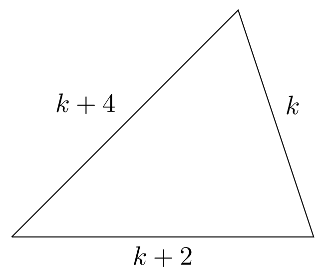
4. *Answer the question.* Let's add a column to our table to list the length of the three pieces. The lengths of the second and third pieces are found by substituting 41 for x in $3x$ and $x + 6$.

Piece	Length (centimeters)	Length (centimeters)
First	x	41
Second	$3x$	123
Third	$x + 6$	47
Total length	211	211

5. *Look back.* The third column of the table above shows that the lengths sum to 211 centimeters, so we have the correct solution.

15. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* An example of three consecutive even integers is 18, 20, and 22. These are not the integers we seek, but they do give us some sense of the meaning of three consecutive even integers. Note that each consecutive even integer is two larger than the preceding integer. Thus, if k is the length of the first side of the triangle, then the next two sides are $k+2$ and $k+4$. In this example, our variable dictionary will take the form of a well-labeled figure.



2. *Set up an Equation.* The perimeter of the triangle is the sum of the three sides. If the perimeter is 450 yards, then:

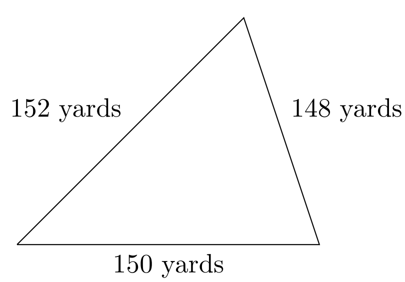
$$k + (k+2) + (k+4) = 450$$

3. *Solve the Equation.* To solve for k, first simplify the left-hand side of the equation by combining like terms.

$$\begin{aligned}
k + (k+2) + (k+4) &= 450 && \text{Original equation.}\\
3k + 6 &= 450 && \text{Combine like terms.}\\
3k + 6 - 6 &= 450 - 6 && \text{Subtract 6 from both sides.}\\
3k &= 444 && \text{Simplify.}\\
\frac{3k}{3} &= \frac{444}{3} && \text{Divide both sides by 3.}\\
k &= 148 && \text{Simplify.}
\end{aligned}$$

4. *Answer the Question.* Thus, the first side has length 148 yards. Because the next two consecutive even integers are $k+2 = 150$ and $k+4 = 152$, the three sides of the triangle measure 148, 150, and 152 yards, respectively.

5. *Look Back.* An image helps our understanding. The three sides are consecutive even integers.



Note that the perimeter (sum of the three sides) is:

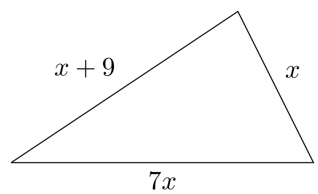
$$148\,\text{yards} + 150\,\text{yards} + 152\,\text{yards} = 450\,\text{yards} \tag{2.2}$$

Thus, the perimeter is 450 yards, as it should be. Our solution is correct.

17. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let x represent the length of the first side of the triangle.

2. *Set up an equation.* The second side is 7 times as long as the first side, so the second side has length $7x$. The third side is 9 yards longer than the first side, so the second side has length $x + 9$. Let's sketch a diagram to summarize the information provided in this problem (the sketch is not drawn to scale).



   The sum of the three sides of the triangle equals the perimeter. Hence, the equation is:

$$x + 7x + (x + 9) = 414$$

3. *Solve the equation.* First, simplify the left-hand side of the equation by combining like terms.

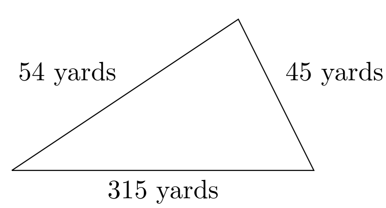
$$\begin{aligned} x + 7x + (x + 9) &= 414 \\ 9x + 9 &= 414 \end{aligned}$$

   Subtract 9 from both sides of the equation, then divide both sides of the resulting equation by 9.

$$\begin{aligned} 9x + 9 - 9 &= 414 - 9 \\ 9x &= 405 \\ \frac{9x}{9} &= \frac{405}{9} \\ x &= 45 \end{aligned}$$

4. *Answer the question.* Because the first side is $x = 45$ yards, the second side is $7x = 315$ yards, and the third side is $x + 9 = 54$ yards.

5. *Look back.* Let's add the lengths of the three sides to our sketch.



Our sketch clearly indicates that the perimeter of the triangle is Perimeter = $45 + 315 + 54$, or 414 yards. Hence, our solution is correct.

19. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let k represent the smallest of three consecutive odd integers.

2. *Set up an equation.* Because k is the smallest of three consecutive odd integers, the next two consecutive odd integers are $k + 2$ and $k + 4$. Therefore, the statement "the sum of three consecutive odd integers is -543" becomes the equation:

$$k + (k + 2) + (k + 4) = -543$$

3. *Solve the equation.* First, combine like terms on the left-hand side of the equaton.

$$\begin{aligned} k + (k + 2) + (k + 4) &= -543 \\ 3k + 6 &= -543 \end{aligned}$$

Subtract 6 from both sides, then divide both sides of the resulting equation by 3.

$$\begin{aligned} 3k + 6 - 6 &= -543 - 6 \\ 3k &= -549 \\ \frac{3k}{3} &= \frac{-549}{3} \\ k &= -183 \end{aligned}$$

4. *Answer the question.* The smallest of three consecutive odd integers is -183.

5. *Look back.* Because the smallest of three consecutive odd integers is -183, the next two consecutive odd integers are -181, and -179. If we sum these integers, we get

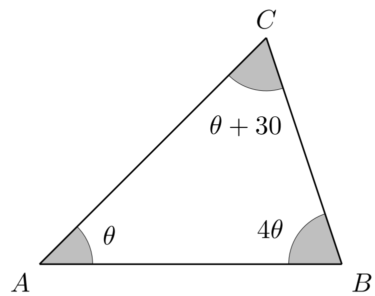
$$-183 + (-181) + (-179) = -543,$$

so our solution is correct.

21. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let θ represent the measure of angle A.

2. *Set up an equation.* A sketch will help summarize the information given in the problem. Because angle B is 4 times the size of angle A, the degree measure of angle B is represented by 4θ. Because angle C is 30 degrees larger than the degree measure of angle A, the degree measure of angle C is represented by $\theta + 30$.



Because the sum of the three angles is 180°, we have the following equation:

$$\theta + 4\theta + (\theta + 30) = 180$$

3. *Solve the equation.* Start by combining like terms on the left-hand side of the equation.

$$\begin{aligned} \theta + 4\theta + (\theta + 30) &= 180 \\ 6\theta + 30 &= 180 \end{aligned}$$

Subtract 30 from both sides of the equation and simplify.

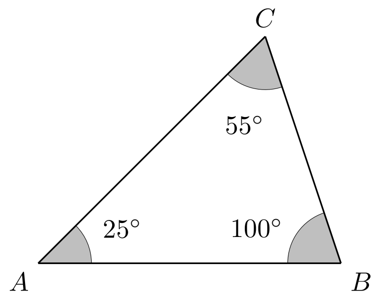
$$\begin{aligned} 6\theta + 30 - 30 &= 180 - 30 \\ 6\theta &= 150 \end{aligned}$$

Divide both sides by 6.

$$\begin{aligned} \frac{6\theta}{6} &= \frac{150}{6} \\ \theta &= 25 \end{aligned}$$

4. *Answer the question.* The degree measure of angle A is $\theta = 25°$. The degree measure of angle B is $4\theta = 100°$. The degree measure of angle C is $\theta + 30 = 55°$.

5. *Look back.* Our figure now looks like the following.



Note that

$$25 + 100 + 55 = 180,$$

so our solution is correct.

23. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let k represent the smallest of three consecutive integers.

2. *Set up an equation.* Because k is the smallest of three consecutive integers, the next two consecutive integers are $k + 1$ and $k + 2$. Therefore, the statement "the sum of three consecutive integers is -384" becomes the equation:

$$k + (k + 1) + (k + 2) = -384$$

3. *Solve the equation.* First, combine like terms on the left-hand side of the equaton.

$$\begin{aligned} k + (k + 1) + (k + 2) &= -384 \\ 3k + 3 &= -384 \end{aligned}$$

Subtract 3 from both sides, then divide both sides of the resulting equation by 3.

$$\begin{aligned} 3k + 3 - 3 &= -384 - 3 \\ 3k &= -387 \\ \frac{3k}{3} &= \frac{-387}{3} \\ k &= -129 \end{aligned}$$

4. *Answer the question.* The smallest of three consecutive integers is $k = -129$, so the next two consecutive integers are -128 and -127. Therefore, the largest of the three consecutive integers is -127.

5. *Look back.* If we sum the integers -129, -128, and -127, we get

$$-129 + (-128) + (-127) = -384,$$

so our solution is correct.

25. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let x represent the unknown number.

2. *Set up an equation.* The statement "seven more than two times a certain number is 181" becomes the equation:

$$7 + 2x = 181$$

3. *Solve the equation.* Subtract 7 from both sides, then divide the resulting equation by 2.

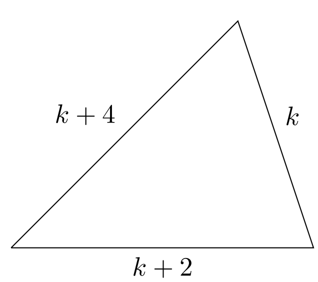
$$\begin{aligned} 7 + 2x &= 181 \\ 7 + 2x - 7 &= 181 - 7 \\ 2x &= 174 \\ \frac{2x}{2} &= \frac{174}{2} \\ x &= 87 \end{aligned}$$

4. *Answer the question.* The unknown number is 87.

5. *Look back.* "Seven more than two times 87" translates as $7+2(87)$, which equals 181. The solution make sense.

27. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* An example of three consecutive odd integers is 19, 21, and 23. These are not the integers we seek, but they do give us some sense of the meaning of three consecutive odd integers. Note that each consecutive odd integer is two larger than the preceding integer. Thus, if k is the length of the first side of the triangle, then the next two sides are $k+2$ and $k+4$. In this example, our variable dictionary will take the form of a well-labeled figure.



2. *Set up an Equation.* The perimeter of the triangle is the sum of the three sides. If the perimeter is 537 feet, then:

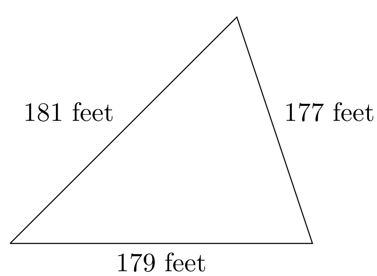
$$k + (k + 2) + (k + 4) = 537$$

3. *Solve the Equation.* To solve for k, first simplify the left-hand side of the equation by combining like terms.

$$\begin{aligned} k + (k + 2) + (k + 4) &= 537 && \text{Original equation.} \\ 3k + 6 &= 537 && \text{Combine like terms.} \\ 3k + 6 - 6 &= 537 - 6 && \text{Subtract 6 from both sides.} \\ 3k &= 531 && \text{Simplify.} \\ \frac{3k}{3} &= \frac{531}{3} && \text{Divide both sides by 3.} \\ k &= 177 && \text{Simplify.} \end{aligned}$$

4. *Answer the Question.* Thus, the first side has length 177 feet. Because the next two consecutive odd integers are $k+2 = 179$ and $k+4 = 181$, the three sides of the triangle measure 177, 179, and 181 feet, respectively.

5. *Look Back.* An image helps our understanding. The three sides are consecutive odd integers.



Note that the perimeter (sum of the three sides) is:

$$177\,\text{feet} + 179\,\text{feet} + 181\,\text{feet} = 537\,\text{feet} \tag{2.3}$$

Thus, the perimeter is 537 feet, as it should be. Our solution is correct.

29. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let M represent the marked price of the article.

2. *Solve the equation.* Because the store offers a 14% discount, Yao pays 86% for the article. Thus, the question becomes "86% of the marked price is $670.8." This translates into the equation

$$86\% \times M = 670.8,$$

or equivalently,

$$0.86M = 670.8$$

Use a calculator to help divide both sides by 0.86.

$$\begin{aligned} \frac{0.86M}{0.86} &= \frac{670.8}{0.86} \\ M &= 780 \end{aligned}$$

3. *Answer the question.* Hence, the original marked price was $780.

4. *Look back.* Because the store offers a 14% discount, Yao has to pay 86% for the article. Check what 86% of the marked price will be.

$$\begin{aligned} 86\% \times 780 &= 0.86 \times 780 \\ &= 670.8 \end{aligned}$$

That's the sales price that Yao paid. Hence, we've got the correct solution.

31. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let k represent the smallest of three consecutive even integers.

2. *Set up an equation.* Because k is the smallest of three consecutive even integers, the next two consecutive even integers are $k + 2$ and $k + 4$. Therefore, the statement "the sum of three consecutive even integers is -486" becomes the equation:

$$k + (k + 2) + (k + 4) = -486$$

3. *Solve the equation.* First, combine like terms on the left-hand side of the equaton.

$$\begin{aligned} k + (k + 2) + (k + 4) &= -486 \\ 3k + 6 &= -486 \end{aligned}$$

Subtract 6 from both sides, then divide both sides of the resulting equation by 3.

$$\begin{aligned} 3k + 6 - 6 &= -486 - 6 \\ 3k &= -492 \\ \frac{3k}{3} &= \frac{-492}{3} \\ k &= -164 \end{aligned}$$

4. *Answer the question.* The smallest of three consecutive even integers is -164.

5. *Look back.* Because the smallest of three consecutive even integers is -164, the next two consecutive even integers are -162, and -160. If we sum these integers, we get

$$-164 + (-162) + (-160) = -486,$$

so our solution is correct.

33. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let M represent the amount invested in the mutal fund.

2. *Set up the equation.* We'll use a table to help summarize the information in this problem. Because the amount invested in the certificate of deposit is \$3,500 more than 6 times the amount invested in the mutual fund, we represent the amount invested in the certificate of deposit with the expression $6M + 3500$.

Investment	Amount invested
Mutual fund	M
Certificate of deposit	$6M + 3500$
Totals	45500

The second column of the table gives us the needed equation. The two investment amounts must total \$45,500.

$$M + (6M + 3500) = 45500$$

3. *Solve the equation.* To solve the equation, first combine like terms on the left-hand side of the equation.

$$\begin{aligned} M + (6M + 3500) &= 45500 \\ 7M + 3500 &= 45500 \end{aligned}$$

Subtract 3500 from both sides of the equation and simplify.

$$\begin{aligned} 7M + 3500 - 3500 &= 45500 - 3500 \\ 7M &= 42000 \end{aligned}$$

Divide both sides of the equation by 7.

$$\begin{aligned} \frac{7M}{7} &= \frac{42000}{7} \\ M &= 6000 \end{aligned}$$

4. *Answer the question.* The amount invested in the mutual fund is $M = \$6,000$. The amount invested in the certificate of deposit is:

$$\begin{aligned} \text{Certificate of deposit} &= 6M + 3500 \\ &= 6(6000) + 3500 \\ &= 36000 + 3500 \\ &= 39500 \end{aligned}$$

Hence, the amount invested in the certificate of deposit is \$39,500.

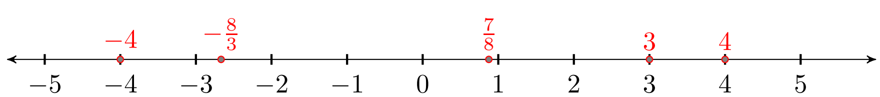
5. *Look back.* Note that the two amounts total

$$6000 + 39500 = 45500,$$

so we have the correct solution.

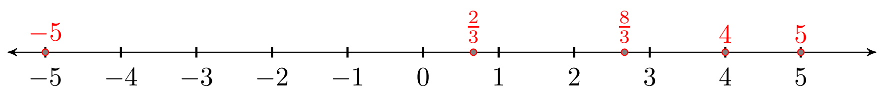
2.6 Inequalities

1. Draw a number line, indicating the scale below the line. For each value 4, 3, -4, $7/8$, and $-8/3$, plot the corresponding point on the line and label it with its value.



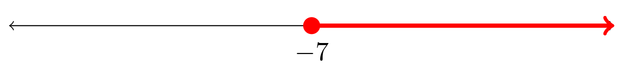
The position of the numbers on the number line give us the ordering from smallest to largest: -4, $-8/3$, $7/8$, 3, and 4.

3. Draw a number line, indicating the scale below the line. For each value -5, 5, 4, 2/3, and 8/3, plot the corresponding point on the line and label it with its value.

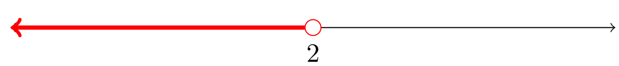


The position of the numbers on the number line give us the ordering from smallest to largest: -5, 2/3, 8/3, 4, and 5.

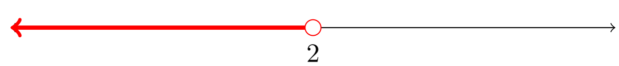
5. $\{x : x \geq -7\}$ is read "the set of all x such that x is greater than or equal to -7"; that is, the set of all x that lie to the "right of" -7 or including -7 on the number line. Note that this does include the number -7.



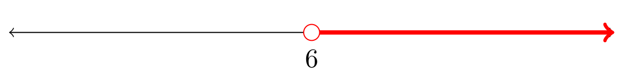
7. $\{x : x < 2\}$ is read "the set of all x such that x is less than 2"; that is, the set of all x that lie to the "left of" 2 on the number line. Note that this does not include the number 2.



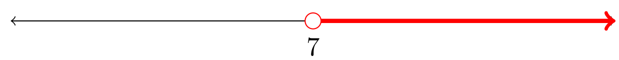
9. $(-\infty, 2) = \{x : x < 2\}$, and so it is read "the set of all x such that x is less than 2"; that is, the set of all x that lie to the "left of" 2 on the number line. Note that this does not include the number 2.



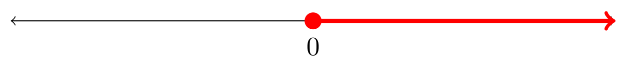
11. $(6, \infty) = \{x : x > 6\}$, and so it is read "the set of all x such that x is greater than 6"; that is, the set of all x that lie to the "right of" 6 on the number line. Note that this does not include the number 6.



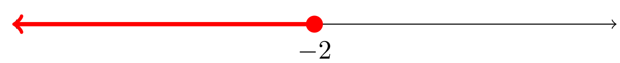
13. $\{x : x < 7\}$ is read "the set of all x such that x is greater than 7"; that is, the set of all x that lie to the "right of" 7 on the number line. Note that this does not include the number 7.



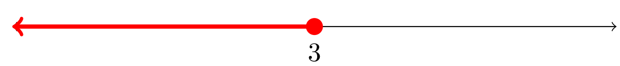
15. $[0, \infty) = \{x : x \geq 0\}$, and so it is read "the set of all x such that x is greater than or equal to 0"; that is, the set of all x that lie to the "right of" 0 or including 0 on the number line. Note that this does include the number 0.



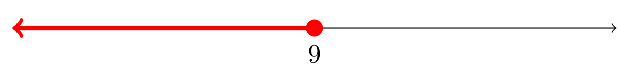
17. $\{x : x \leq -2\}$ is read "the set of all x such that x is less than or equal to -2"; that is, the set of all x that lie to the "left of" -2 or including -2 on the number line. Note that this does include the number -2.



19. $(-\infty, 3] = \{x : x \leq 3\}$, and so it is read "the set of all x such that x is less than or equal to 3"; that is, the set of all x that lie to the "left of" 3 or including 3 on the number line. Note that this does include the number 3.

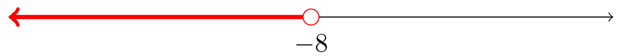


21. Consider the shaded region on the given number line.



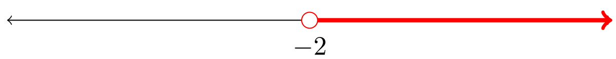
Note that every number to the left of 9, including 9, (i.e., "less than or equal to 9") is shaded. Using set-builder notation, the shaded region is described by $\{x : x \leq 9\}$.

23. Consider the shaded region on the given number line.



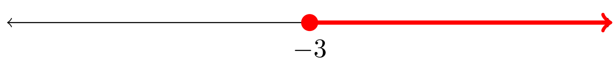
Note that every number to the left of -8 (i.e., "less than -8") is shaded. Using set-builder notation, the shaded region is described by $\{x : x < -8\}$.

25. Consider the shaded region on the given number line.



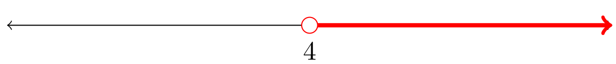
Note that every number to the right of -2 (i.e., "greater than -2") is shaded. Using set-builder notation, the shaded region is described by $\{x : x > -2\}$.

27. Consider the shaded region on the given number line.



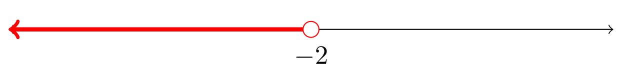
Note that every number to the right of -3, including -3, (i.e., "greater than or equal to -3") is shaded. Using set-builder notation, the shaded region is described by $\{x : x \geq -3\}$.

29. Consider the shaded region on the given number line.



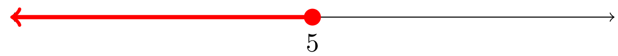
Note that every number to the right of 4 (i.e., "greater than 4") is shaded. Using set-builder notation, the shaded region is described by $(4, \infty)$.

31. Consider the shaded region on the given number line.



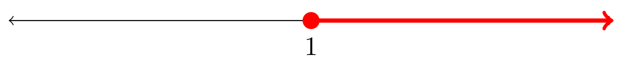
Note that every number to the left of -2 (i.e., "less than -2") is shaded. Using interval notation, the shaded region is described by $(-\infty, -2)$.

33. Consider the shaded region on the given number line.



Note that every number to the left of 5, including 5, (i.e., "less than or equal to 5") is shaded. Using interval notation, the shaded region is described by $(-\infty, 5]$.

35. Consider the shaded region on the given number line.

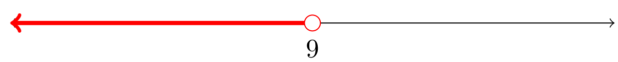


Note that every number to the right of 1, including 1, (i.e., "greater than or equal to 1") is shaded. Using interval notation, the shaded region is described by $[1, \infty)$.

37. To "undo" adding 10, we subtract 10 from both sides of the inequality.

$$\begin{aligned} x + 10 &< 19 && \text{Original inquality.} \\ x + 10 - 10 &< 19 - 10 && \text{Subtract 10 from both sides.} \\ x &< 9 && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "less than" or "left of" 9 on a number line.



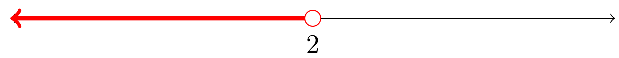
Thus, using set-builder and interval notation, the solution is:

$$\{x : x < 9\} = (-\infty, 9)$$

39. To "undo" multiplying by 4, we divide both sides of the inequality by 4. Because we are dividing by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} 4x &< 8 && \text{Original inquality.} \\ \frac{4x}{4} &< \frac{8}{4} && \text{Divide both sides by 4.} \\ x &< 2 && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "less than" or "left of" 2 on a number line.



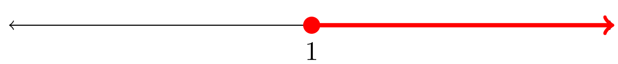
Thus, using set-builder and interval notation, the solution is:

$$\{x : x < 2\} = (-\infty, 2)$$

41. To "undo" multiplying by -2, we divide both sides of the inequality by -2. Because we are dividing by a negative number, we reverse the inequality sign.

$$\begin{aligned} -2x &\le -2 && \text{Original inquality.} \\ \frac{-2x}{-2} &\ge \frac{-2}{-2} && \text{Divide both sides by } -2 \\ & && \text{and reverse the inequality sign.} \\ x &\ge 1 && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" 1 on a number line.



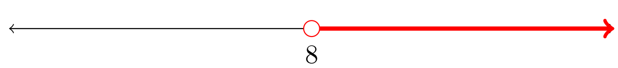
Thus, using set-builder and interval notation, the solution is:

$$\{x : x \ge 1\} = [1, \infty)$$

43. To "undo" subtracting 18, we add 18 to both sides of the inequality.

$$\begin{aligned} x - 18 &> -10 && \text{Original inquality.} \\ x - 18 + 18 &> -10 + 18 && \text{Add 18 to both sides.} \\ x &> 8 && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "greater than" or "right of" 8 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x > 8\} = (8, \infty)$$

45. We need to isolate terms containing x on one side of the inequality. We begin by adding $9x$ to both sides of the inequality.

$$\begin{aligned} -5x - 6 &\ge 4 - 9x && \text{Original inquality.} \\ -5x - 6 + 9x &\ge 4 - 9x + 9x && \text{Add } 9x \text{ to both sides.} \\ 4x - 6 &\ge 4 && \text{Simplify both sides.} \end{aligned}$$

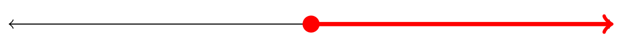
We continue to isolate terms containing x on one side of the inequality. We next add 6 to both sides of the inequality.

$$\begin{aligned} 4x - 6 + 6 &\ge 4 + 6 && \text{Add 6 to both sides.} \\ 4x &\ge 10 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 4, divide both sides by 4. Since we are dividing both sides by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{4x}{4} &\ge \frac{10}{4} && \text{Divide both sides by 4.} \\ x &\ge \frac{5}{2} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" 5/2 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \ge 5/2\} = [5/2, \infty)$$

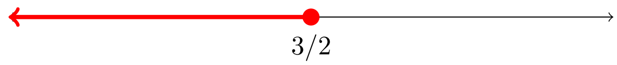
47. To "undo" subtracting 6, add 6 to both sides of the inequality.

$$\begin{aligned} 16x - 6 &\le 18 && \text{Original inquality.} \\ 16x - 6 + 6 &\le 18 + 6 && \text{Add 6 to both sides.} \\ 16x &\le 24 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 16, divide both sides by 16. Since we are dividing both sides by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{16x}{16} &\le \frac{24}{16} && \text{Divide both sides by 16.} \\ x &\le \frac{3}{2} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "less than or equal to" or "left of or including" 3/2 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \leq 3/2\} = (-\infty, 3/2]$$

49. We need to isolate terms containing x on one side of the inequality. We begin by adding $4x$ to both sides of the inequality.

$$\begin{aligned} -14x - 6 &\geq -10 - 4x && \text{Original inquality.} \\ -14x - 6 + 4x &\geq -10 - 4x + 4x && \text{Add } 4x \text{ to both sides.} \\ -10x - 6 &\geq -10 && \text{Simplify both sides.} \end{aligned}$$

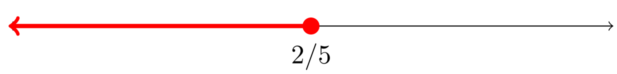
We continue to isolate terms containing x on one side of the inequality. We next add 6 to both sides of the inequality.

$$\begin{aligned} -10x - 6 + 6 &\geq -10 + 6 && \text{Add 6 to both sides.} \\ -10x &\geq -4 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -10, divide both sides by -10. Since we are dividing both sides by a negative number, we reverse the inequality sign.

$$\begin{aligned} \frac{-10x}{-10} &\leq \frac{-4}{-10} && \text{Divide both sides by } -10 \\ & && \text{and reverse the inequality sign.} \\ x &\leq \frac{2}{5} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "less than or equal to" or "left of or including" 2/5 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \leq 2/5\} = (-\infty, 2/5]$$

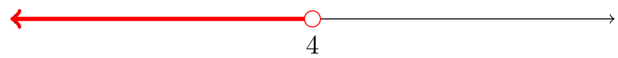
51. To "undo" adding 18, subtract 18 from both sides of the inequality.

$$\begin{aligned} 5x + 18 &< 38 && \text{Original inquality.} \\ 5x + 18 - 18 &< 38 - 18 && \text{Subtract 18 from both sides.} \\ 5x &< 20 && \text{Simplify both sides.} \end{aligned}$$

To “undo” multiplying by 5, divide both sides by 5. Since we are dividing both sides by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{5x}{5} &< \frac{20}{5} && \text{Divide both sides by 5.} \\ x &< 4 && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are “less than” or “left of” 4 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x < 4\} = (-\infty, 4)$$

53. We need to isolate terms containing x on one side of the inequality. We begin by adding $6x$ to both sides of the inequality.

$$\begin{aligned} -16x - 5 &\geq -11 - 6x && \text{Original inquality.} \\ -16x - 5 + 6x &\geq -11 - 6x + 6x && \text{Add } 6x \text{ to both sides.} \\ -10x - 5 &\geq -11 && \text{Simplify both sides.} \end{aligned}$$

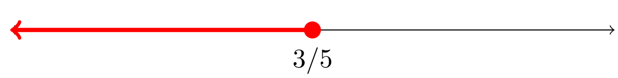
We continue to isolate terms containing x on one side of the inequality. We next add 5 to both sides of the inequality.

$$\begin{aligned} -10x - 5 + 5 &\geq -11 + 5 && \text{Add 5 to both sides.} \\ -10x &\geq -6 && \text{Simplify both sides.} \end{aligned}$$

To “undo” multiplying by -10, divide both sides by -10. Since we are dividing both sides by a negative number, we reverse the inequality sign.

$$\begin{aligned} \frac{-10x}{-10} &\leq \frac{-6}{-10} && \text{Divide both sides by } -10 \text{ and reverse the inequality sign.} \\ x &\leq \frac{3}{5} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are “less than or equal to” or “left of or including” 3/5 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \leq 3/5\} = (-\infty, 3/5]$$

55. We need to isolate terms containing x on one side of the inequality. We begin by adding $8x$ to both sides of the inequality.

$$\begin{aligned} 2x - 9 &\geq 5 - 8x && \text{Original inquality.} \\ 2x - 9 + 8x &\geq 5 - 8x + 8x && \text{Add } 8x \text{ to both sides.} \\ 10x - 9 &\geq 5 && \text{Simplify both sides.} \end{aligned}$$

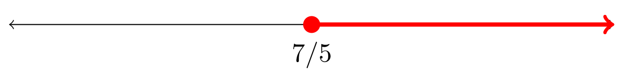
We continue to isolate terms containing x on one side of the inequality. We next add 9 to both sides of the inequality.

$$\begin{aligned} 10x - 9 + 9 &\geq 5 + 9 && \text{Add 9 to both sides.} \\ 10x &\geq 14 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 10, divide both sides by 10. Since we are dividing both sides by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{10x}{10} &\geq \frac{14}{10} && \text{Divide both sides by 10.} \\ x &\geq \frac{7}{5} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" 7/5 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \geq 7/5\} = [7/5, \infty)$$

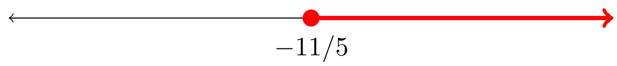
57. To "undo" subtracting 4, add 4 to both sides of the inequality.

$$\begin{aligned} -10x - 4 &\leq 18 && \text{Original inquality.} \\ -10x - 4 + 4 &\leq 18 + 4 && \text{Add 4 to both sides.} \\ -10x &\leq 22 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -10, divide both sides by -10. Since we are dividing both sides by a negative number, we reverse the inequality sign.

$$\begin{aligned} \frac{-10x}{-10} &\geq \frac{22}{-10} && \text{Divide both sides by } -10 \\ & && \text{and reverse the inequality sign.} \\ x &\geq -\frac{11}{5} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" $-11/5$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \geq -11/5\} = [-11/5, \infty)$$

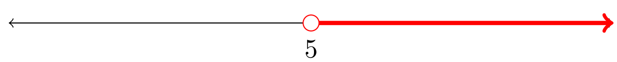
59. To "undo" adding 4, subtract 4 from both sides of the inequality.

$$\begin{aligned} -12x + 4 &< -56 && \text{Original inquality.} \\ -12x + 4 - 4 &< -56 - 4 && \text{Subtract 4 from both sides.} \\ -12x &< -60 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -12, divide both sides by -12. Since we are dividing both sides by a negative number, we reverse the inequality sign.

$$\begin{aligned} \frac{-12x}{-12} &> \frac{-60}{-12} && \text{Divide both sides by } -12 \text{ and reverse the inequality sign.} \\ x &> 5 && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "greater than" or "right of" 5 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x > 5\} = (5, \infty)$$

61. We need to isolate terms containing x on one side of the inequality. We begin by subtracting $6x$ from both sides of the inequality.

$$\begin{aligned} 15x + 5 &< 6x + 2 && \text{Original inquality.} \\ 15x + 5 - 6x &< 6x + 2 - 6x && \text{Subtract } 6x \text{ from both sides.} \\ 9x + 5 &< 2 && \text{Simplify both sides.} \end{aligned}$$

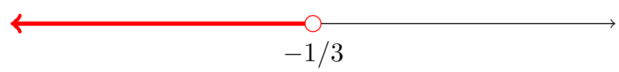
We continue to isolate terms containing x on one side of the inequality. We next subtract 5 from both sides of the inequality.

$$\begin{aligned} 9x + 5 - 5 &< 2 - 5 && \text{Subtract 5 from both sides.} \\ 9x &< -3 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 9, divide both sides by 9. Since we are dividing both sides by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{9x}{9} &< \frac{-3}{9} && \text{Divide both sides by 9.} \\ x &< -\frac{1}{3} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "less than" or "left of" $-1/3$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x < -1/3\} = (-\infty, -1/3)$$

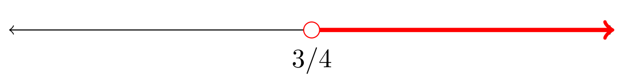
63. The common denominator is 8. Clear the fractions from the inequality by multiplying both sides of the inequality by 8.

$$\begin{aligned} \frac{3}{2}x &> \frac{9}{8} && \text{Original inequality.} \\ 8\left(\frac{3}{2}x\right) &> \left(\frac{9}{8}\right)8 && \text{Multiply both sides by 8.} \\ 12x &> 9 && \text{Cancel and multiply.} \end{aligned}$$

To "undo" multiplying by 12, divide both sides of the inequality by 12. Because we are dividing by a positive number, we do **not** reverse the inequality.

$$\begin{aligned} \frac{12x}{12} &> \frac{9}{12} && \text{Divide both sides by 12.} \\ x &> \frac{3}{4} && \text{Simplify.} \end{aligned}$$

Shade all real numbers that are "greater than" or "right of" $3/4$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x > 3/4\} = (3/4, \infty)$$

65. The common denominator is 10. We will now clear the fractions from the inequality by multiplying both sides of the inequality by 10.

$$\begin{aligned} x + \frac{3}{2} &< \frac{9}{5} && \text{Original inequality.} \\ 10\left(x + \frac{3}{2}\right) &< 10\left(\frac{9}{5}\right) && \text{Multiply both sides by 10.} \\ 10x + 10\left(\frac{3}{2}\right) &< 10\left(\frac{9}{5}\right) && \text{On the left, distribute the 10.} \\ 10x + 15 &< 18 && \text{Multiply.} \end{aligned}$$

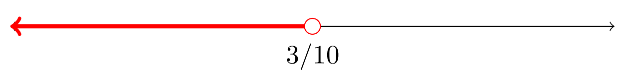
To "undo" the effect of adding 15, subtract 15 from both sides of the inequality.

$$\begin{aligned} 10x + 15 - 15 &< 18 - 15 && \text{Subtract 15 from both sides.} \\ 10x &< 3 && \text{Simplify both sides.} \end{aligned}$$

To "undo" the effect of multiplying by 10, divide both sides of the inequality by 10.

$$\begin{aligned} \frac{10x}{10} &< \frac{3}{10} && \text{Divide both sides by 10.} \\ x &< \frac{3}{10} && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "less than" or "left of" 3/10 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x < 3/10\} = (-\infty, 3/10)$$

67. Thus, using set-builder and interval notation, the solution is:

$$\{x : x \geq 5/7\} = [5/7, \infty)$$

69. The common denominator is 56. We will now clear the fractions from the inequality by multiplying both sides of the inequality by 56.

$$\begin{aligned} x - \frac{3}{8} &\geq -\frac{9}{7} && \text{Original inequality.} \\ 56\left(x - \frac{3}{8}\right) &\geq 56\left(-\frac{9}{7}\right) && \text{Multiply both sides by 56.} \\ 56x - 56\left(\frac{3}{8}\right) &\geq 56\left(-\frac{9}{7}\right) && \text{On the left, distribute the 56.} \\ 56x - 21 &\geq -72 && \text{Multiply.} \end{aligned}$$

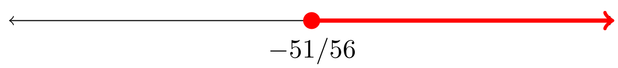
To "undo" the effect of subtracting 21, add 21 to both sides of the inequality.

$$\begin{aligned} 56x - 21 + 21 &\geq -72 + 21 && \text{Add 21 to both sides.} \\ 56x &\geq -51 && \text{Simplify both sides.} \end{aligned}$$

To "undo" the effect of multiplying by 56, divide both sides of the inequality by 56.

$$\begin{aligned} \frac{56x}{56} &\geq \frac{-51}{56} && \text{Divide both sides by 56.} \\ x &\geq -\frac{51}{56} && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" $-51/56$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \geq -51/56\} = [-51/56, \infty)$$

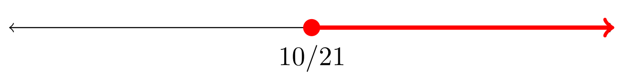
71. The common denominator is 35. Clear the fractions from the inequality by multiplying both sides of the inequality by 35.

$$\begin{aligned} -\frac{6}{5}x &\leq -\frac{4}{7} && \text{Original inequality.} \\ 35\left(-\frac{6}{5}x\right) &\leq \left(-\frac{4}{7}\right)35 && \text{Multiply both sides by 35.} \\ -42x &\leq -20 && \text{Cancel and multiply.} \end{aligned}$$

To "undo" multiplying by -42, divide both sides of the inequality by -42. Because we are dividing by a negative number, we reverse the inequality.

$$\begin{aligned} \frac{-42x}{-42} &\geq \frac{-20}{-42} && \text{Divide both sides by } -42. \\ x &\geq \frac{10}{21} && \text{Simplify.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" $10/21$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \geq 10/21\} = [10/21, \infty)$$

73. The common denominator is 45. Clear the fractions from the inequality by multiplying both sides of the inequality by 45.

$$\begin{aligned} -\frac{6}{5}x - \frac{7}{3} &\le \frac{5}{9} - \frac{2}{9} && \text{Original inequality.} \\ 45\left(-\frac{6}{5}x - \frac{7}{3}\right) &\le \left(\frac{5}{9} - \frac{2}{9}\right)45 && \text{Multiply both sides by 45.} \\ -54x - 105 &\le 25 - 10x && \text{Cancel and multiply.} \end{aligned}$$

We need to isolate all terms containing x on one side of the equation. To "undo" subtracting $10x$, add $10x$ from both sides of the inequality.

$$\begin{aligned} -54x - 105 + 10x &\le 25 - 10x + 10x && \text{Add } 10x \text{ to both sides.} \\ -44x - 105 &\le 25 && \text{Simplify both sides.} \end{aligned}$$

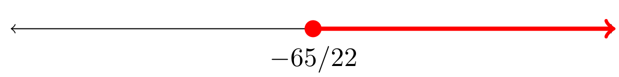
We continue to isolate terms containing x on one side of the inequality. To "undo" subtracting 105, add 105 from both sides of the inequality.

$$\begin{aligned} -44x - 105 + 105 &\le 25 + 105 && \text{Add 105 to both sides.} \\ -44x &\le 130 && \text{Simplify both sides.} \end{aligned}$$

To "undo" the effect of multiplying by -44, divide both sides of the inequality by -44. Because we are dividing both sides by a negative number, we reverse the inequality sign.

$$\begin{aligned} \frac{-44x}{-44} &\ge \frac{130}{-44} && \text{Divide both sides by } -44 \\ & && \text{and reverse the inequality sign.} \\ x &\ge -\frac{65}{22} && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" $-65/22$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \ge -65/22\} = [-65/22, \infty)$$

75. The common denominator is 14. Clear the fractions from the inequality by multiplying both sides of the inequality by 14.

$$\begin{aligned} \frac{9}{7}x + \frac{9}{2} &> \frac{1}{7}x + \frac{7}{2} && \text{Original inequality.} \\ 14\left(\frac{9}{7}x + \frac{9}{2}\right) &> \left(\frac{1}{7}x + \frac{7}{2}\right)14 && \text{Multiply both sides by 14.} \\ 18x + 63 &> 2x + 49 && \text{Cancel and multiply.} \end{aligned}$$

We need to isolate all terms containing x on one side of the equation. To "undo" adding $2x$, subtract $2x$ from both sides of the inequality.

$$\begin{aligned} 18x + 63 - 2x &> 2x + 49 - 2x && \text{Subtract } 2x \text{ from both sides.} \\ 16x + 63 &> 49 && \text{Simplify both sides.} \end{aligned}$$

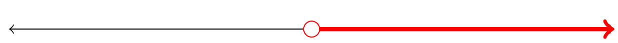
We continue to isolate terms containing x on one side of the inequality. To "undo" adding 63, subtract 63 from both sides of the inequality.

$$\begin{aligned} 16x + 63 - 63 &> 49 - 63 && \text{Subtract 63 from both sides.} \\ 16x &> -14 && \text{Simplify both sides.} \end{aligned}$$

To "undo" the effect of multiplying by 16, divide both sides of the inequality by 16. Because we are dividing both sides by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{16x}{16} &> \frac{-14}{16} && \text{Divide both sides by 16.} \\ x &> -\frac{7}{8} && \text{Simplify both sides.} \end{aligned}$$

Shade all real numbers that are "greater than" or "right of" $-7/8$ on a number line.



$-7/8$

Thus, using set-builder and interval notation, the solution is:

$$\{x : x > -7/8\} = (-7/8, \infty)$$

77. We'll first clear the decimals by multiplying both sides by 100, which will move each decimal point two places to the right.

$$\begin{aligned} -3.7x - 1.98 &\le 3.2 && \text{Original inequality.} \\ -370x - 198 &\le 320 && \text{Multiply both sides by 100.} \end{aligned}$$

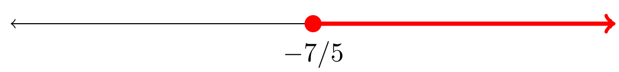
To "undo" subtracting 198, add 198 to both sides of the inequality.

$$\begin{aligned} -370x - 198 + 198 &\le 320 + 198 && \text{Add 198 to both sides.} \\ -370x &\le 518 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -370, divide both sides of the inequality by -370. Because we are dividing by a negative number, so we reverse the inequality sign.

$$\begin{aligned} \frac{-370x}{-370} &\ge \frac{518}{-370} && \text{Simplify both sides.} \\ x &\ge -\frac{7}{5} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "greater than or equal to" or "right of or including" $-7/5$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \ge -7/5\} = [-7/5, \infty)$$

79. We'll first clear the decimals by multiplying both sides by 10, which will move each decimal point one place to the right.

$$\begin{aligned} -3.4x + 3.5 &\ge 0.9 - 2.2x && \text{Original inequality.} \\ -34x + 35 &\ge 9 - 22x && \text{Multiply both sides by 10.} \end{aligned}$$

We need to isolate terms containing x on one side of the inequality. Start by adding $22x$ to both sides of the inequality.

$$\begin{aligned} -34x + 35 + 22x &\ge 9 - 22x + 22x && \text{Add } 22x \text{ to both sides.} \\ -12x + 35 &\ge 9 && \text{Simplify both sides.} \end{aligned}$$

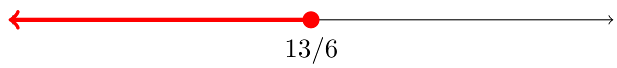
We continue to isolate terms containing x on one side of the inequality. Subtract 35 from both sides of the inequality.

$$\begin{aligned} -12x + 35 - 35 &\ge 9 - 35 && \text{Subtract 35 from both sides.} \\ -12x &\ge -26 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by -12, divide both sides of the inequality by -12. Because we are dividing by a negative number, we reverse the inequality sign.

$$\begin{aligned} \frac{-12x}{-12} &\le \frac{-26}{-12} && \text{Simplify both sides.} \\ x &\le \frac{13}{6} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "less than or equal" or "left of or including " 13/6 on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x \le 13/6\} = (-\infty, 13/6]$$

81. We'll first clear the decimals by multiplying both sides by 10, which will move each decimal point one place to the right.

$$\begin{aligned} -1.3x + 2.9 &> -2.6 - 3.3x && \text{Original inequality.} \\ -13x + 29 &> -26 - 33x && \text{Multiply both sides by 10.} \end{aligned}$$

We need to isolate terms containing x on one side of the inequality. Start by adding $33x$ to both sides of the inequality.

$$\begin{aligned} -13x + 29 + 33x &> -26 - 33x + 33x && \text{Add } 33x \text{ to both sides.} \\ 20x + 29 &> -26 && \text{Simplify both sides.} \end{aligned}$$

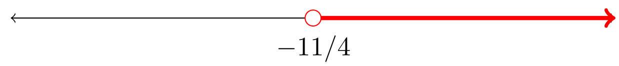
We continue to isolate terms containing x on one side of the inequality. Subtract 29 from both sides of the inequality.

$$\begin{aligned} 20x + 29 - 29 &> -26 - 29 && \text{Subtract 29 from both sides.} \\ 20x &> -55 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 20, divide both sides of the inequality by 20. Because we are dividing by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{20x}{20} &> \frac{-55}{20} && \text{Simplify both sides.} \\ x &> -\frac{11}{4} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "greater than" or "right of " $-11/4$ on a number line.



Thus, using set-builder and interval notation, the solution is:

$$\{x : x > -11/4\} = (-11/4, \infty)$$

83. We'll first clear the decimals by multiplying both sides by 10, which will move each decimal point one place to the right.

$$\begin{aligned} 2.2x + 1.9 &< -2.3 && \text{Original inequality.} \\ 22x + 19 &< -23 && \text{Multiply both sides by 10.} \end{aligned}$$

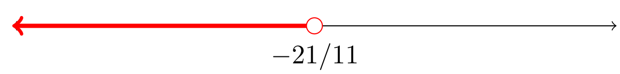
To "undo" adding 19, subtract 19 from both sides of the inequality.

$$\begin{aligned} 22x + 19 - 19 &< -23 - 19 && \text{Subtract 19 from both sides.} \\ 22x &< -42 && \text{Simplify both sides.} \end{aligned}$$

To "undo" multiplying by 22, divide both sides of the inequality by 22. Because we are dividing by a positive number, we do **not** reverse the inequality sign.

$$\begin{aligned} \frac{22x}{22} &< \frac{-42}{22} && \text{Simplify both sides.} \\ x &< -\frac{21}{11} && \text{Reduce to lowest terms.} \end{aligned}$$

Shade all real numbers that are "less than" or "left of " −21/11 on a number line.



Thus, using set-builder and interval notation, the solution is:

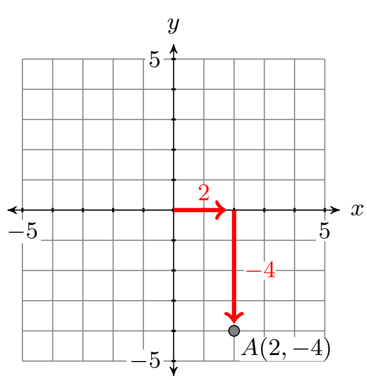
$$\{x : x < -21/11\} = (-\infty, -21/11)$$

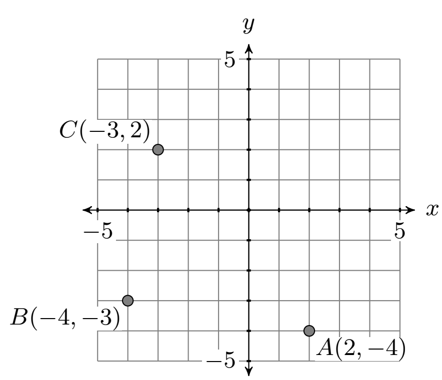
Chapter 3

Introduction to Graphing

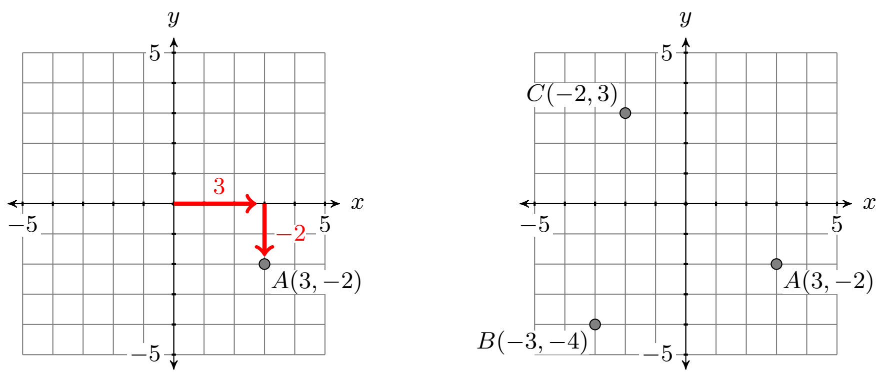
3.1 Graphing Equations by Hand

1. To plot the point $A(2, -4)$, move 2 units to the right, then move 4 units down. The remaining points are plotted in a similar manner.

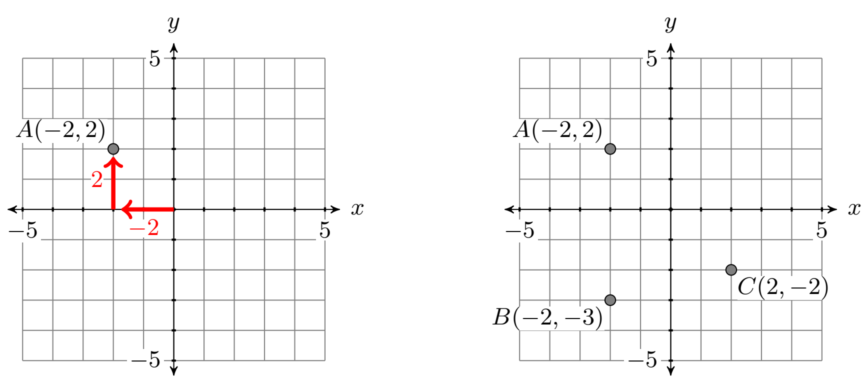




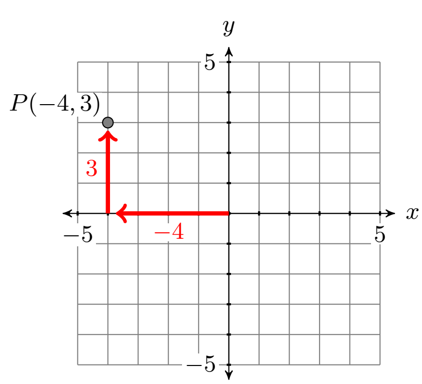
3. To plot the point $A(3, -2)$, move 3 units to the right, then move 2 units down. The remaining points are plotted in a similar manner.



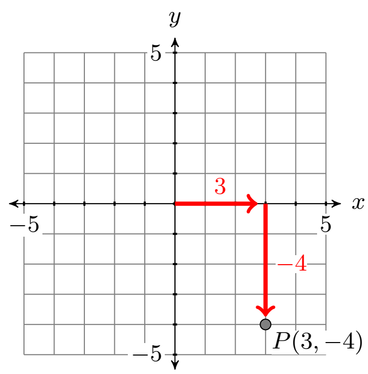
5. To plot the point $A(-2, 2)$, move 2 units to the left, then move 2 units up. The remaining points are plotted in a similar manner.



7. Start at the origin, move 4 units to the left then move 3 units up to reach the point P.



9. Start at the origin, move 3 units to the right then move 4 units down to reach the point P.



11. Substitute $(x, y) = (5, 20)$ into the equation $y = 5x - 5$.

$$\begin{aligned} y &= 5x - 5 && \text{Original equation.} \\ 20 &= 5(5) - 5 && \text{Substitute: 5 for } x\text{, 20 for } y. \\ 20 &= 25 - 5 && \text{Multiply: } 5(5) = 25. \\ 20 &= 20 && \text{Subtract: } 25 - 5 = 20. \end{aligned}$$

Because this last statement is a true statement, $(5, 20)$ satisfies (is a solution of) the equation $y = 5x - 5$.

For contrast, consider the point $(8, 36)$.

$$\begin{aligned} y &= 5x - 5 && \text{Original equation.} \\ 36 &= 5(8) - 5 && \text{Substitute: 8 for } x\text{, 36 for } y. \\ 36 &= 40 - 5 && \text{Multiply: } 5(8) = 40. \\ 36 &= 35 && \text{Subtract: } 40 - 5 = 35. \end{aligned}$$

Note that this last statement is false. Hence, the pair $(8, 36)$ is **not** a solution of $y = 5x - 5$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

13. Substitute $(x, y) = (1, 1)$ into the equation $y = -5x + 6$.

$$\begin{aligned} y &= -5x + 6 && \text{Original equation.} \\ 1 &= -5(1) + 6 && \text{Substitute: 1 for } x\text{, 1 for } y. \\ 1 &= -5 + 6 && \text{Multiply: } -5(1) = -5. \\ 1 &= 1 && \text{Add: } -5 + 6 = 1. \end{aligned}$$

Because this last statement is a true statement, $(1, 1)$ satisfies (is a solution of) the equation $y = -5x + 6$.

For contrast, consider the point $(2, -3)$.

$$\begin{aligned}
y &= -5x + 6 && \text{Original equation.}\\
-3 &= -5(2) + 6 && \text{Substitute: 2 for } x\text{, } -3 \text{ for } y.\\
-3 &= -10 + 6 && \text{Multiply: } -5(2) = -10.\\
-3 &= -4 && \text{Add: } -10 + 6 = -4.
\end{aligned}$$

Note that this last statement is false. Hence, the pair $(2, -3)$ is **not** a solution of $y = -5x+6$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

15. Substitute $(x, y) = (7, 395)$ into the equation $y = 8x^2 + 3$.

$$\begin{aligned}
y &= 8x^2 + 3 && \text{Original equation.}\\
395 &= 8(7)^2 + 3 && \text{Substitute: 7 for } x\text{, 395 for } y.\\
395 &= 8(49) + 3 && \text{Square: } (7)^2 = 49.\\
395 &= 392 + 3 && \text{Multiply: } 8(49) = 392.\\
395 &= 395 && \text{Add: } 392 + 3 = 395.
\end{aligned}$$

Because this last statement is a true statement, $(7, 395)$ satisfies (is a solution of) the equation $y = 8x^2 + 3$.

For contrast, consider the point $(1, 12)$.

$$\begin{aligned}
y &= 8x^2 + 3 && \text{Original equation.}\\
12 &= 8(1)^2 + 3 && \text{Substitute: 1 for } x\text{, 12 for } y.\\
12 &= 8(1) + 3 && \text{Square: } (1)^2 = 1.\\
12 &= 8 + 3 && \text{Multiply: } 8(1) = 8.\\
12 &= 11 && \text{Add: } 8 + 3 = 11.
\end{aligned}$$

Note that this last statement is false. Hence, the pair $(1, 12)$ is **not** a solution of $y = 8x^2+3$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

17. Substitute $(x, y) = (8, 400)$ into the equation $y = 6x^2 + 2x$.

$$\begin{aligned}
y &= 6x^2 + 2x && \text{Original equation.}\\
400 &= 6(8)^2 + 2(8) && \text{Substitute: 8 for } x\text{, 400 for } y.\\
400 &= 6(64) + 2(8) && \text{Square: } (8)^2 = 64.\\
400 &= 384 + 16 && \text{Multiply: } 6(64) = 384;\ 2(8) = 16\\
400 &= 400 && \text{Add: } 384 + 16 = 400.
\end{aligned}$$

Because this last statement is a true statement, $(8, 400)$ satisfies (is a solution of) the equation $y = 6x^2 + 2x$.

For contrast, consider the point $(2, 29)$.

$$\begin{aligned}
y &= 6x^2 + 2x && \text{Original equation.}\\
29 &= 6(2)^2 + 2(2) && \text{Substitute: 2 for } x\text{, 29 for } y.\\
29 &= 6(4) + 2(2) && \text{Square: } (2)^2 = 4.\\
29 &= 24 + 4 && \text{Multiply: } 6(4) = 24;\ 2(2) = 4\\
29 &= 28 && \text{Add: } 24 + 4 = 28.
\end{aligned}$$

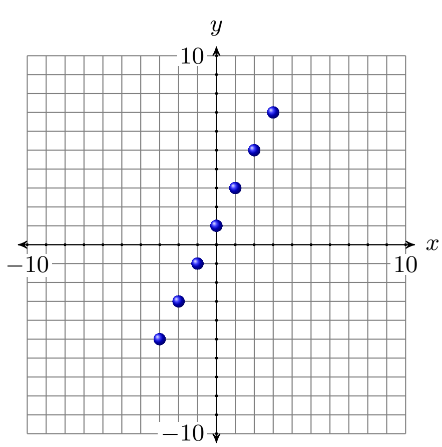
Note that this last statement is false. Hence, the pair $(2, 29)$ is **not** a solution of $y = 6x^2 + 2x$. In similar fashion, readers should also check that the remaining two points are **not** solutions.

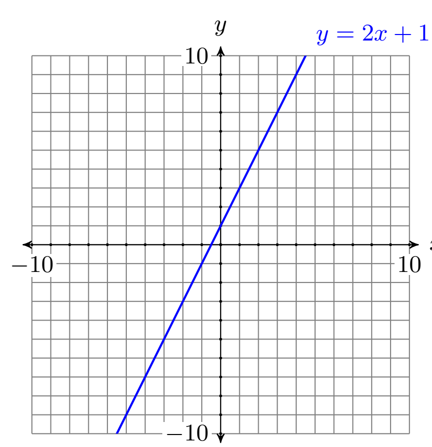
19. First, complete the table of points that satisfy the equation $y = 2x + 1$.

$$\begin{aligned}
y &= 2(-3) + 1 = -5\\
y &= 2(-2) + 1 = -3\\
y &= 2(-1) + 1 = -1\\
y &= 2(0) + 1 = 1\\
y &= 2(1) + 1 = 3\\
y &= 2(2) + 1 = 5\\
y &= 2(3) + 1 = 7
\end{aligned}$$

x	$y = 2x + 1$	(x, y)
-3	-5	$(-3, -5)$
-2	-3	$(-2, -3)$
-1	-1	$(-1, -1)$
0	1	$(0, 1)$
1	3	$(1, 3)$
2	5	$(2, 5)$
3	7	$(3, 7)$

Next, plot the points in the table, as shown in the image on the right. This first image gives us enough evidence to believe that if we plotted **all** points satisfying the equation $y = 2x + 1$, the result would be the graph on the right.



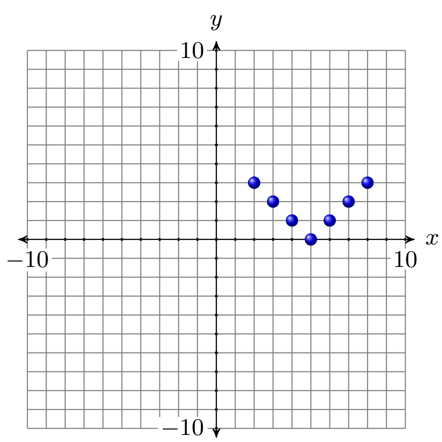


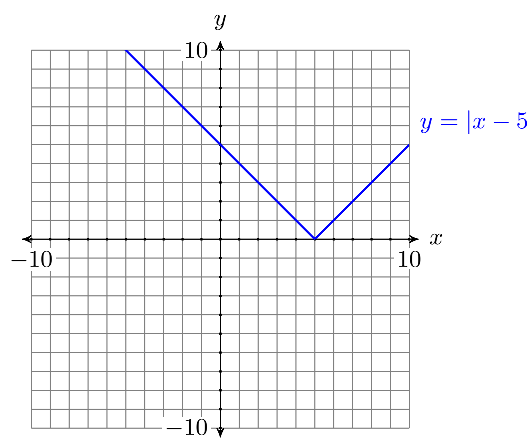
21. First, complete the table of points that satisfy the equation $y = |x - 5|$.

$$\begin{aligned} y &= |2-5| = 3 \\ y &= |3-5| = 2 \\ y &= |4-5| = 1 \\ y &= |5-5| = 0 \\ y &= |6-5| = 1 \\ y &= |7-5| = 2 \\ y &= |8-5| = 3 \end{aligned}$$

x	$y = \|x - 5\|$	(x, y)
2	3	$(2, 3)$
3	2	$(3, 2)$
4	1	$(4, 1)$
5	0	$(5, 0)$
6	1	$(6, 1)$
7	2	$(7, 2)$
8	3	$(8, 3)$

Next, plot the points in the table, as shown in the image on the right. This first image gives us enough evidence to believe that if we plotted **all** points satisfying the equation $y = |x - 5|$, the result would be the graph on the right.





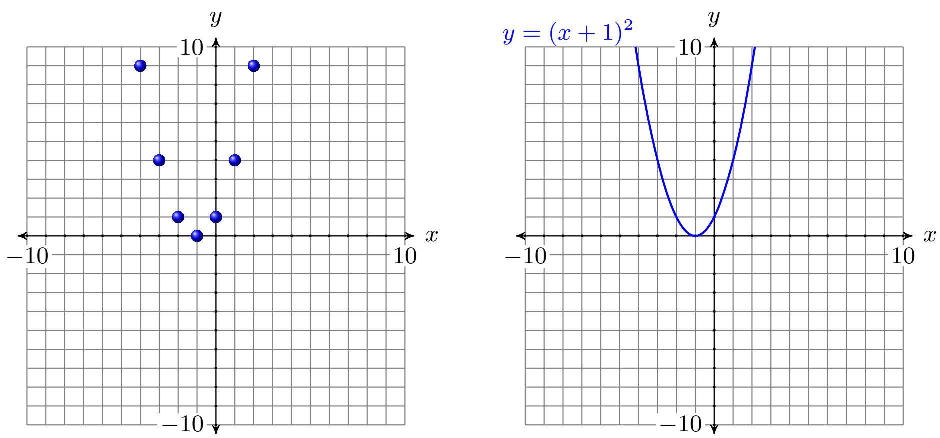
23. First, complete the table of points that satisfy the equation $y = (x + 1)^2$.

$$\begin{aligned} y &= (-4+1)^2 = 9 \\ y &= (-3+1)^2 = 4 \\ y &= (-2+1)^2 = 1 \\ y &= (-1+1)^2 = 0 \\ y &= (0+1)^2 = 1 \\ y &= (1+1)^2 = 4 \\ y &= (2+1)^2 = 9 \end{aligned}$$

x	$y = (x + 1)^2$	(x, y)
-4	9	$(-4, 9)$
-3	4	$(-3, 4)$
-2	1	$(-2, 1)$
-1	0	$(-1, 0)$
0	1	$(0, 1)$
1	4	$(1, 4)$
2	9	$(2, 9)$

Next, plot the points in the table, as shown in the image on the left. This first image gives us enough evidence to believe that if we plotted **all** points

satisfying the equation $y = (x+1)^2$, the result would be the graph on the right.

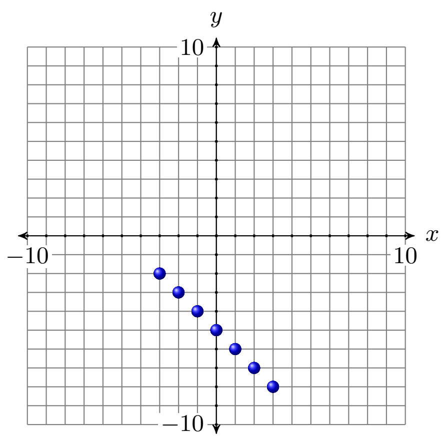


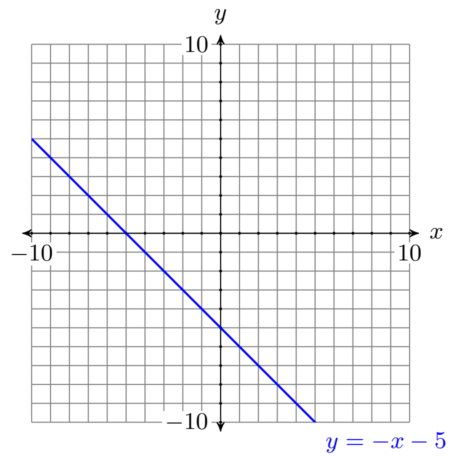
25. First, complete the table of points that satisfy the equation $y = -x - 5$.

$$\begin{aligned}
y &= -(-3) - 5 = -2\\
y &= -(-2) - 5 = -3\\
y &= -(-1) - 5 = -4\\
y &= -(0) - 5 = -5\\
y &= -(1) - 5 = -6\\
y &= -(2) - 5 = -7\\
y &= -(3) - 5 = -8
\end{aligned}$$

x	$y = -x - 5$	(x, y)
-3	-2	$(-3, -2)$
-2	-3	$(-2, -3)$
-1	-4	$(-1, -4)$
0	-5	$(0, -5)$
1	-6	$(1, -6)$
2	-7	$(2, -7)$
3	-8	$(3, -8)$

Next, plot the points in the table, as shown in the image on the right. This first image gives us enough evidence to believe that if we plotted **all** points satisfying the equation $y = -x - 5$, the result would be the graph on the right.

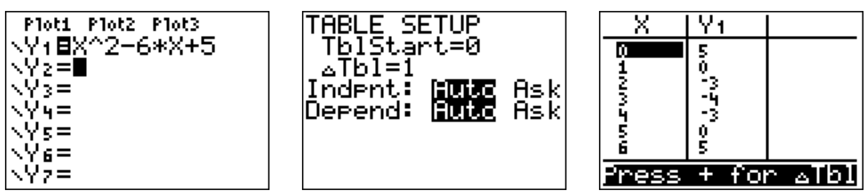




27. Enter the equation $y = x^2 - 6x + 5$ in `Y1` in the Y= menu, using the following keystrokes.

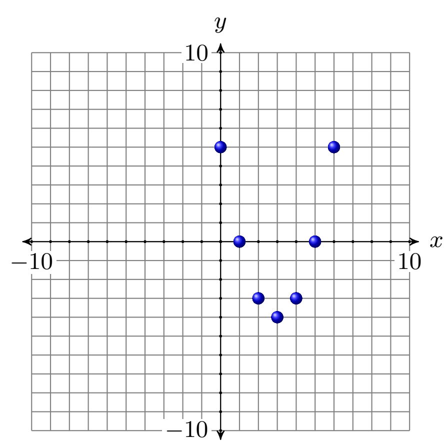
X,T,θ,n | $\wedge$ | 2 | $-$ | 6 | $\times$ | X,T,θ,n | $+$ | 5 | ENTER

Press 2ND WINDOW, then set **TblStart** $= 0$ and $\boldsymbol{\Delta}$**Tbl** $= 1$. Press 2ND GRAPH to produce the table.

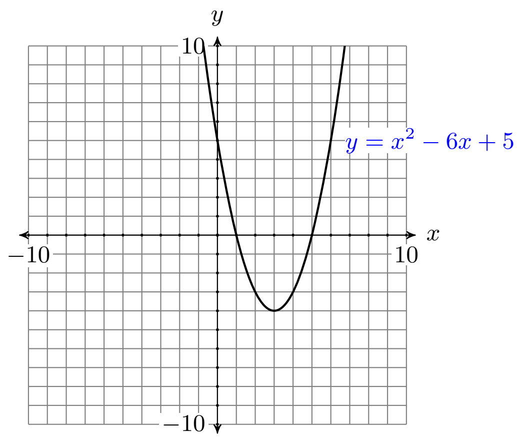


Enter the data from your calculator into a table on your graph paper. Use a ruler to construct the axes of your coordinate system, label and scale each axis. Plot the points in the table, as shown in the image on the right.

x	y	(x, y)
0	5	$(0, 5)$
1	0	$(1, 0)$
2	-3	$(2, -3)$
3	-4	$(3, -4)$
4	-3	$(4, -3)$
5	0	$(5, 0)$
6	5	$(6, 5)$



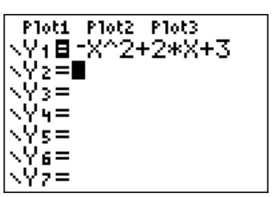
The plotted points provide enough evidence to convince us that if we plotted all points that satisfied the equation $y = x^2 - 6x + 5$, we would get the following result.

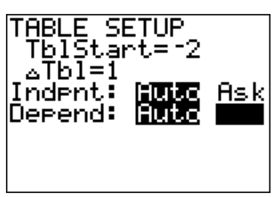


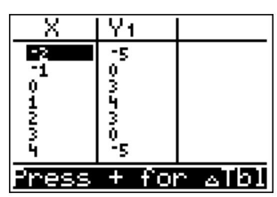
29. Enter the equation $y = -x^2 + 2x + 3$ in `Y1` in the Y= menu using the following keystrokes.

$\boxed{(-)}$ $\boxed{X,T,\theta,n}$ $\boxed{\wedge}$ $\boxed{2}$ $\boxed{+}$ $\boxed{2}$ $\boxed{\times}$ $\boxed{X,T,\theta,n}$ $\boxed{+}$ $\boxed{3}$ $\boxed{\text{ENTER}}$

Press 2ND WINDOW, then set **TblStart** $= -2$ and $\mathbf{\Delta Tbl} = 1$. Press 2ND GRAPH to produce the table.

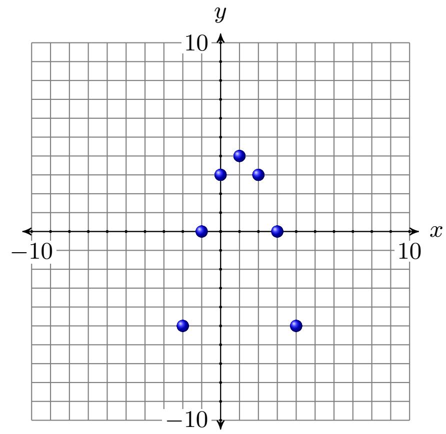




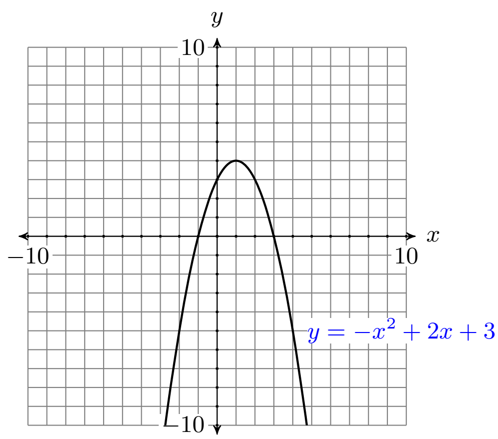


Enter the data from your calculator into a table on your graph paper. Use a ruler to construct the axes of your coordinate system, label and scale each axis. Plot the points in the table, as shown in the image on the right.

x	y	(x, y)
-2	-5	$(-2, -5)$
-1	0	$(-1, 0)$
0	3	$(0, 3)$
1	4	$(1, 4)$
2	3	$(2, 3)$
3	0	$(3, 0)$
4	-5	$(4, -5)$



The plotted points provide enough evidence to convince us that if we plotted all points that satisfied the equation $y = -x^2+2x+3$, we would get the following result.

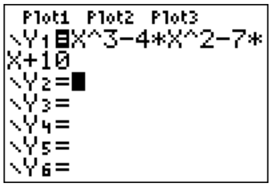


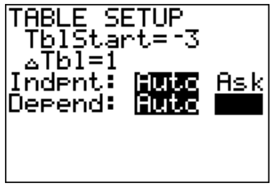
31. Enter the equation $y = x^3 - 4x^2 - 7x + 10$ in `Y1` in the Y= menu using the following keystrokes.

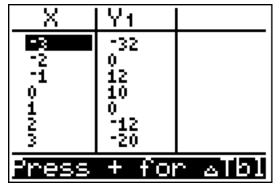
X,T,θ,n $\wedge$ 3 $-$ 4 $\times$ X,T,θ,n $\wedge$ 2 $-$ 7 $\times$ X,T,θ,n

$+$ 1 0 ENTER

Press 2ND WINDOW, then set **TblStart** $= -3$ and **ΔTbl** $= 1$. Press 2ND GRAPH to produce the table.

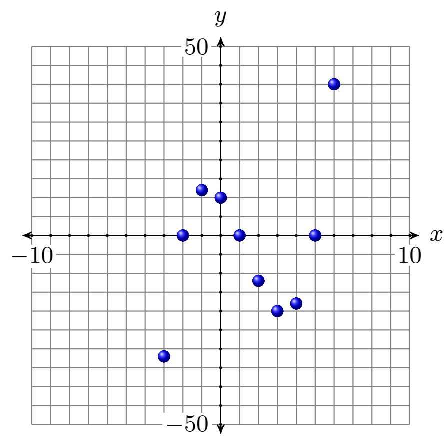




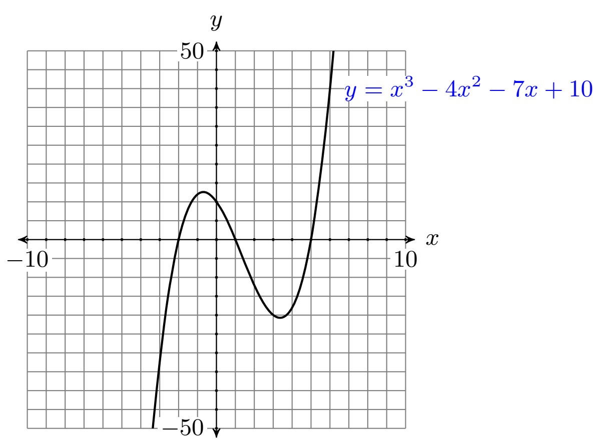


Enter the data from your calculator into a table on your graph paper. Use a ruler to construct the axes of your coordinate system, label and scale each axis. Plot the points in the table, as shown in the image on the right.

x	y	(x, y)
-3	-32	$(-3, -32)$
-2	0	$(-2, 0)$
-1	12	$(-1, 12)$
0	10	$(0, 10)$
1	0	$(1, 0)$
2	-12	$(2, -12)$
3	-20	$(3, -20)$
4	-18	$(4, -18)$
5	0	$(5, 0)$
6	40	$(6, 40)$



The plotted points provide enough evidence to convince us that if we plotted all points that satisfied the equation $y = x^3 - 4x^2 - 7x + 10$, we would get the following result.

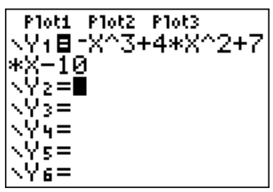


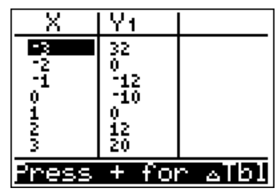
33. Enter the equation $y = -x^3 + 4x^2 + 7x - 10$ in `Y1` in the Y= menu using the following keystrokes.

(-) X,T,θ,n ∧ 3 + 4 × X,T,θ,n ∧ 2 + 7 × X,T,θ,n

− 1 0 ENTER

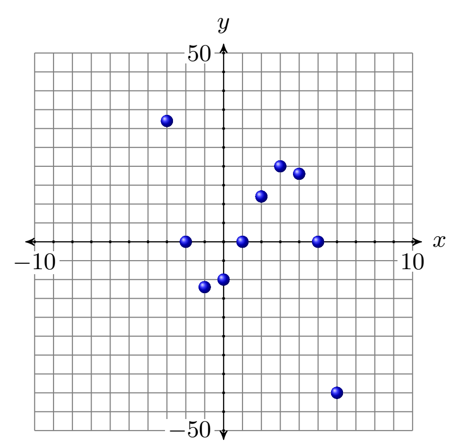
Press 2ND WINDOW, then set **TblStart** $= -3$ and **ΔTbl** $= 1$. Press 2ND GRAPH to produce the table.



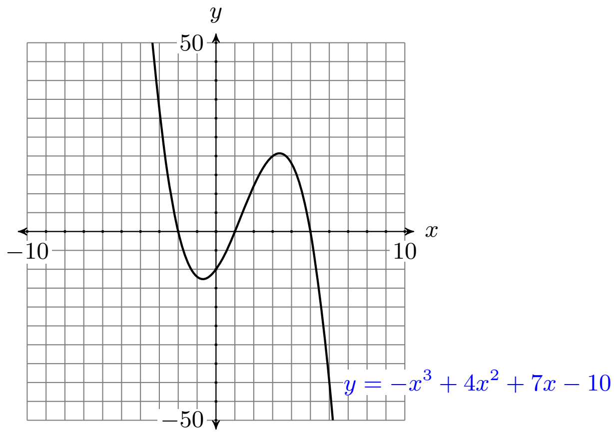


Enter the data from your calculator into a table on your graph paper. Use a ruler to construct the axes of your coordinate system, label and scale each axis. Plot the points in the table, as shown in the image on the right.

x	y	(x, y)
-3	32	$(-3, 32)$
-2	0	$(-2, 0)$
-1	-12	$(-1, -12)$
0	-10	$(0, -10)$
1	0	$(1, 0)$
2	12	$(2, 12)$
3	20	$(3, 20)$
4	18	$(4, 18)$
5	0	$(5, 0)$
6	-40	$(6, -40)$



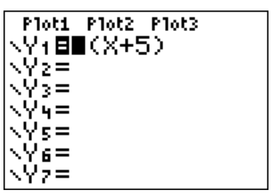
The plotted points provide enough evidence to convince us that if we plotted all points that satisfied the equation $y = -x^3 + 4x^2 + 7x - 10$, we would get the following result.

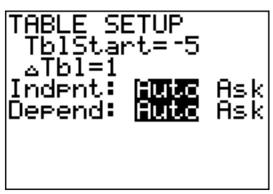


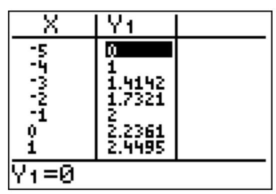
35. First, enter the equation $y = \sqrt{x+5}$ into **Y1** in the **Y=** menu using the following keystrokes. The results are shown in the first image below.

2ND $\sqrt{}$ X,T,θ,n +) ENTER

Next, select 2ND WINDOW and adjust the settings to match those in the second image. below. Finally, select 2ND GRAPH to open the TABLE. Copy the results from the third image below into the tables on your graph paper.



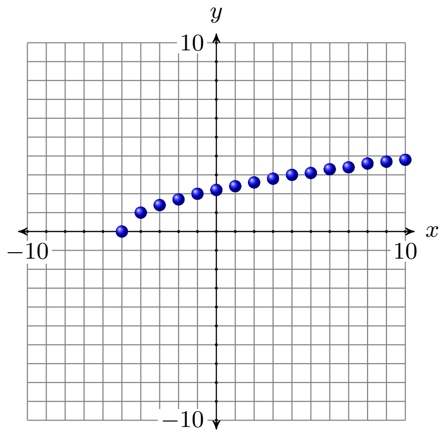


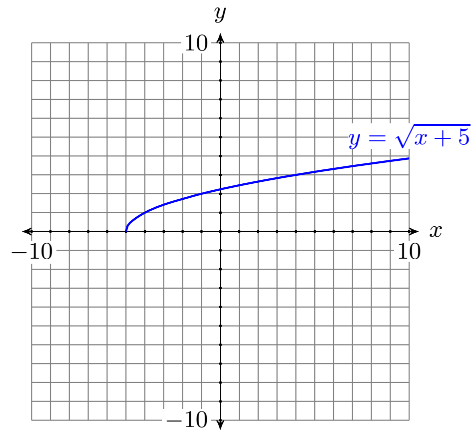


x	$y = \sqrt{x+5}$	(x, y)
-5	0	$(-5, 0)$
-4	1	$(-4, 1)$
-3	1.4	$(-3, 1.4)$
-2	1.7	$(-2, 1.7)$
-1	2	$(-1, 2)$
0	2.2	$(0, 2.2)$
1	2.4	$(1, 2.4)$
2	2.6	$(2, 2.6)$

x	$y = \sqrt{x+5}$	(x, y)
3	2.8	$(3, 2.8)$
4	3	$(4, 3)$
5	3.2	$(5, 3.2)$
6	3.3	$(6, 3.3)$
7	3.5	$(7, 3.5)$
8	3.6	$(8, 3.6)$
9	3.7	$(9, 3.7)$
10	3.9	$(10, 3.9)$

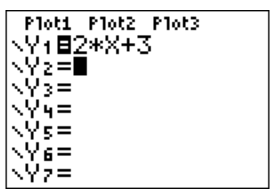
Next, plot the points in the tables, as shown in the image on the left. This first image gives us enough evidence to believe that if we plotted **all** points satisfying the equation $y = \sqrt{x+5}$, the result would be the graph on the right.

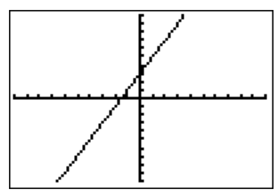




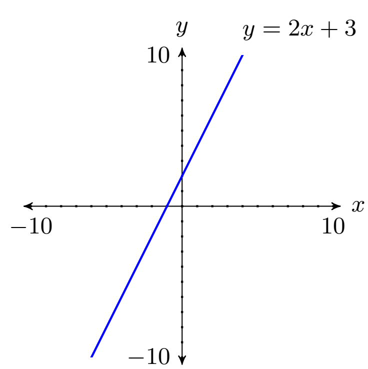
3.2 The Graphing Calculator

1. Enter the equation $y = 2x+3$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

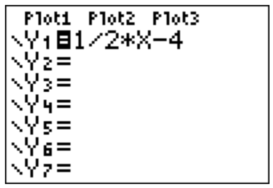


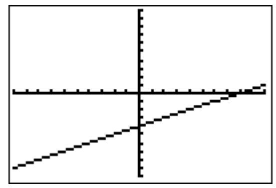


Report the graph on your homework as follows:

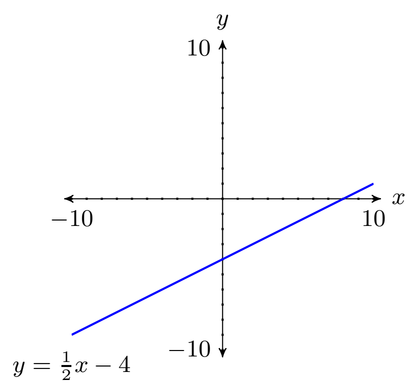


3. Enter the equation $y = \frac{1}{2}x - 4$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

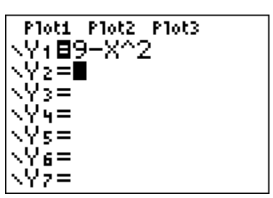


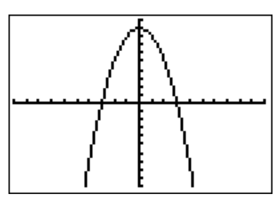


Report the graph on your homework as follows:

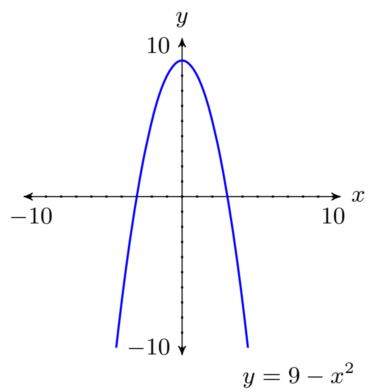


5. Enter the equation $y = 9 - x^2$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

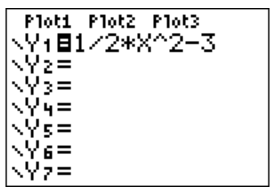


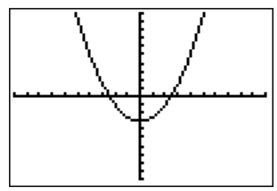


Report the graph on your homework as follows:

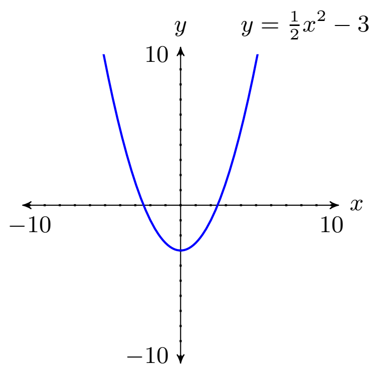


7. Enter the equation $y = \frac{1}{2}x^2 - 3$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

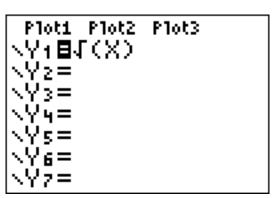


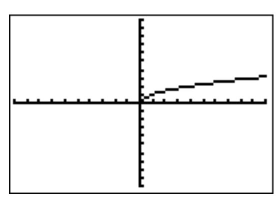


Report the graph on your homework as follows:

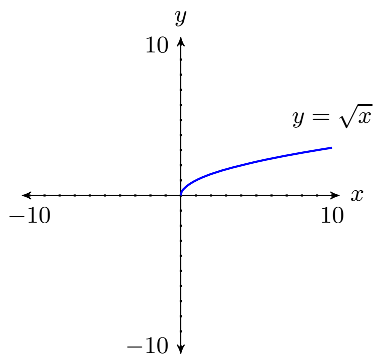


9. Enter the equation $y = \sqrt{x}$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

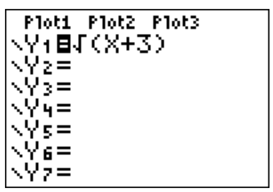


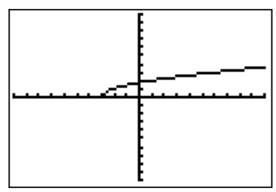


Report the graph on your homework as follows:

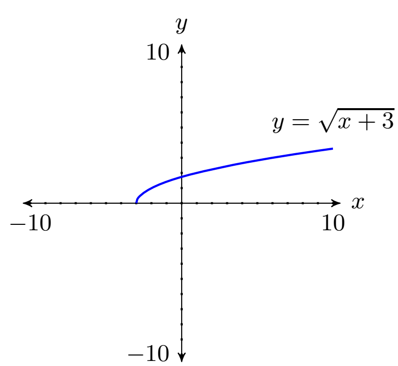


11. Enter the equation $y = \sqrt{x+3}$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

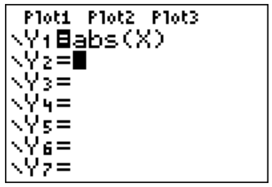


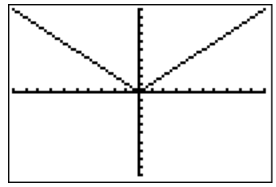


Report the graph on your homework as follows:

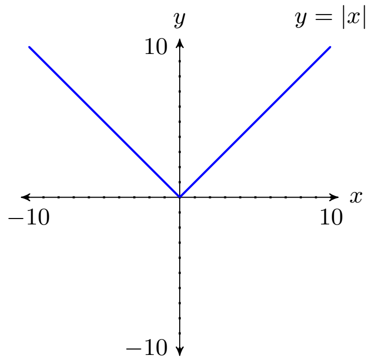


13. Enter the equation $y = |x|$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

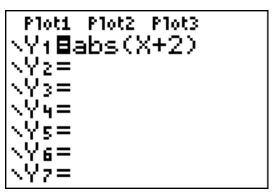


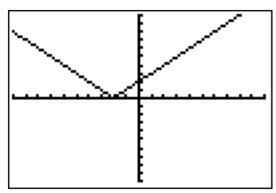


Report the graph on your homework as follows:

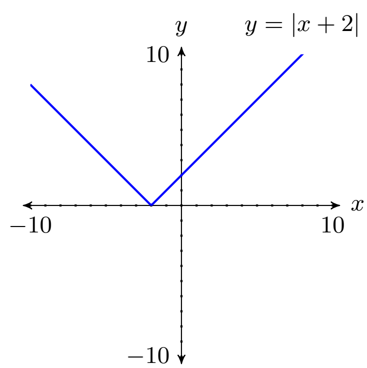


15. Enter the equation $y = |x+2|$ into the **Y=** menu, then select **6:ZStandard** from the ZOOM menu.

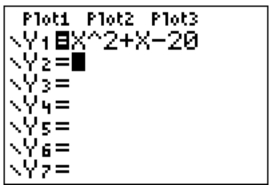


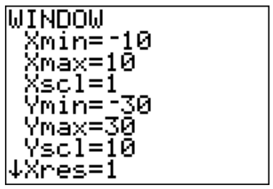


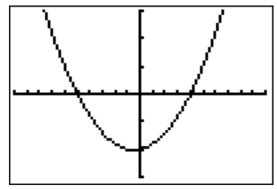
Report the graph on your homework as follows:



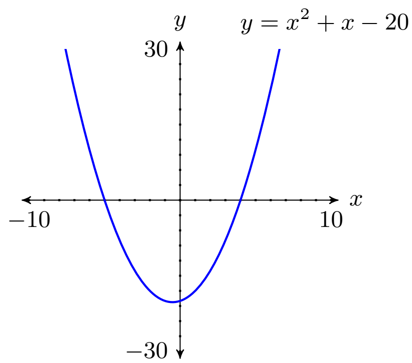
17. Enter the equation $y = x^2 + x - 20$ into **Y1=** of the **Y=** menu, as shown in the first image below, then select **6:ZStandard** from the ZOOM menu. In the WINDOW menu, make the changes shown in the second image to Ymin and Ymax, then push the GRAPH button to produce the graph shown in the third image.



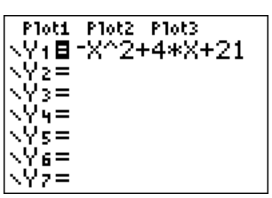


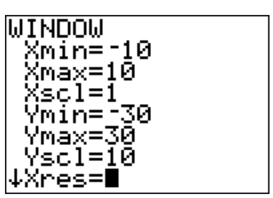


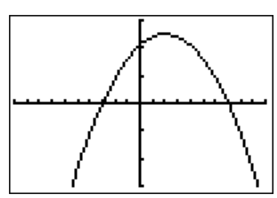
Report the graph on your homework as follows:



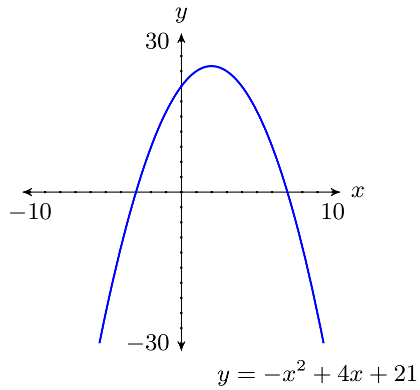
19. Enter the equation $y = -x^2+4x+21$ into **Y1=** of the **Y=** menu, as shown in the first image below, then select **6:ZStandard** from the ZOOM menu. In the WINDOW menu, make the changes shown in the second image to Ymin and Ymax, then push the GRAPH button to produce the graph shown in the third image.



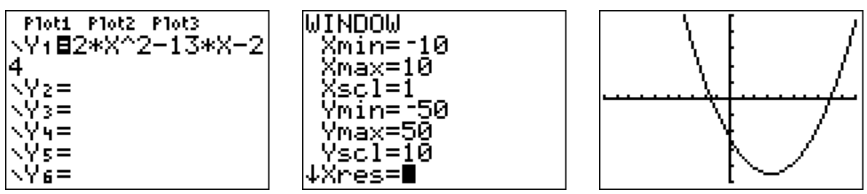




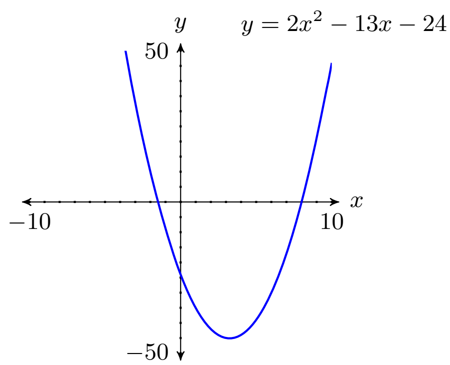
Report the graph on your homework as follows:



21. Enter the equation $y = 2x^2 - 13x - 24$ into **Y1=** of the **Y=** menu, as shown in the first image below, then select **6:ZStandard** from the ZOOM menu. In the WINDOW menu, make the changes shown in the second image to Ymin and Ymax, then push the GRAPH button to produce the graph shown in the third image.

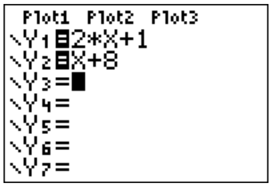


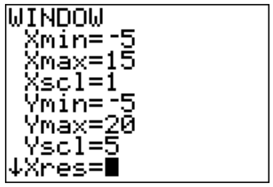
Report the graph on your homework as follows:

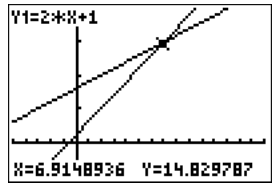


23. Enter the equations $y = 2x + 1$ and $y = x + 8$ into **Y1=** and **Y2=** of the **Y=** menu, as shown in the first image below, then select **6:ZStandard** from the ZOOM menu. In the WINDOW menu, make the changes shown in

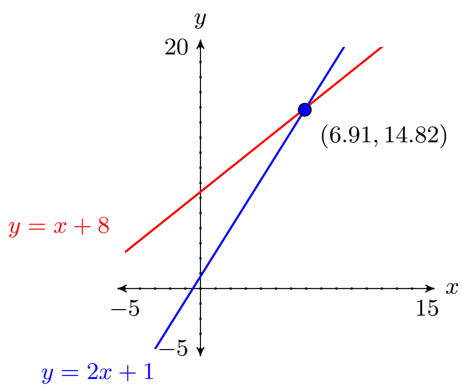
the second image to Ymin and Ymax, then push the GRAPH button. Press the TRACE button, then use the arrow keys to move the cursor as close as possible to the point of intersection, as shown in the third image.



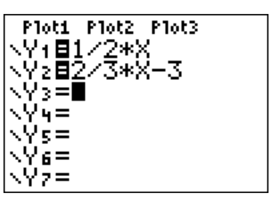


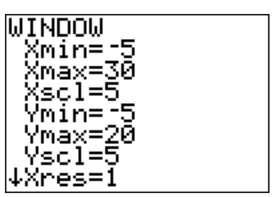


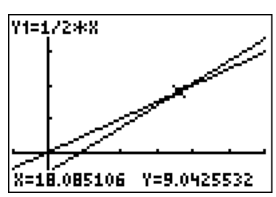
Report the graph on your homework as follows. As the TRACE key is only an approximation, estimates of the point of intersection may vary a bit.



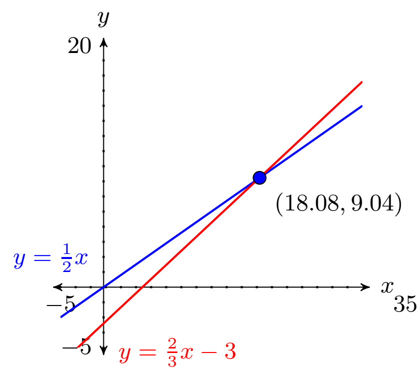
25. Enter the equations $y = \frac{1}{2}x$ and $y = \frac{2}{3}x - 3$ into **Y1=** and **Y2=** of the **Y=** menu, as shown in the first image below, then select **6:ZStandard** from the ZOOM menu. In the WINDOW menu, make the changes shown in the second image to Ymin and Ymax, then push the GRAPH button. Press the TRACE button, then use the arrow keys to move the cursor as close as possible to the point of intersection, as shown in the third image.



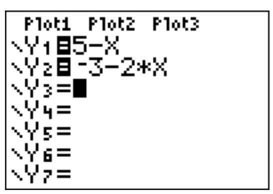


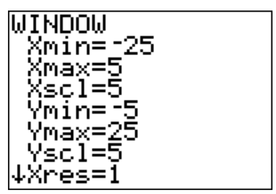


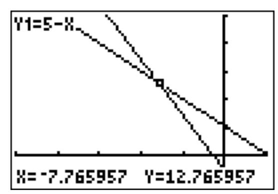
Report the graph on your homework as follows. As the TRACE key is only an approximation, estimates of the point of intersection may vary a bit.



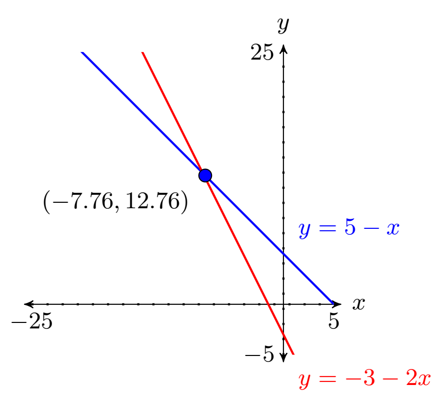
27. Enter the equations $y = 5 - x$ and $y = -3 - 2x$ into **Y1=** and **Y2=** of the **Y=** menu, as shown in the first image below, then select **6:ZStandard** from the ZOOM menu. In the WINDOW menu, make the changes shown in the second image to Ymin and Ymax, then push the GRAPH button. Press the TRACE button, then use the arrow keys to move the cursor as close as possible to the point of intersection, as shown in the third image.





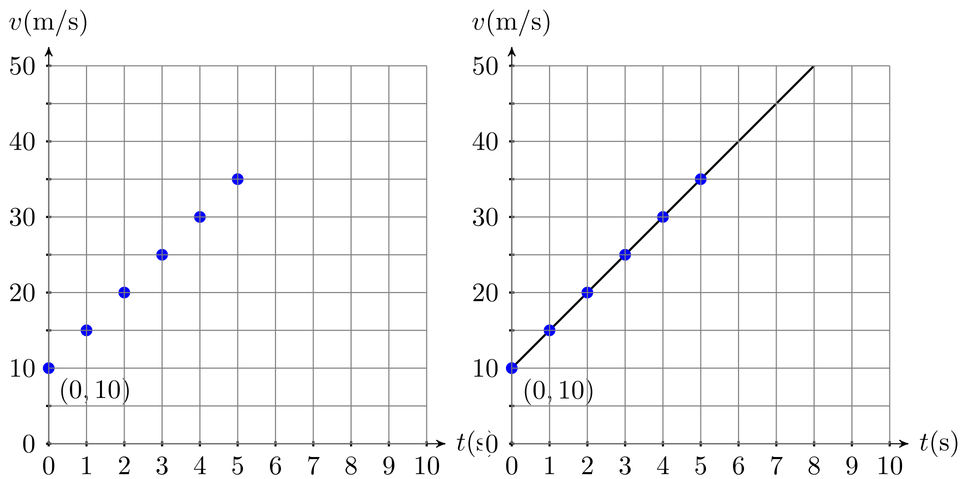


Report the graph on your homework as follows. As the TRACE key is only an approximation, estimates of the point of intersection may vary a bit.



3.3 Rates and Slope

1. Start by setting up a Cartesian Coordinate System, labeling and scaling each axis. Plot the point $(0, 10)$. This represents the fact that the initial velocity is $v = 10$ m/s at time $t = 0$ s. Because the object accelerates at a rate of 5 m/s each second, start at the point (0,10), then every time you move 1 second to the right, move 5 m/s upward. Do this for 5 consecutive points. Finally, as evidenced from the initial points plotted on the left, the constant acceleration of 5 m/s/s guarantees that the graph of the object's velocity versus time will be a line, shown on the right.



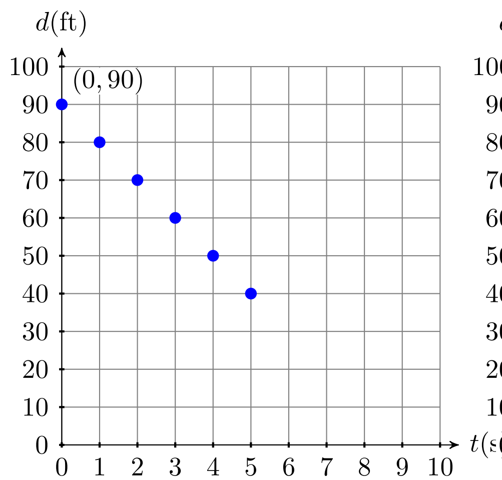
We know that every time the time increases by 1 second ($\Delta t = 1\,\text{s}$), the object's speed increases by 5 m/s ($\Delta v = 5\,\text{m/s}$). Thus, the slope of the line is:

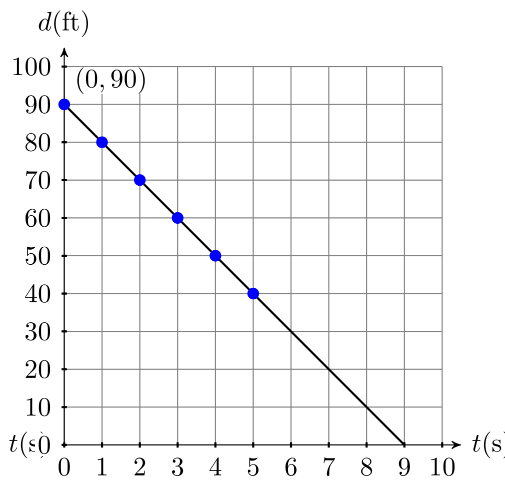
$$\begin{aligned}\text{Slope} &= \frac{\Delta v}{\Delta t}\\ &= \frac{5\,\text{m/s}}{1\,\text{s}}\\ &= 5\,\text{m/s/s}\end{aligned}$$

Note that the slope of the line is identical to the rate at which the objects's velocity is increasing with respect to time.

3. Start by setting up a Cartesian Coordinate System, labeling and scaling each axis. Plot the point $(0, 90)$. This represents the fact that the David's initial distance from his brother is $d = 90$ feet at time $t = 0$ seconds. Because David's distance from his brother decreases at a constant rate of 10 feet per second (10 ft/s), start at the point (0,90), then every time you move 1 second to the right, move 10 ft downward. Do this for 5 consecutive points. Finally,

as evidenced from the initial points plotted on the left, the decrease of David's distance from his brother at a constant rate of 10ft/s guarantees that the graph of the David's distance from his brother versus time will be a line, shown on the right.





We know that every time the time increases by 1 second ($\Delta t = 1\,\mathrm{s}$), David's distance from his brother decreases by 10 ft ($\Delta d = -10\,\mathrm{ft}$). Thus, the slope of the line is:

$$\begin{aligned}\text{Slope} &= \frac{\Delta d}{\Delta t}\\ &= \frac{-10\,\mathrm{ft}}{1\,\mathrm{s}}\\ &= -10\,\mathrm{ft/s}\end{aligned}$$

Note that the slope of the line is identical to the rate at which the David's distance from his brother is decreasing with respect to time. Note that the minus sign indicates the the David's distance from his brother is decreasing with respect to time.

5. To calculate the slope, we'll subtract the coordinates of point $P(9, 0)$ from

the point $Q(-9, 15)$.

$$\begin{aligned}
\text{Slope} &= \frac{\Delta y}{\Delta x} && \text{Slope is the change in } y \text{ divided by the change in } x.\\
&= \frac{15 - 0}{-9 - 9} && \text{Divide the difference in } y \text{ by the difference in } x.\\
&= \frac{15}{-18} && \text{Simplify numerator and denominator.}\\
&= -\frac{5}{6} && \text{Reduce to lowest terms.}
\end{aligned}$$

7. To calculate the slope, we'll subtract the coordinates of point $P(0, 11)$ from the point $Q(16, -11)$.

$$\begin{aligned}
\text{Slope} &= \frac{\Delta y}{\Delta x} && \text{Slope is the change in } y \text{ divided by the change in } x.\\
&= \frac{-11 - 11}{16 - 0} && \text{Divide the difference in } y \text{ by the difference in } x.\\
&= \frac{-22}{16} && \text{Simplify numerator and denominator.}\\
&= -\frac{11}{8} && \text{Reduce to lowest terms.}
\end{aligned}$$

9. To calculate the slope, we'll subtract the coordinates of point $P(11, 1)$ from the point $Q(-1, -1)$.

$$\begin{aligned}
\text{Slope} &= \frac{\Delta y}{\Delta x} && \text{Slope is the change in } y \text{ divided by the change in } x.\\
&= \frac{-1 - 1}{-1 - 11} && \text{Divide the difference in } y \text{ by the difference in } x.\\
&= \frac{-2}{-12} && \text{Simplify numerator and denominator.}\\
&= \frac{1}{6} && \text{Reduce to lowest terms.}
\end{aligned}$$

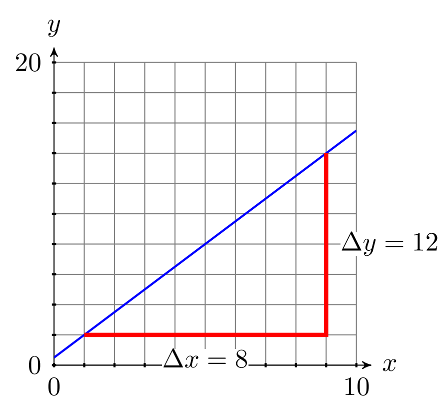
11. To calculate the slope, we'll subtract the coordinates of point $P(-18, 8)$ from the point $Q(3, -10)$.

$$\begin{aligned}
\text{Slope} &= \frac{\Delta y}{\Delta x} && \text{Slope is the change in } y \text{ divided by the change in } x. \\
&= \frac{-10 - 8}{3 - (-18)} && \text{Divide the difference in } y \text{ by the difference in } x. \\
&= \frac{-18}{21} && \text{Simplify numerator and denominator.} \\
&= -\frac{6}{7} && \text{Reduce to lowest terms.}
\end{aligned}$$

13. To calculate the slope, we'll subtract the coordinates of point $P(-18, 10)$ from the point $Q(-9, 7)$.

$$\begin{aligned}
\text{Slope} &= \frac{\Delta y}{\Delta x} && \text{Slope is the change in } y \text{ divided by the change in } x. \\
&= \frac{7 - 10}{-9 - (-18)} && \text{Divide the difference in } y \text{ by the difference in } x. \\
&= \frac{-3}{9} && \text{Simplify numerator and denominator.} \\
&= -\frac{1}{3} && \text{Reduce to lowest terms.}
\end{aligned}$$

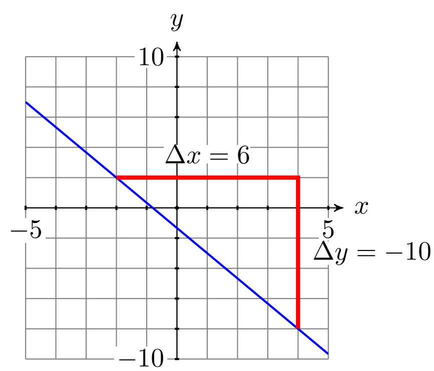
15. Because each gridline on the x-axis represents 1 unit, $\Delta x = 8$. Because each gridline on the y-axis represents 2 units, $\Delta y = 12$.



Therefore, the slope is:

$$\begin{aligned}\text{Slope} &= \frac{\Delta y}{\Delta x}\\ &= \frac{12}{8}\\ &= \frac{3}{2}\end{aligned}$$

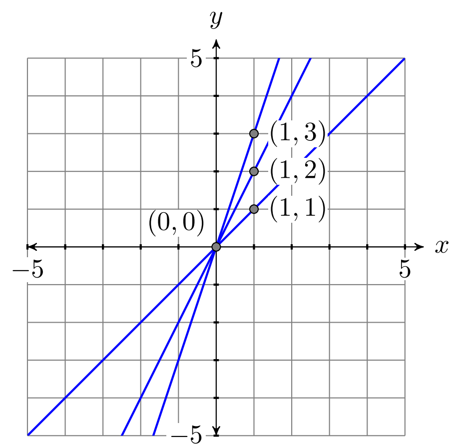
17. Because each gridline on the x-axis represents 1 unit, $\Delta x = 6$. Because each gridline on the y-axis represents 2 units, $\Delta y = -10$.



Therefore, the slope is:

$$\begin{aligned}\text{Slope} &= \frac{\Delta y}{\Delta x}\\ &= \frac{-10}{6}\\ &= -\frac{5}{3}\end{aligned}$$

19. First, sketch each of the lines passing through the given points.



Next, calculate the slope of each line.

Slope through $(0,0)$ and $(1,1)$.

$$\begin{aligned} m_1 &= \frac{\Delta y}{\Delta x} \\ &= \frac{1-0}{1-0} \\ &= 1 \end{aligned}$$

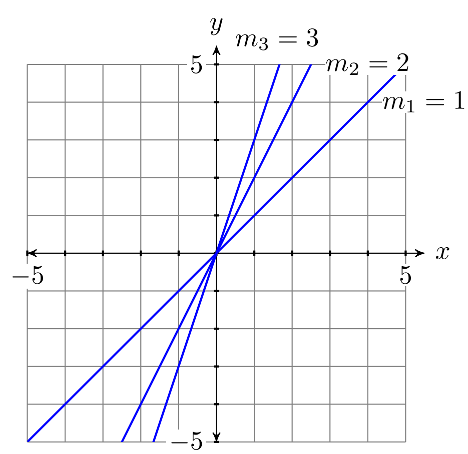
Slope through $(0,0)$ and $(1,2)$.

$$\begin{aligned} m_1 &= \frac{\Delta y}{\Delta x} \\ &= \frac{2-0}{1-0} \\ &= 2 \end{aligned}$$

Slope through $(0,0)$ and $(1,3)$.

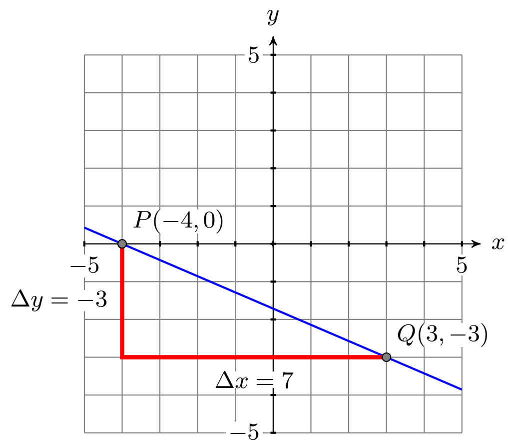
$$\begin{aligned} m_1 &= \frac{\Delta y}{\Delta x} \\ &= \frac{3-0}{1-0} \\ &= 3 \end{aligned}$$

Label each line with its slope.

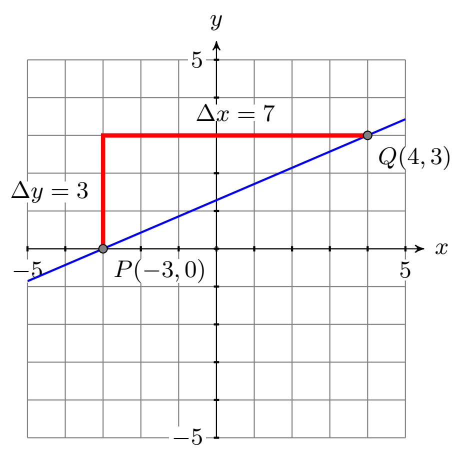


As the slope gets larger, the line gets steeper.

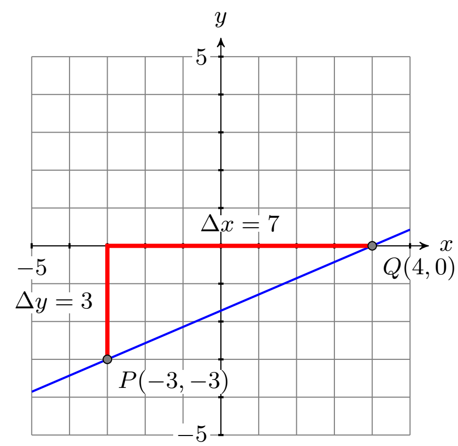
21. First, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the point $P(-4, 0)$. Because the slope is $\Delta y/\Delta x = -3/7$, start at the point $P(-4, 0)$, then move 3 units downward and 7 units to the right, reaching the point $Q(3, -3)$. Draw the line through the points P and Q.



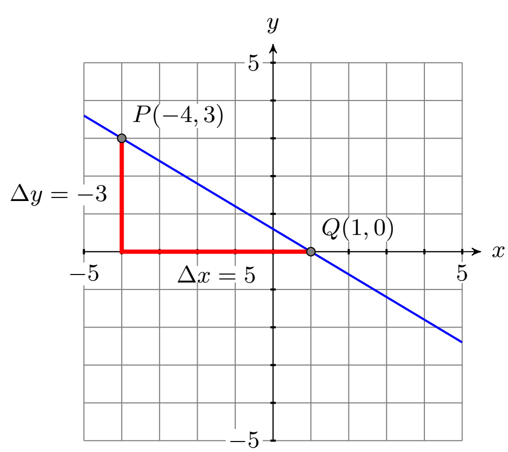
23. First, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the point $P(-3, 0)$. Because the slope is $\Delta y/\Delta x = 3/7$, start at the point $P(-3, 0)$, then move 3 units upward and 7 units to the right, reaching the point $Q(4, 3)$. Draw the line through the points P and Q.



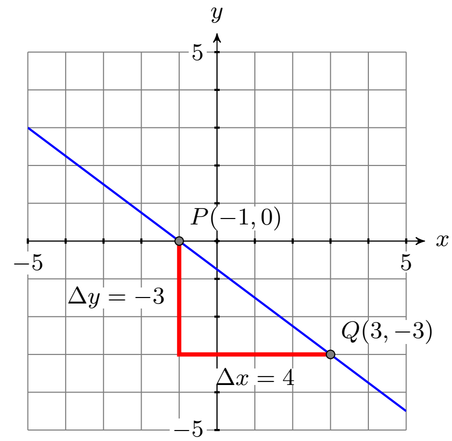
25. First, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the point $P(-3, -3)$. Because the slope is $\Delta y/\Delta x = 3/7$, start at the point $P(-3, -3)$, then move 3 units upward and 7 units to the right, reaching the point $Q(4, 0)$. Draw the line through the points P and Q.



27. First, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the point $P(-4, 3)$. Because the slope is $\Delta y/\Delta x = -3/5$, start at the point $P(-4, 3)$, then move 3 units downward and 5 units to the right, reaching the point $Q(1, 0)$. Draw the line through the points P and Q.



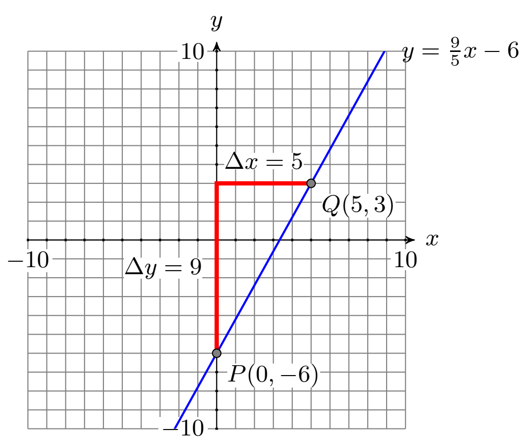
29. First, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the point $P(-1, 0)$. Because the slope is $\Delta y/\Delta x = -3/4$, start at the point $P(-1, 0)$, then move 3 units downward and 4 units to the right, reaching the point $Q(3, -3)$. Draw the line through the points P and Q.



3.4 Slope-Intercept Form of a Line

1. First, compare $y = \frac{9}{5}x - 6$ with $y = mx + b$ and note that $m = 9/5$ and $b = -6$. Therefore, the slope of the line is $9/5$ and the y-intercept is $(0, -6)$.

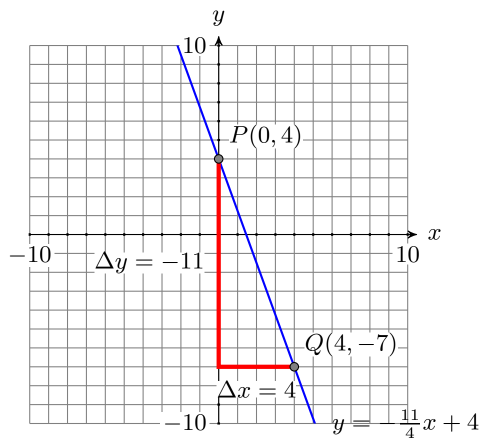
Next, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the y-intercept $P(0, -6)$. Because the slope is $\Delta y/\Delta x = 9/5$, start at the y-intercept $P(0, -6)$, then move 9 units upward and 5 units to the right, reaching the point $Q(5, 3)$. Draw the line through the points P and Q and label it with its equation $y = \frac{9}{5}x - 6$.



3. First, compare $y = -\frac{11}{4}x + 4$ with $y = mx + b$ and note that $m = -11/4$ and $b = 4$. Therefore, the slope of the line is $-11/4$ and the y-intercept is $(0, 4)$.

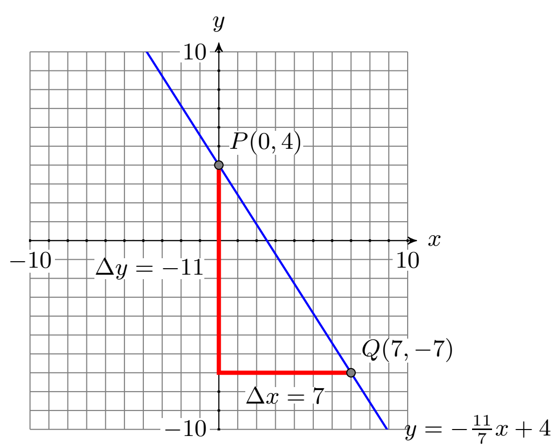
Next, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the y-intercept $P(0, 4)$. Because the slope is

$\Delta y/\Delta x = -11/4$, start at the y-intercept $P(0, 4)$, then move 11 units downward and 4 units to the right, reaching the point $Q(4, -7)$. Draw the line through the points P and Q and label it with its equation $y = -\frac{11}{4}x + 4$.



5. First, compare $y = -\frac{11}{7}x+4$ with $y = mx+b$ and note that $m = -11/7$ and $b = 4$. Therefore, the slope of the line is $-11/7$ and the y-intercept is $(0, 4)$.

Next, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the y-intercept $P(0, 4)$. Because the slope is $\Delta y/\Delta x = -11/7$, start at the y-intercept $P(0, 4)$, then move 11 units downward and 7 units to the right, reaching the point $Q(7, -7)$. Draw the line through the points P and Q and label it with its equation $y = -\frac{11}{7}x + 4$.

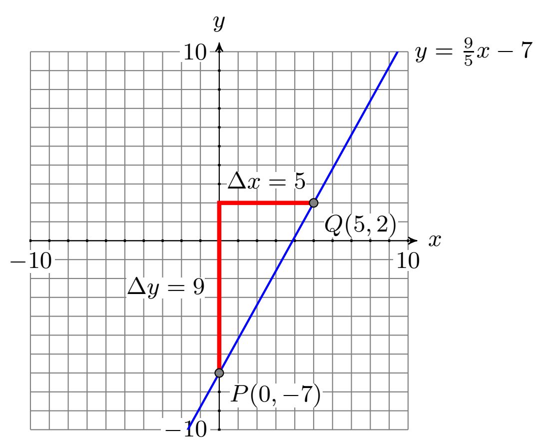


7. The y-intercept is $(0, -7)$, so $b = -7$. Further, the slope is $9/5$, so $m = 9/5$. Substitute these numbers into the slope-intercept form of the line.

$$y = mx + b \qquad \text{Slope-intercept form.}$$
$$y = \frac{9}{5}x - 7 \qquad \text{Substitute: } 9/5 \text{ for } m,\ -7 \text{ for } b.$$

Therefore, the slope-intercept form of the line is $y = \frac{9}{5}x - 7$.

Next, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the y-intercept $P(0, -7)$. Because the slope is $\Delta y/\Delta x = 9/5$, start at the y-intercept $P(0, -7)$, then move 9 units upward and 5 units to the right, reaching the point $Q(5, 2)$. Draw the line through the points P and Q and label it with its equation $y = \frac{9}{5}x - 7$.

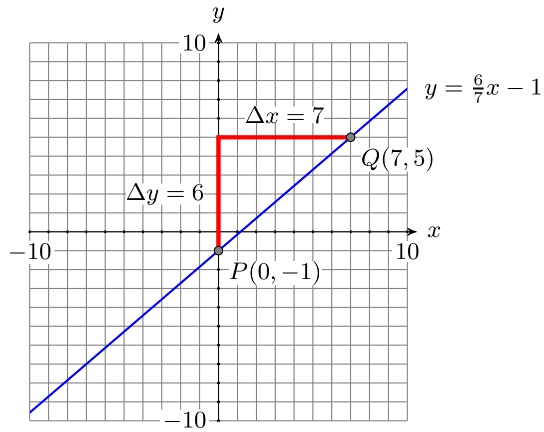


9. The y-intercept is $(0, -1)$, so $b = -1$. Further, the slope is $6/7$, so $m = 6/7$. Substitute these numbers into the slope-intercept form of the line.

$$y = mx + b \qquad \text{Slope-intercept form.}$$
$$y = \frac{6}{7}x - 1 \qquad \text{Substitute: } 6/7 \text{ for } m,\ -1 \text{ for } b.$$

Therefore, the slope-intercept form of the line is $y = \frac{6}{7}x - 1$.

Next, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the y-intercept $P(0, -1)$. Because the slope is $\Delta y/\Delta x = 6/7$, start at the y-intercept $P(0, -1)$, then move 6 units upward and 7 units to the right, reaching the point $Q(7, 5)$. Draw the line through the points P and Q and label it with its equation $y = \frac{6}{7}x - 1$.

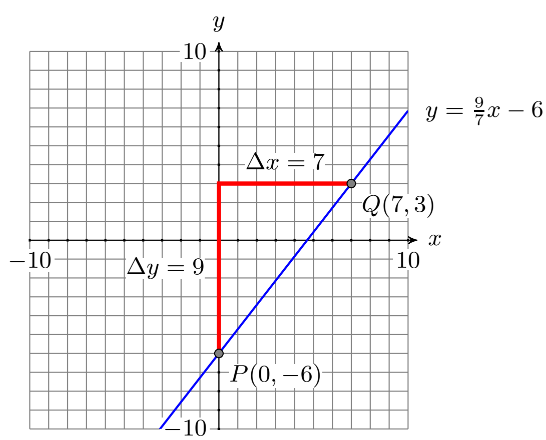


11. The y-intercept is $(0, -6)$, so $b = -6$. Further, the slope is $9/7$, so $m = 9/7$. Substitute these numbers into the slope-intercept form of the line.

$$
\begin{aligned}
y &= mx + b && \text{Slope-intercept form.} \\
y &= \frac{9}{7}x - 6 && \text{Substitute: } 9/7 \text{ for } m,\ -6 \text{ for } b.
\end{aligned}
$$

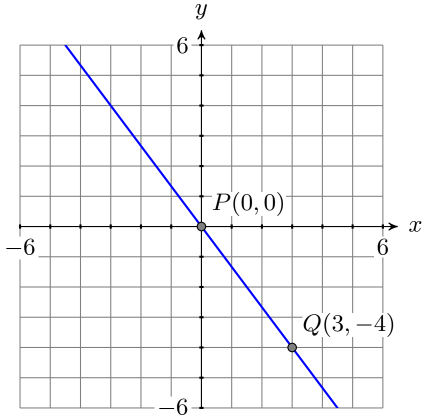
Therefore, the slope-intercept form of the line is $y = \frac{9}{7}x - 6$.

Next, set up a coordinate system on a sheet of graph paper. Label and scale each axis, then plot the y-intercept $P(0, -6)$. Because the slope is $\Delta y/\Delta x = 9/7$, start at the y-intercept $P(0, -6)$, then move 9 units upward and 7 units to the right, reaching the point $Q(7, 3)$. Draw the line through the points P and Q and label it with its equation $y = \frac{9}{7}x - 6$.



13. First, note that the y-intercept of the line (where it crosses the y-axis) is the point $P(0,0)$. This means that $b = 0$ in the slope-intercept formula $y = mx + b$.

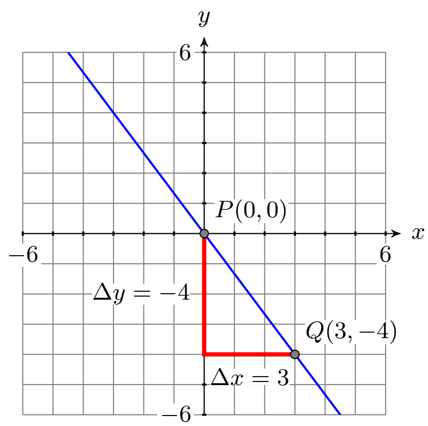
Next, we need to determine the slope of the line. Try to locate a second point on the line that passes directly through a lattice point, a point where a horizontal and vertical gridline intersect. It appears that the point $Q(3,-4)$ qualifies.



Subtract the coordinates of $P(0,0)$ from the coordinates of $Q(3,-4)$ to determine the slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4-(0)}{3-0} \\ &= -\frac{4}{3} \end{aligned}$$

Alternate technique for finding the slope: Start at the y-intercept $P(0,0)$, then move 4 units downward and 3 units to the right, reaching the point $Q(3,-4)$.



This also indicates that the slope of the line is:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4}{3} \end{aligned}$$

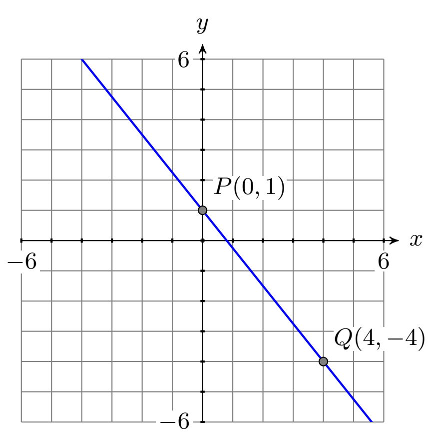
Finally, substitute $m = -4/3$ and $b = 0$ in the slope-intercept form of the line:

$$\begin{aligned} y &= mx + b \\ y &= -\frac{4}{3}x + (0) \end{aligned}$$

Hence, the equation of the line is $y = -\frac{4}{3}x$.

15. First, note that the y-intercept of the line (where it crosses the y-axis) is the point $P(0, 1)$. This means that $b = 1$ in the slope-intercept formula $y = mx + b$.

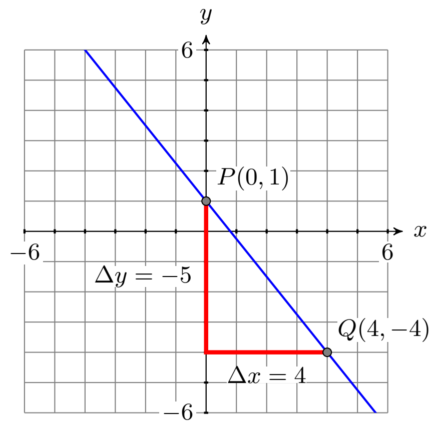
Next, we need to determine the slope of the line. Try to locate a second point on the line that passes directly through a lattice point, a point where a horizontal and vertical gridline intersect. It appears that the point $Q(4, -4)$ qualifies.



Subtract the coordinates of $P(0, 1)$ from the coordinates of $Q(4, -4)$ to determine the slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4 - 1}{4 - 0} \\ &= -\frac{5}{4} \end{aligned}$$

Alternate technique for finding the slope: Start at the y-intercept $P(0, 1)$, then move 5 units downward and 4 units to the right, reaching the point $Q(4, -4)$.



This also indicates that the slope of the line is:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-5}{4} \end{aligned}$$

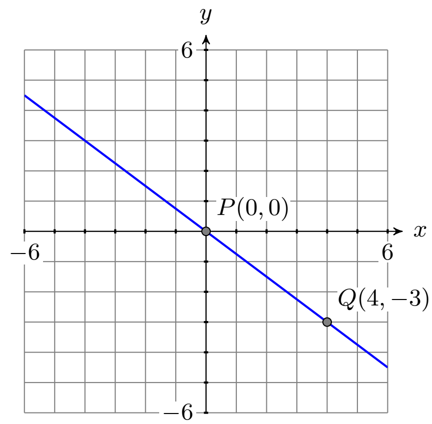
Finally, substitute $m = -5/4$ and $b = 1$ in the slope-intercept form of the line:

$$\begin{aligned} y &= mx + b \\ y &= -\frac{5}{4}x + 1 \end{aligned}$$

Hence, the equation of the line is $y = -\frac{5}{4}x + 1$.

17. First, note that the y-intercept of the line (where it crosses the y-axis) is the point $P(0, 0)$. This means that $b = 0$ in the slope-intercept formula $y = mx + b$.

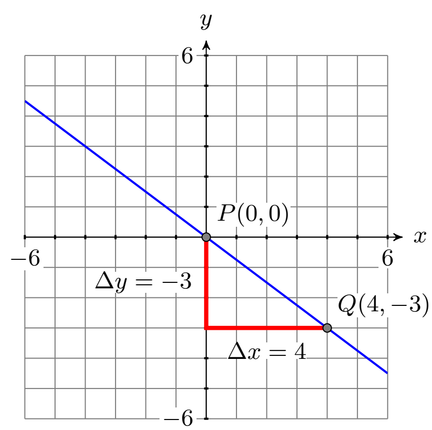
Next, we need to determine the slope of the line. Try to locate a second point on the line that passes directly through a lattice point, a point where a horizontal and vertical gridline intersect. It appears that the point $Q(4, -3)$ qualifies.



Subtract the coordinates of $P(0,0)$ from the coordinates of $Q(4,-3)$ to determine the slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-3-(0)}{4-0} \\ &= -\frac{3}{4} \end{aligned}$$

Alternate technique for finding the slope: Start at the y-intercept $P(0,0)$, then move 3 units downward and 4 units to the right, reaching the point $Q(4,-3)$.



This also indicates that the slope of the line is:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-3}{4} \end{aligned}$$

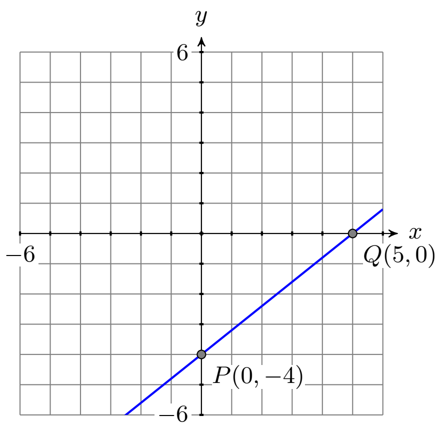
Finally, substitute $m = -3/4$ and $b = 0$ in the slope-intercept form of the line:

$$\begin{aligned} y &= mx + b \\ y &= -\frac{3}{4}x + (0) \end{aligned}$$

Hence, the equation of the line is $y = -\frac{3}{4}x$.

19. First, note that the y-intercept of the line (where it crosses the y-axis) is the point $P(0,-4)$. This means that $b=-4$ in the slope-intercept formula $y=mx+b$.

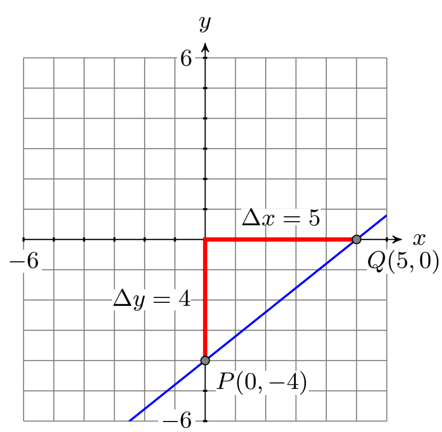
Next, we need to determine the slope of the line. Try to locate a second point on the line that passes directly through a lattice point, a point where a horizontal and vertical gridline intersect. It appears that the point $Q(5,0)$ qualifies.



Subtract the coordinates of $P(0,-4)$ from the coordinates of $Q(5,0)$ to determine the slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{0-(-4)}{5-0} \\ &= \frac{4}{5} \end{aligned}$$

Alternate technique for finding the slope: Start at the y-intercept $P(0,-4)$, then move 4 units upward and 5 units to the right, reaching the point $Q(5,0)$.



This also indicates that the slope of the line is:

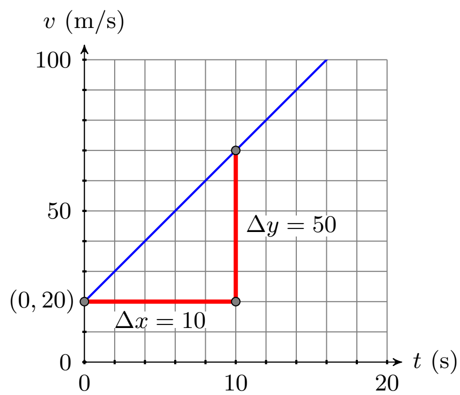
$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{4}{5} \end{aligned}$$

Finally, substitute $m = 4/5$ and $b = -4$ in the slope-intercept form of the line:

$$y = mx + b$$
$$y = \frac{4}{5}x + (-4)$$

Hence, the equation of the line is $y = \frac{4}{5}x - 4$.

21. Set up a coordinate system. Label the horizontal axis with the time t (in seconds) and the vertical axis with the velocity v (in meters per second). At time $t = 0$, the initial velocity is 20 m/s. This gives us the point (0,20). The velocity is increasing at a rate of 5 m/s each second. Let's start at the point (0,20), then move right 10 seconds and upward 50 m/s, arriving at the point (10,70). Draw the line through these two points. Note that this makes the slope $\Delta v/\Delta t = (50\,\mathrm{m/s})/(10\,\mathrm{s})$, or 2 m/s per second.



Because we know the intercept is (0,20) and the slope is 5, we use the slope-intercept form to produce the equation of the line.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form.} \\ y &= 5x + 20 && \text{Substitute: 5 for } m\text{, 20 for } b. \end{aligned}$$

Replace y with v and x with t to produce the following result:

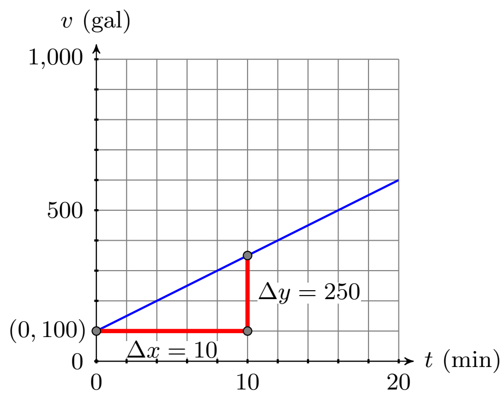
$$v = 5t + 20$$

To find the velocity at time $t = 14$ seconds, substitute 14 for t and simplify.

$$v = 5(14) + 20$$
$$v = 90$$

Thus, at $t = 14$ seconds, the velocity is $v = 90$ m/s.

23. Set up a coordinate system. Label the horizontal axis with the time t (in minutes) and the vertical axis with the volume V (in gallons). At time $t = 0$, the initial volume of water is 100 gallons. This gives us the point (0,100). The volume of water is increasing at a rate of 25 gal/min. Start at the point (0,100), then move right 10 minutes and upward 250 gallons, arriving at the point (10,350). Draw the line through these two points. Note that this makes the slope $\Delta V/\Delta t = (250\,\text{gal})/(10\,\text{min})$, or 25 gal/min, as required.



Because we know the intercept is (0,100) and the slope is 25, we use the slope-intercept form to produce the equation of the line.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form.} \\ y &= 25x + 100 && \text{Substitute: 25 for } m\text{, 100 for } b. \end{aligned}$$

Replace y with V and x with t to produce the following result:

$$V = 25t + 100$$

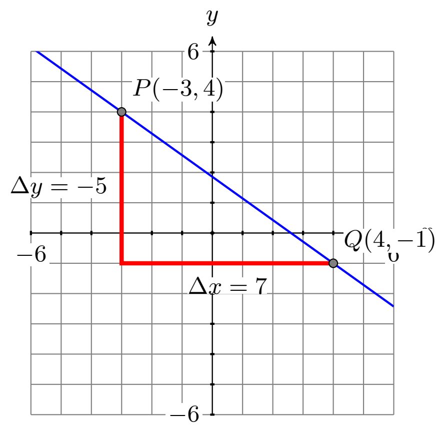
To find the time it takes the volume of water to reach 400 gallons, substitute 400 for V and solve for t.

$$\begin{aligned} 400 &= 25t + 100 && \text{Substitute 400 for } V. \\ 400 - 100 &= 25t + 100 - 100 && \text{Subtract 100 from both sides.} \\ 300 &= 25t && \text{Simplify.} \\ \frac{300}{25} &= \frac{25t}{25} && \text{Divide both sides by 25.} \\ 12 &= t && \text{Simplify.} \end{aligned}$$

Thus, it takes 12 minutes for the volume of water to reach 400 gallons.

3.5 Point-Slope Form of a Line

1. First, plot the point $P(-3, 4)$. Then, because the slope is $\Delta y/\Delta x = -5/7$, start at the point $P(-3, 4)$, then move 5 units downward and 7 units to the right, arriving at the point $Q(4, -1)$.

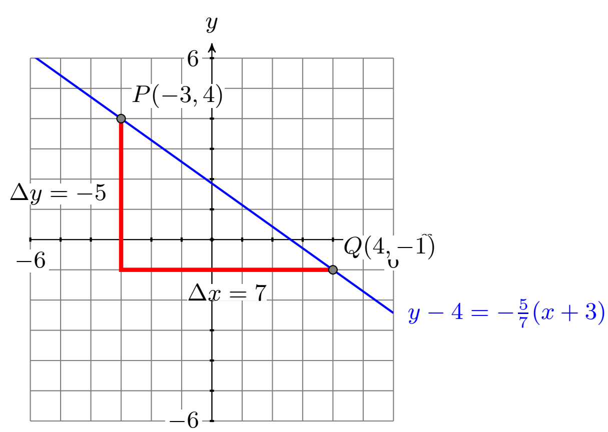


Next, substitute $m = -5/7$ and $(x_0, y_0) = (-3, 4)$ in the point-slope form of the line:

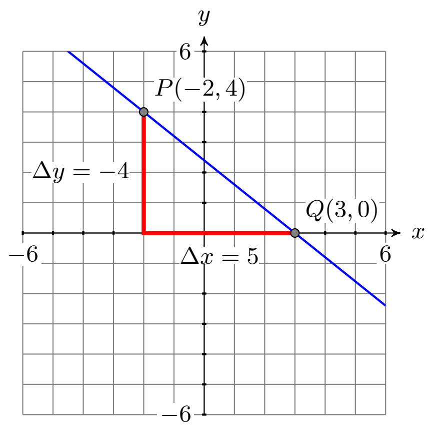
$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

$$y - 4 = -\frac{5}{7}(x - (-3)) \qquad \text{Substitute: } -5/7 \text{ for } m, -3 \text{ for } x_0, \text{ and } 4 \text{ for } y_0$$

Hence, the equation of the line is $y - 4 = -\frac{5}{7}(x + 3)$. Label the line with its equation.



3. First, plot the point $P(-2, 4)$. Then, because the slope is $\Delta y/\Delta x = -4/5$, start at the point $P(-2, 4)$, then move 4 units downward and 5 units to the right, arriving at the point $Q(3, 0)$.

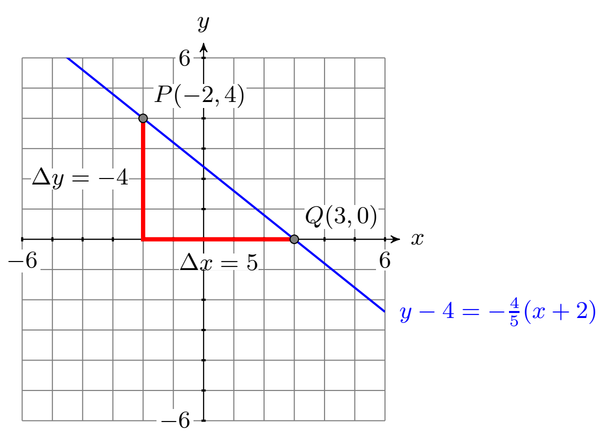


Next, substitute $m = -4/5$ and $(x_0, y_0) = (-2, 4)$ in the point-slope form of the line:

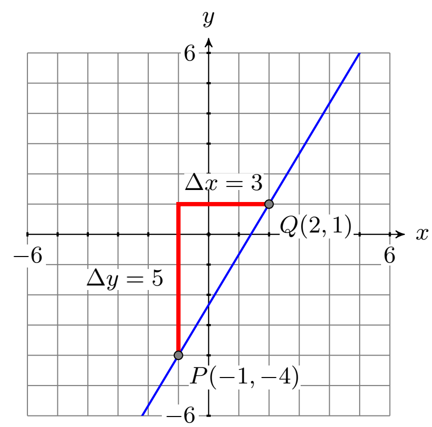
$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

$$y - 4 = -\frac{4}{5}(x - (-2)) \qquad \text{Substitute: } -4/5 \text{ for } m,\ -2 \text{ for } x_0, \text{ and 4 for } y_0$$

Hence, the equation of the line is $y - 4 = -\frac{4}{5}(x + 2)$. Label the line with its equation.



5. First, plot the point $P(-1, -4)$. Then, because the slope is $\Delta y/\Delta x = 5/3$, start at the point $P(-1, -4)$, then move 5 units upward and 3 units to the right, arriving at the point $Q(2, 1)$.

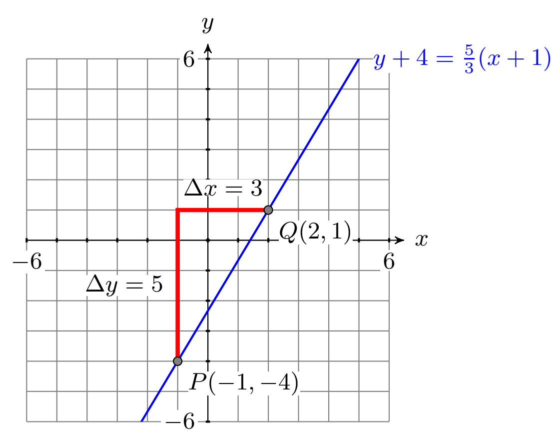


Next, substitute $m = 5/3$ and $(x_0, y_0) = (-1, -4)$ in the point-slope form of the line:

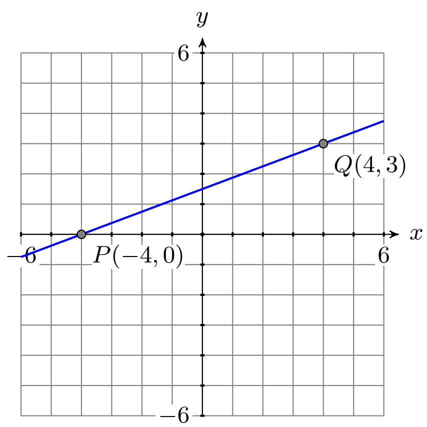
$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

$$y - (-4) = \frac{5}{3}(x - (-1)) \qquad \text{Substitute: } 5/3 \text{ for } m, -1 \text{ for } x_0, \text{ and } -4 \text{ for } y_0$$

Hence, the equation of the line is $y + 4 = \frac{5}{3}(x + 1)$. Label the line with its equation.



7. Plot the points $P(-4, 0)$ and $Q(4, 3)$ and draw a line through them.



Subtract the coordinates of poing $P(-4,0)$ from the point $Q(4,3)$.

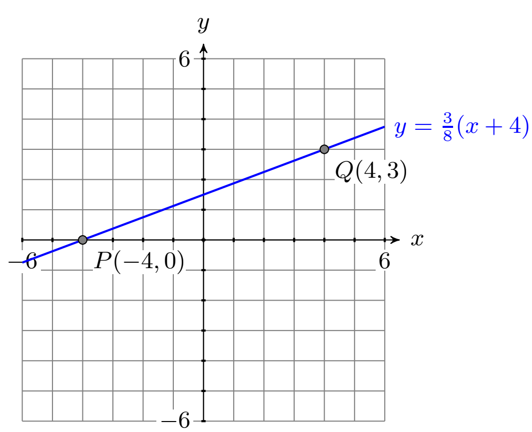
$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{3-(0)}{4-(-4)} \\ &= \frac{3}{8} \end{aligned}$$

Next, substitute $m = 3/8$, then substitute either point $P(-4,0)$ or point $Q(4,3)$ for (x_0, y_0) in the point-slope form of the line. We'll substitute $P(-4,0)$ for (x_0, y_0).

$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

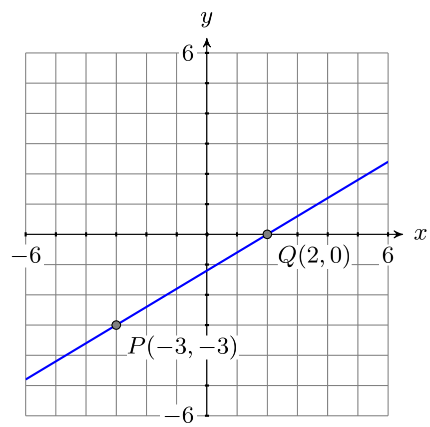
$$y - (0) = \frac{3}{8}(x - (-4)) \qquad \text{Substitute: } 3/8 \text{ for } m, -4 \text{ for } x_0, \text{ and } 0 \text{ for } y_0$$

Hence, the equation of the line is $y = \frac{3}{8}(x+4)$. Label the line with its equation.



Alternately, if you substitute the point $Q(4,3)$ for (x_0, y_0) in the point-slope form $y - y_0 = m(x - x_0)$, you get an equivalent equation $y - 3 = \frac{3}{8}(x - 4)$.

9. Plot the points $P(-3,-3)$ and $Q(2,0)$ and draw a line through them.



Subtract the coordinates of poing $P(-3,-3)$ from the point $Q(2,0)$.

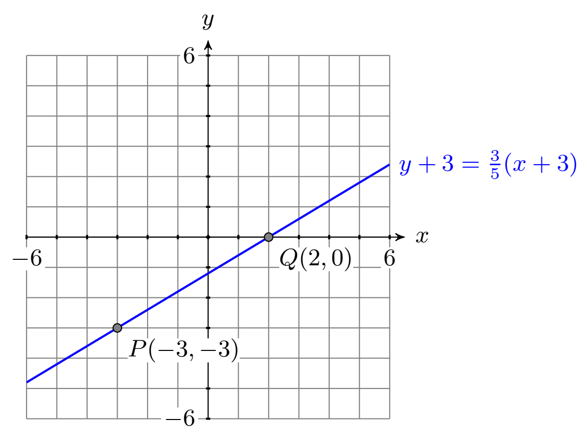
$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{0-(-3)}{2-(-3)} \\ &= \frac{3}{5} \end{aligned}$$

Next, substitute $m = 3/5$, then substitute either point $P(-3,-3)$ or point $Q(2,0)$ for (x_0, y_0) in the point-slope form of the line. We'll substitute $P(-3,-3)$ for (x_0, y_0).

$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

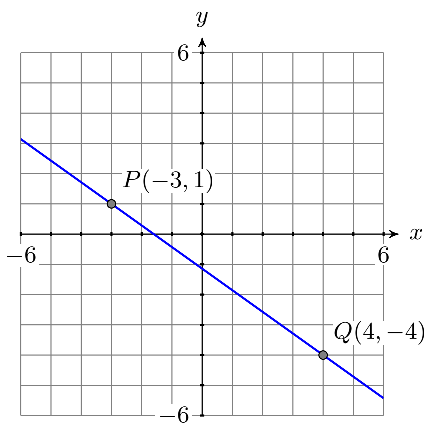
$$y - (-3) = \frac{3}{5}(x - (-3)) \qquad \text{Substitute: } 3/5 \text{ for } m,\ -3 \text{ for } x_0, \text{ and } -3 \text{ for } y_0$$

Hence, the equation of the line is $y + 3 = \frac{3}{5}(x + 3)$. Label the line with its equation.



Alternately, if you substitute the point $Q(2,0)$ for (x_0, y_0) in the point-slope form $y - y_0 = m(x - x_0)$, you get an equivalent equation $y = \frac{3}{5}(x - 2)$.

11. Plot the points $P(-3,1)$ and $Q(4,-4)$ and draw a line through them.



Subtract the coordinates of poing $P(-3,1)$ from the point $Q(4,-4)$.

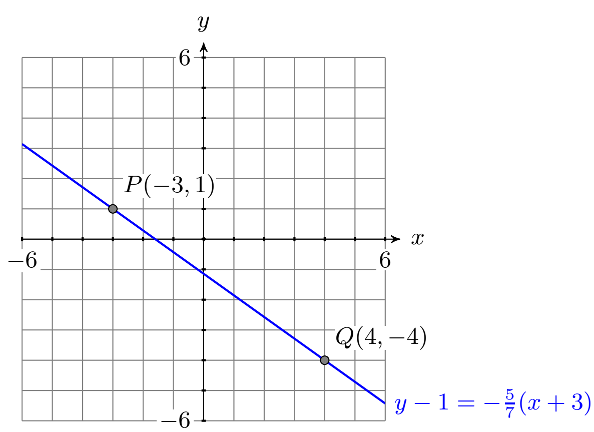
$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4-1}{4-(-3)} \\ &= -\frac{5}{7} \end{aligned}$$

Next, substitute $m = -5/7$, then substitute either point $P(-3,1)$ or point $Q(4,-4)$ for (x_0, y_0) in the point-slope form of the line. We'll substitute $P(-3,1)$ for (x_0, y_0).

$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

$$y - 1 = -\frac{5}{7}(x - (-3)) \qquad \text{Substitute: } -5/7 \text{ for } m,\ -3 \text{ for } x_0, \text{ and 1 for } y_0$$

Hence, the equation of the line is $y - 1 = -\frac{5}{7}(x+3)$. Label the line with its equation.

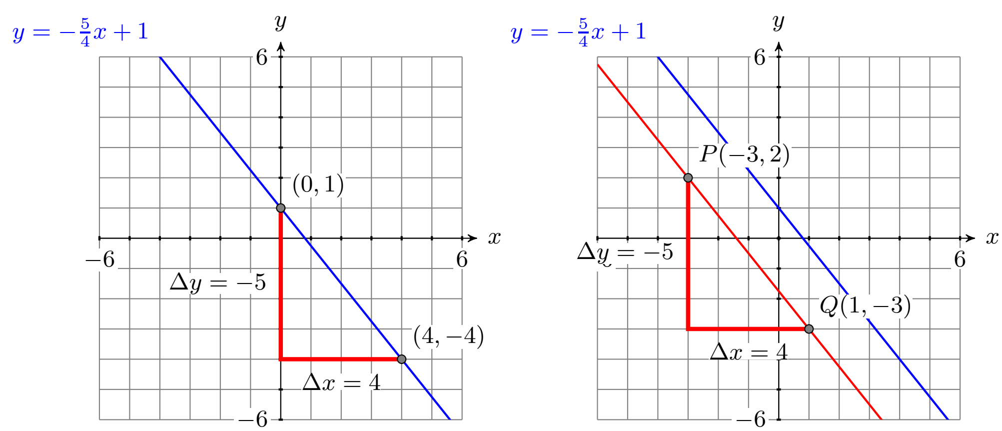


Alternately, if you substitute the point $Q(4,-4)$ for (x_0, y_0) in the point-slope form $y - y_0 = m(x - x_0)$, you get an equivalent equation $y + 4 = -\frac{5}{7}(x-4)$.

13. Start by plotting the line $y = -\frac{5}{4}x + 1$. Comparing this equation with the slope-intercept form $y = mx + b$, we note that $m = -5/4$ and $b = 1$. Thus,

the slope is $-5/4$ and the y-intercept is $(0,1)$. First plot the y-intercept $(0,1)$. Because the slope is $\Delta y/\Delta x = -5/4$, start at the y-intercept, then move -5 units downward and 4 units to the right, arriving at the point $(4,-4)$. Draw the line $y = -\frac{5}{4}x + 1$ through these two points.

Next, plot the point $P(-3,2)$. Because the line through P must be parallel to the first line, it must have the same slope $-5/4$. Start at the point $P(-3,2)$, then move -5 units downward and 4 units to the right, arriving at the point $Q(1,-3)$. Draw the line through these two points.

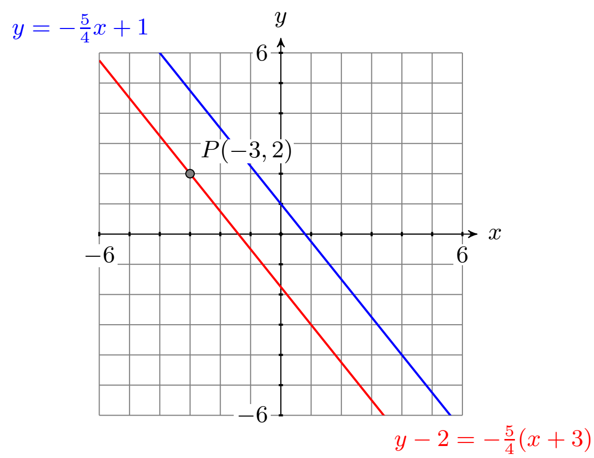


To find the equation of the second line, we'll substitute the point $(x_0, y_0) = (-3, 2)$ and slope $m = -5/4$ into the point-slope form of the line.

$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

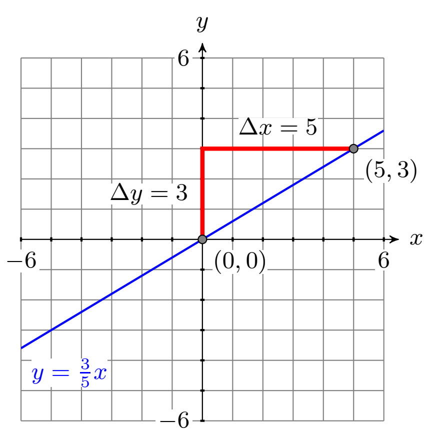
$$y - 2 = -\frac{5}{4}(x - (-3)) \qquad \text{Substitute: } -5/4 \text{ for } m,\ -3 \text{ for } x_0, \text{ and 2 for } y_0.$$

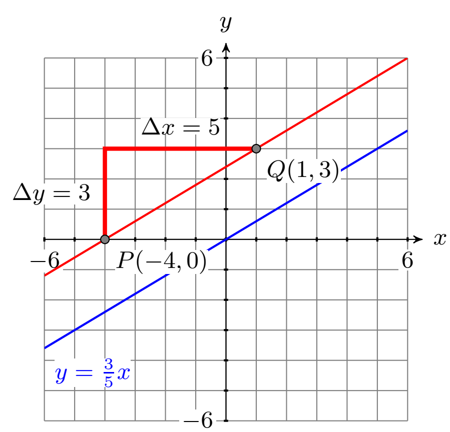
Hence, the equation of the line is $y - 2 = -\frac{5}{4}(x+3)$. Label the line with its equation.



15. Start by plotting the line $y = \frac{3}{5}x$. Comparing this equation with the slope-intercept form $y = mx + b$, we note that $m = 3/5$ and $b = 0$. Thus, the slope is $3/5$ and the y-intercept is $(0, 0)$. First plot the y-intercept $(0, 0)$. Because the slope is $\Delta y/\Delta x = 3/5$, start at the y-intercept, then move 3 units upward and 5 units to the right, arriving at the point $(5, 3)$. Draw the line $y = \frac{3}{5}x$ through these two points.

Next, plot the point $P(-4, 0)$. Because the line through P must be parallel to the first line, it must have the same slope $3/5$. Start at the point $P(-4, 0)$, then move 3 units upward and 5 units to the right, arriving at the point $Q(1, 3)$. Draw the line through these two points.



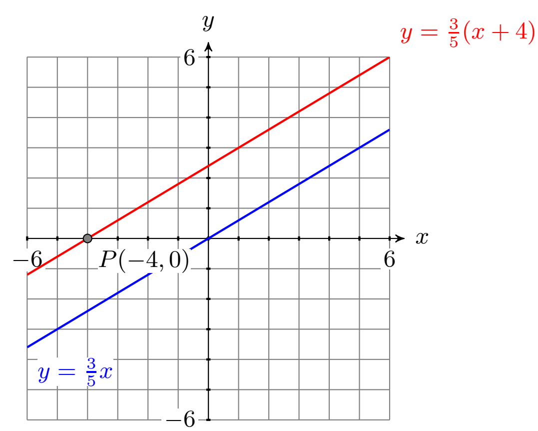


To find the equation of the second line, we'll substitute the point $(x_0, y_0) =$

$(-4, 0)$ and slope $m = 3/5$ into the point-slope form of the line.

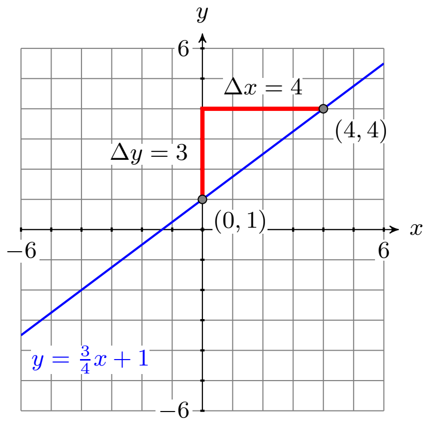
$$\begin{aligned} y - y_0 &= m(x - x_0) && \text{Point-slope form.} \\ y - 0 &= \frac{3}{5}(x - (-4)) && \text{Substitute: } 3/5 \text{ for } m, -4 \text{ for } x_0, \\ & && \text{and } 0 \text{ for } y_0. \end{aligned}$$

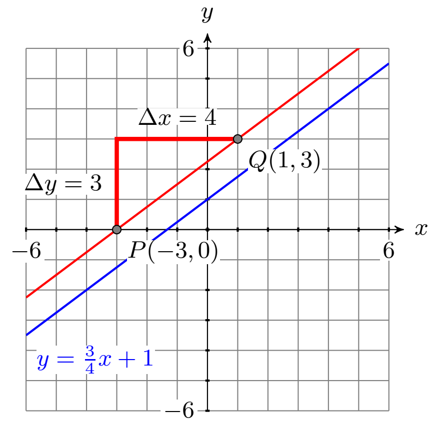
Hence, the equation of the line is $y = \frac{3}{5}(x+4)$. Label the line with its equation.



17. Start by plotting the line $y = \frac{3}{4}x + 1$. Comparing this equation with the slope-intercept form $y = mx + b$, we note that $m = 3/4$ and $b = 1$. Thus, the slope is $3/4$ and the y-intercept is $(0, 1)$. First plot the y-intercept $(0, 1)$. Because the slope is $\Delta y/\Delta x = 3/4$, start at the y-intercept, then move 3 units upward and 4 units to the right, arriving at the point $(4, 4)$. Draw the line $y = \frac{3}{4}x + 1$ through these two points.

Next, plot the point $P(-3, 0)$. Because the line through P must be parallel to the first line, it must have the same slope $3/4$. Start at the point $P(-3, 0)$, then move 3 units upward and 4 units to the right, arriving at the point $Q(1, 3)$. Draw the line through these two points.



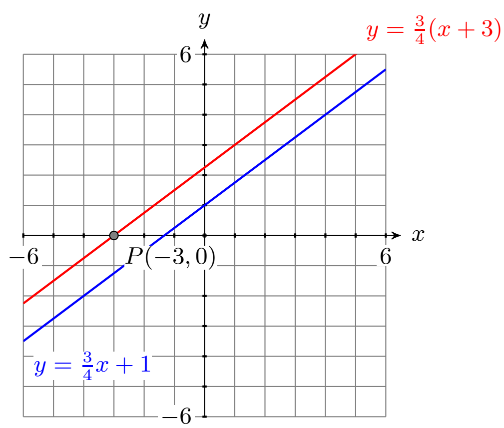


To find the equation of the second line, we'll substitute the point $(x_0, y_0) = (-3, 0)$ and slope $m = 3/4$ into the point-slope form of the line.

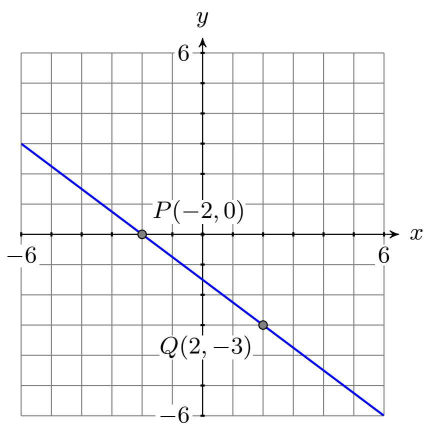
$$y - y_0 = m(x - x_0) \qquad \text{Point-slope form.}$$

$$y - 0 = \frac{3}{4}(x - (-3)) \qquad \text{Substitute: } 3/4 \text{ for } m,\ -3 \text{ for } x_0, \text{ and } 0 \text{ for } y_0.$$

Hence, the equation of the line is $y = \frac{3}{4}(x+3)$. Label the line with its equation.



19. Start by plotting the line passing through the points $P(-2, 0)$ and $Q(2, -3)$ and then calculating its slope.

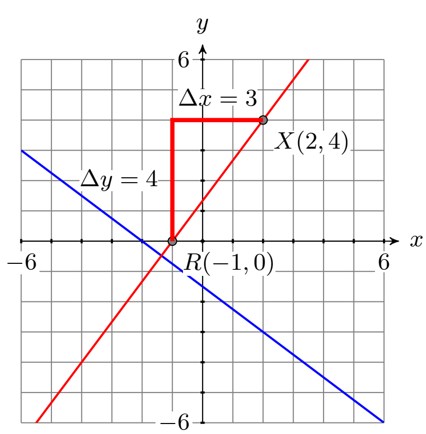


Subtract coordinates of point $P(-2, 0)$ from the coordinates of the point $Q(2, -3)$.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-3-(0)}{2-(-2)} \\ &= -\frac{3}{4} \end{aligned}$$

Thus, the slope of the line through points P and Q is $-3/4$. The slope of any line perpendicular to this line is the negative reciprocal of this number, that is $4/3$.

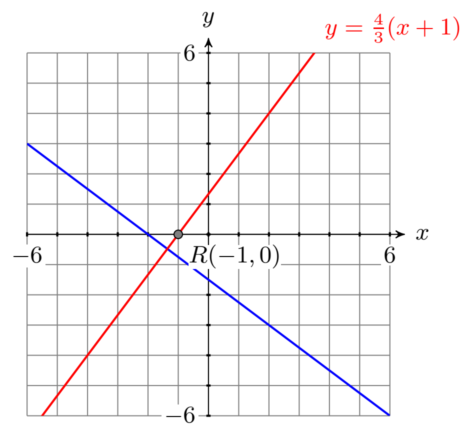
The next step is to plot the point $R(-1, 0)$. To draw a line through R with slope $4/3$, start at the point R, then move upward 4 units and right 3 units, arriving at the point $X(2, 4)$.



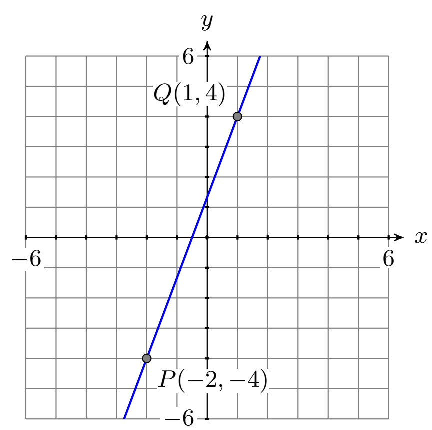
To find the equation of the perpendicular line, substitute $4/3$ for m and $(-1, 0)$ for (x_0, y_0) in the point-slope form.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - (0) &= \frac{4}{3}(x - (-1)) \end{aligned}$$

Hence, the equation line through the point R perpendicular to the line through points P and Q is $y = \frac{4}{3}(x + 1)$. Label the line with its equation.



21. Start by plotting the line passing through the points $P(-2, -4)$ and $Q(1, 4)$ and then calculating its slope.

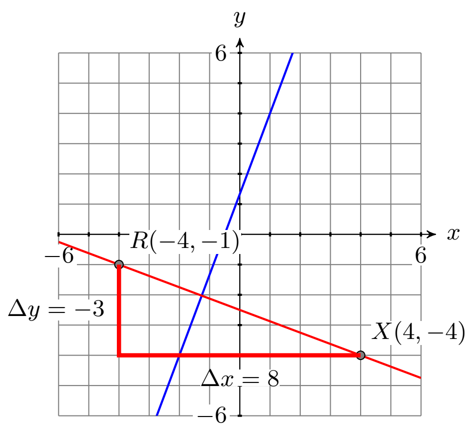


Subtract coordinates of point $P(-2, -4)$ from the coordinates of the point $Q(1, 4)$.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{4 - (-4)}{1 - (-2)} \\ &= \frac{8}{3} \end{aligned}$$

Thus, the slope of the line through points P and Q is $8/3$. The slope of any line perpendicular to this line is the negative reciprocal of this number, that is $-3/8$.

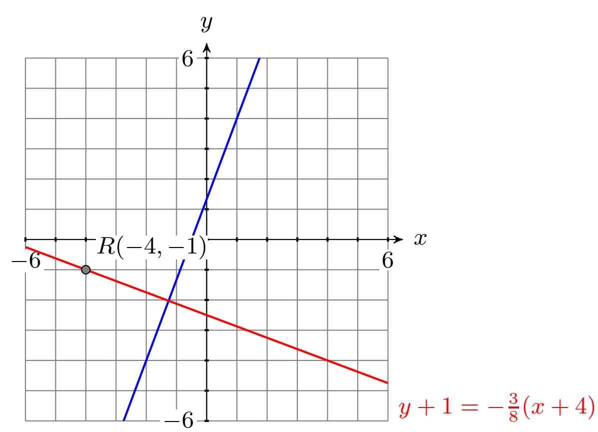
The next step is to plot the point $R(-4, -1)$. To draw a line through R with slope $-3/8$, start at the point R, then move downward 3 units and right 8 units, arriving at the point $X(4, -4)$.



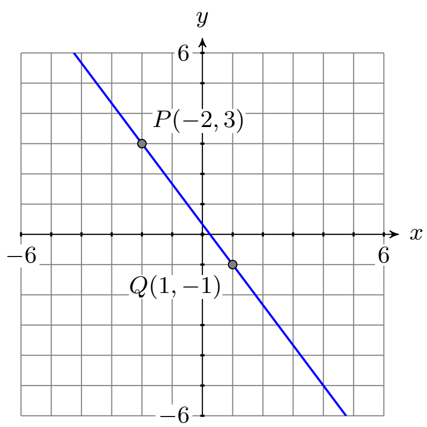
To find the equation of the perpendicular line, substitute $-3/8$ for m and $(-4, -1)$ for (x_0, y_0) in the point-slope form.

$$y - y_0 = m(x - x_0)$$
$$y - (-1) = -\frac{3}{8}(x - (-4))$$

Hence, the equation line through the point R perpendicular to the line through points P and Q is $y + 1 = -\frac{3}{8}(x + 4)$. Label the line with its equation.



23. Start by plotting the line passing through the points $P(-2, 3)$ and $Q(1, -1)$ and then calculating its slope.

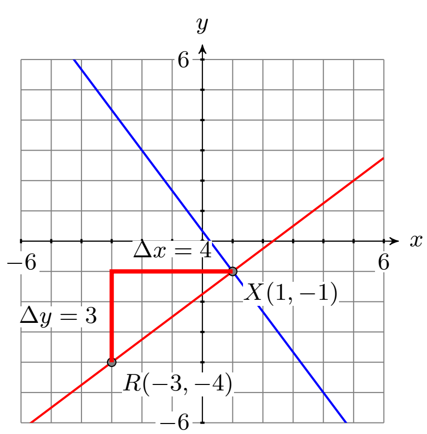


Subtract coordinates of point $P(-2, 3)$ from the coordinates of the point $Q(1, -1)$.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-1-3}{1-(-2)} \\ &= -\frac{4}{3} \end{aligned}$$

Thus, the slope of the line through points P and Q is $-4/3$. The slope of any line perpendicular to this line is the negative reciprocal of this number, that is $3/4$.

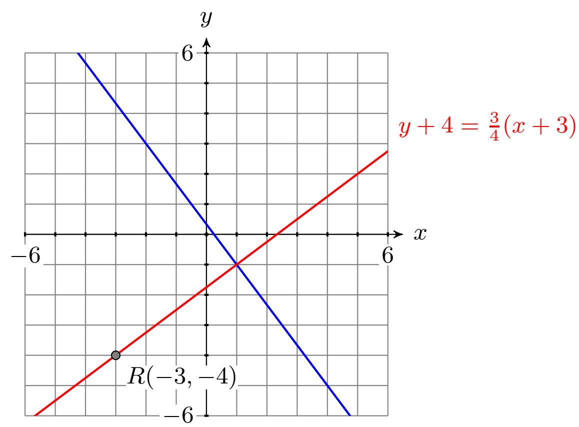
The next step is to plot the point $R(-3, -4)$. To draw a line through R with slope $3/4$, start at the point R, then move upward 3 units and right 4 units, arriving at the point $X(1, -1)$.



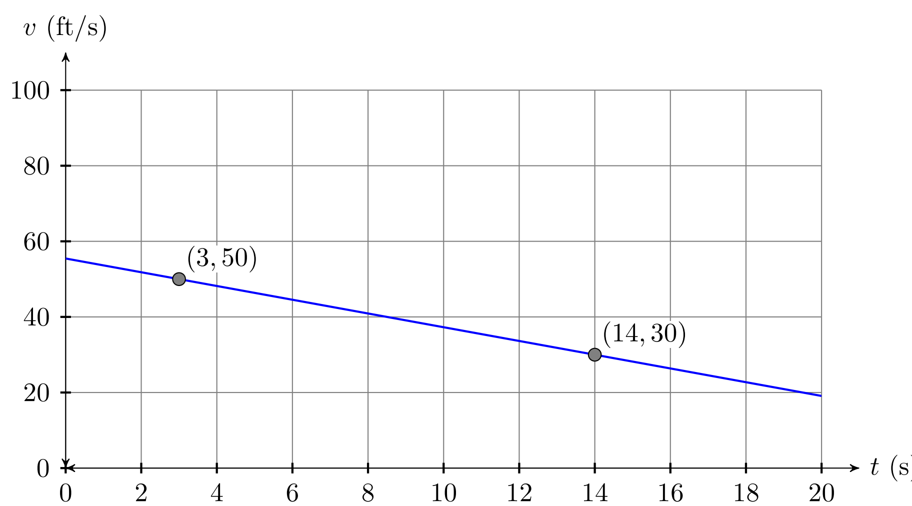
To find the equation of the perpendicular line, substitute $3/4$ for m and $(-3, -4)$ for (x_0, y_0) in the point-slope form.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - (-4) &= \frac{3}{4}(x - (-3)) \end{aligned}$$

Hence, the equation line through the point R perpendicular to the line through points P and Q is $y + 4 = \frac{3}{4}(x + 3)$. Label the line with its equation.



25. Set up a coordinate system, placing the time t on the horizontal axis and velocity v on the vertical axis. Label and scale each axis, including units in your labels. Plot the points $(3, 50)$ and $(14, 30)$ and draw a line through the points.



Compute the slope.

$$\begin{aligned}\text{Slope} &= \frac{30 - 50}{14 - 3}\\ &= \frac{-20}{11}\end{aligned}$$

Substitute $-20/11$ for m and $(3, 50)$ for (x_0, y_0) in the point-slope form.

$$\begin{aligned}y - y_0 &= m(x - x_0)\\ y - 50 &= -\frac{20}{11}(x - 3)\end{aligned}$$

Replace x with t and y with v.

$$v - 50 = -\frac{20}{11}(t - 3) \qquad \text{Point-slope form.}$$

Solve for v.

$$\begin{aligned}
v - 50 &= -\frac{20}{11}t + \frac{60}{11} && \text{Distribute } -20/11. \\
v - 50 + 50 &= -\frac{20}{11}t + \frac{60}{11} + 50 && \text{Add 50 to both sides.} \\
v &= -\frac{20}{11}t + \frac{60}{11} + \frac{550}{11} && \text{On the left, simplify. On the right make equivalent fractions, with a common denominator.} \\
v &= -\frac{20}{11}t + \frac{610}{11} && \text{Simplify.}
\end{aligned}$$

If the time is 6 seconds, then:

$$\begin{aligned}
v &= -\frac{20}{11}(6) + \frac{610}{11} \\
v &= 44.54545454545
\end{aligned}$$

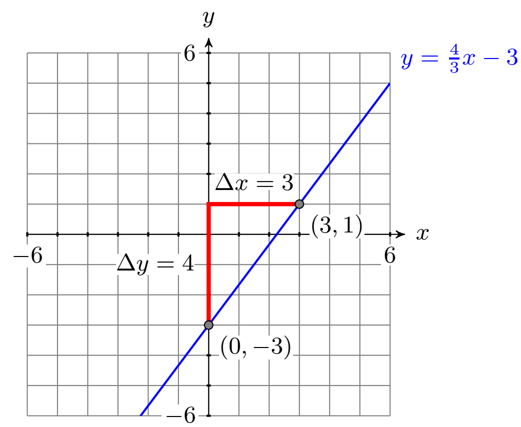
A calculator was used to approximate the last computation. Rounded to the nearest second, the velocity of the object is $v = 44.5$ seconds.

3.6 Standard Form of a Line

1. First, solve the equation $4x - 3y = 9$ for y:

$$\begin{aligned}
4x - 3y &= 9 && \text{Standard form of line.} \\
4x - 3y - 4x &= 9 - 4x && \text{Subtract } 4x \text{ from both sides.} \\
-3y &= -4x + 9 && \text{Simplify.} \\
\frac{-3y}{-3} &= \frac{-4x + 9}{-3} && \text{Divide both sides by } -3. \\
y &= \frac{4}{3}x - 3 && \text{Distribute } -3 \text{ and simplify.}
\end{aligned}$$

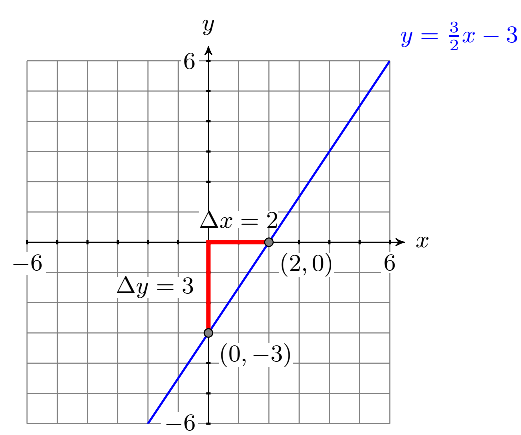
Therefore, the slope of the line is $4/3$ and the y-intercept is $(0, -3)$. To sketch the graph of the line, first plot the y-intercept, then move upward 4 units and right 3 units, arriving at the point $(3, 1)$. Draw the line through $(0, -3)$ and $(3, 1)$ and label it with its equation in slope-intercept form.



3. First, solve the equation $3x - 2y = 6$ for y:

$$\begin{aligned} 3x - 2y &= 6 && \text{Standard form of line.}\\ 3x - 2y - 3x &= 6 - 3x && \text{Subtract } 3x \text{ from both sides.}\\ -2y &= -3x + 6 && \text{Simplify.}\\ \frac{-2y}{-2} &= \frac{-3x+6}{-2} && \text{Divide both sides by } -2.\\ y &= \frac{3}{2}x - 3 && \text{Distribute } -2 \text{ and simplify.} \end{aligned}$$

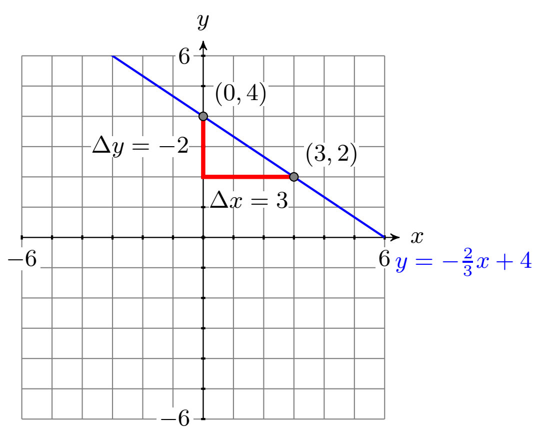
Therefore, the slope of the line is $3/2$ and the y-intercept is $(0, -3)$. To sketch the graph of the line, first plot the y-intercept, then move upward 3 units and right 2 units, arriving at the point $(2, 0)$. Draw the line through $(0, -3)$ and $(2, 0)$ and label it with its equation in slope-intercept form.



5. First, solve the equation $2x + 3y = 12$ for y:

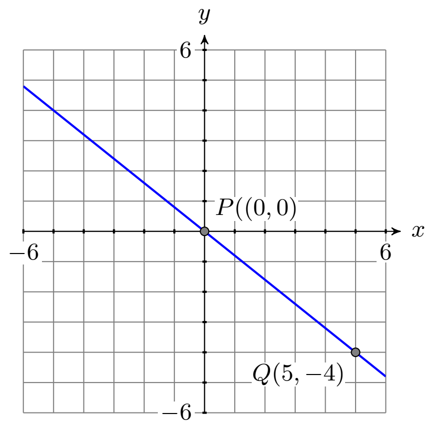
$$\begin{aligned} 2x + 3y &= 12 && \text{Standard form of line.} \\ 2x + 3y - 2x &= 12 - 2x && \text{Subtract } 2x \text{ from both sides.} \\ 3y &= -2x + 12 && \text{Simplify.} \\ \frac{3y}{3} &= \frac{-2x + 12}{3} && \text{Divide both sides by 3.} \\ y &= -\frac{2}{3}x + 4 && \text{Distribute 3 and simplify.} \end{aligned}$$

Therefore, the slope of the line is $-2/3$ and the y-intercept is $(0, 4)$. To sketch the graph of the line, first plot the y-intercept, then move downward 2 units and right 3 units, arriving at the point $(3, 2)$. Draw the line through $(0, 4)$ and $(3, 2)$ and label it with its equation in slope-intercept form.



7. First, note that the y-intercept of the line (where it crosses the y-axis) is the point $P(0, 0)$. This means that $b = 0$ in the slope-intercept formula $y = mx + b$.

Next, we need to determine the slope of the line. Try to locate a second point on the line that passes directly through a lattice point, a point where a horizontal and vertical gridline intersect. It appears that the point $Q(5, -4)$ qualifies.



Subtract the coordinates of $P(0,0)$ from the coordinates of $Q(5,-4)$ to determine the slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4-(0)}{5-0} \\ &= -\frac{4}{5} \end{aligned}$$

Finally, substitute $m = -4/5$ and $b = 0$ in the slope-intercept form of the line:

$$\begin{aligned} y &= mx + b \\ y &= -\frac{4}{5}x + (0) \end{aligned}$$

Hence, the equation of the line in slope-intercept form is $y = -\frac{4}{5}x$.

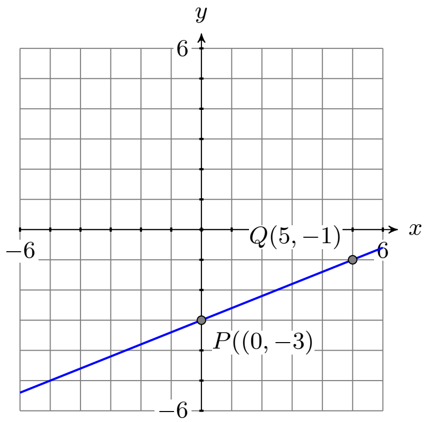
Standard form $Ax+By = C$ does not allow fractional coefficients. Thus, to put this equation into standard form, we must first clear the fractions from the equation by multiplying both sides of the equation by the least common denominator.

$$\begin{aligned} y &= -\frac{4}{5}x && \text{Slope-intercept form.} \\ 5y &= \left(-\frac{4}{5}x\right)5 && \text{Multiply both sides by 5.} \\ 5y &= -4x && \text{Distribute 5 and simplify.} \\ 5y + 4x &= -4x + 4x && \text{Add } 4x \text{ from both sides.} \\ 4x + 5y &= 0 && \text{Simplify.} \end{aligned}$$

Thus, the standard form of the line is $4x + 5y = 0$.

9. First, note that the y-intercept of the line (where it crosses the y-axis) is the point $P(0,-3)$. This means that $b = -3$ in the slope-intercept formula $y = mx + b$.

Next, we need to determine the slope of the line. Try to locate a second point on the line that passes directly through a lattice point, a point where a horizontal and vertical gridline intersect. It appears that the point $Q(5,-1)$ qualifies.



Subtract the coordinates of $P(0, -3)$ from the coordinates of $Q(5, -1)$ to determine the slope:

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-1-(-3)}{5-0} \\ &= \frac{2}{5} \end{aligned}$$

Finally, substitute $m = 2/5$ and $b = -3$ in the slope-intercept form of the line:

$$\begin{aligned} y &= mx + b \\ y &= \frac{2}{5}x + (-3) \end{aligned}$$

Hence, the equation of the line in slope-intercept form is $y = \frac{2}{5}x - 3$.

Standard form $Ax+By = C$ does not allow fractional coefficients. Thus, to put this equation into standard form, we must first clear the fractions from the equation by multiplying both sides of the equation by the least common denominator.

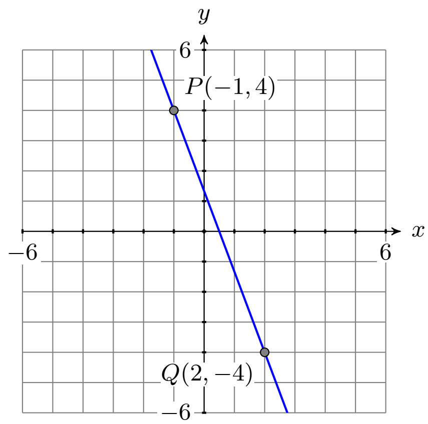
$$\begin{aligned} y &= \frac{2}{5}x - 3 && \text{Slope-intercept form.} \\ 5y &= \left(\frac{2}{5}x - 3\right)5 && \text{Multiply both sides by 5.} \\ 5y &= 2x - 15 && \text{Distribute 5 and simplify.} \\ 5y - 2x &= 2x - 15 - 2x && \text{Subtract } 2x \text{ from both sides.} \\ -2x + 5y &= -15 && \text{Simplify.} \end{aligned}$$

Standard form $Ax + By = C$ requires that $A \geq 0$, so we multiply both sides by -1 to finish.

$$\begin{aligned} -1\,(-2x + 5y) &= (-15)(-1) && \text{Multiply both sides by } -1. \\ 2x - 5y &= 15 && \text{Distribute } -1. \end{aligned}$$

Thus, the standard form of the line is $2x - 5y = 15$.

11. First, plot the points $P(-1, 4)$ and $Q(2, -4)$ and draw a line through them.



Next, determine the slope then use the point-slope form to determine the equation of the line.

Subtract the coordinates of the point $P(-1,4)$ from the coordinates of the point $Q(2,-4)$.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-4-4}{2-(-1)} \\ &= -\frac{8}{3} \end{aligned}$$

Substitute $-8/3$ for m and $(-1,4)$ for (x_0, y_0) in the point-slope form.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 4 &= -\frac{8}{3}(x - (-1)) \\ y - 4 &= -\frac{8}{3}(x+1) \end{aligned}$$

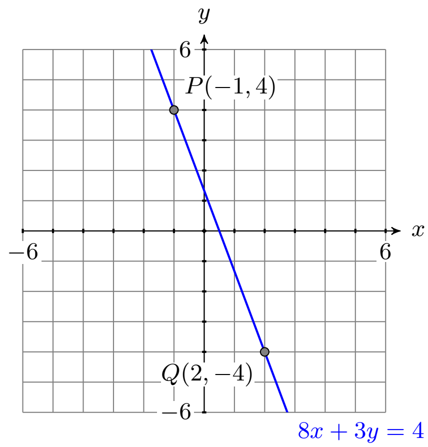
Next, we need to put $y - 4 = -\frac{8}{3}(x+1)$ into standard form. Standard form does not allow fractions as coefficients, so the first step is to clear fractions from the equation.

$$\begin{aligned} y - 4 &= -\frac{8}{3}(x+1) && \text{Point-slope form.} \\ y - 4 &= -\frac{8}{3}x - \frac{8}{3} && \text{Distribute } -\frac{8}{3}. \end{aligned}$$

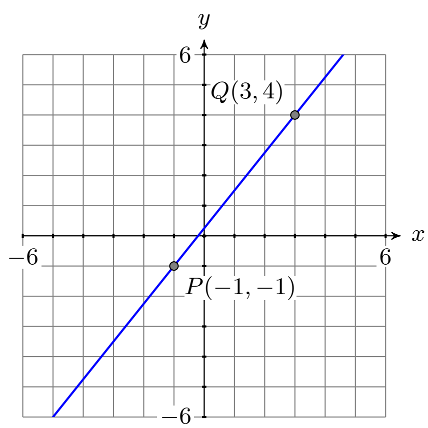
Clear fractions by multiplying both sides by the least common denominator.

$$\begin{aligned} 3(y-4) &= \left(-\frac{8}{3}x - \frac{8}{3}\right)3 && \text{Multiply both sides by 3.} \\ 3y - 12 &= -8x - 8 && \text{Distribute and simplify.} \\ 3y - 12 + 8x &= -8x - 8 + 8x && \text{Add } 8x \text{ to both sides.} \\ 8x + 3y - 12 &= -8 && \text{Simplify.} \\ 8x + 3y - 12 + 12 &= -8 + 12 && \text{Add 12 to both sides.} \\ 8x + 3y &= 4 \end{aligned}$$

Label the graph of the line with its equation in standard form.



13. First, plot the points $P(-1,-1)$ and $Q(3,4)$ and draw a line through them.



Next, determine the slope then use the point-slope form to determine the equation of the line.

Subtract the coordinates of the point $P(-1,-1)$ from the coordinates of the point $Q(3,4)$.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{4-(-1)}{3-(-1)} \\ &= \frac{5}{4} \end{aligned}$$

Substitute $5/4$ for m and $(-1,-1)$ for (x_0,y_0) in the point-slope form.

$$\begin{aligned} y - y_0 &= m(x-x_0) \\ y-(-1) &= \frac{5}{4}(x-(-1)) \\ y+1 &= \frac{5}{4}(x+1) \end{aligned}$$

Next, we need to put $y+1=\frac{5}{4}(x+1)$ into standard form. Standard form does not allow fractions as coefficients, so the first step is to clear fractions from the equation.

$$\begin{aligned} y+1 &= \frac{5}{4}(x+1) && \text{Point-slope form.} \\ y+1 &= \frac{5}{4}x+\frac{5}{4} && \text{Distribute } \frac{5}{4}. \end{aligned}$$

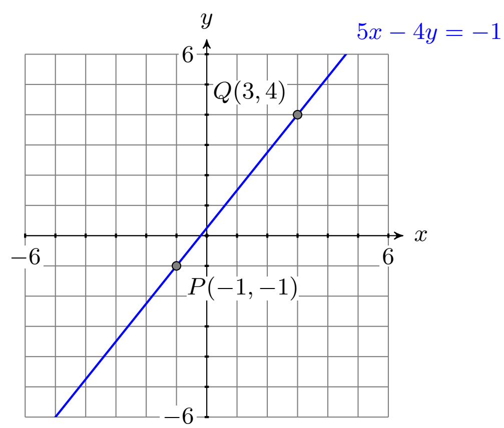
Clear fractions by multiplying both sides by the least common denominator.

$$\begin{aligned} 4(y+1) &= \left(\frac{5}{4}x+\frac{5}{4}\right)4 && \text{Multiply both sides by 4.} \\ 4y+4 &= 5x+5 && \text{Distribute and simplify.} \\ 4y+4-5x &= 5x+5-5x && \text{Subtract } 5x \text{ to both sides.} \\ -5x+4y+4 &= 5 && \text{Simplify.} \\ -5x+4y+4-4 &= 5-4 && \text{Subtract 4 from both sides.} \\ -5x+4y &= 1 && \text{Simplify.} \end{aligned}$$

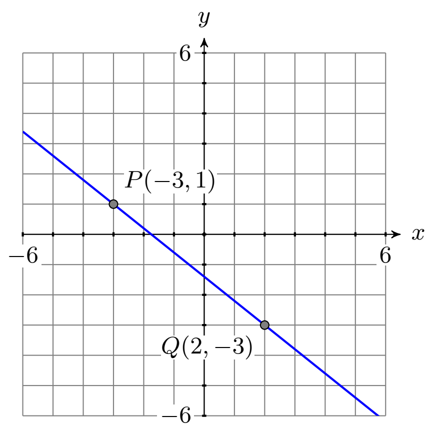
Standard form $Ax+By=C$ requires that $A \geq 0$. Thus, we need to multiply both sides by -1 so that the coefficient of x is greater than or equal to zero.

$$\begin{aligned} -1(-5x+4y) &= (1)(-1) && \text{Multiply both sides by } -1. \\ 5x-4y &= -1 && \text{Simplify.} \end{aligned}$$

Label the graph of the line with its equation in standard form.



15. First, plot the points $P(-3,1)$ and $Q(2,-3)$ and draw a line through them.



Next, determine the slope then use the point-slope form to determine the equation of the line.

Subtract the coordinates of the point $P(-3, 1)$ from the coordinates of the point $Q(2, -3)$.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{-3-1}{2-(-3)} \\ &= -\frac{4}{5} \end{aligned}$$

Substitute $-4/5$ for m and $(-3, 1)$ for (x_0, y_0) in the point-slope form.

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 1 &= -\frac{4}{5}(x - (-3)) \\ y - 1 &= -\frac{4}{5}(x + 3) \end{aligned}$$

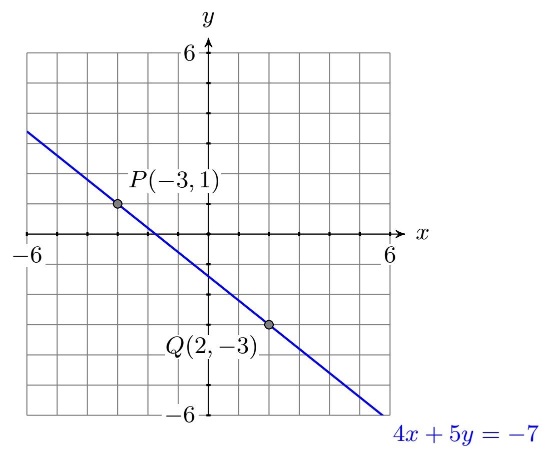
Next, we need to put $y - 1 = -\frac{4}{5}(x + 3)$ into standard form. Standard form does not allow fractions as coefficients, so the first step is to clear fractions from the equation.

$$\begin{aligned} y - 1 &= -\frac{4}{5}(x + 3) && \text{Point-slope form.} \\ y - 1 &= -\frac{4}{5}x - \frac{12}{5} && \text{Distribute } -\frac{4}{5}. \end{aligned}$$

Clear fractions by multiplying both sides by the least common denominator.

$$\begin{aligned} 5(y - 1) &= \left(-\frac{4}{5}x - \frac{12}{5}\right)5 && \text{Multiply both sides by 5.} \\ 5y - 5 &= -4x - 12 && \text{Distribute and simplify.} \\ 5y - 5 + 4x &= -4x - 12 + 4x && \text{Add } 4x \text{ to both sides.} \\ 4x + 5y - 5 &= -12 && \text{Simplify.} \\ 4x + 5y - 5 + 5 &= -12 + 5 && \text{Add 5 to both sides.} \\ 4x + 5y &= -7 \end{aligned}$$

Label the graph of the line with its equation in standard form.



17. First, find the x-and y-intercepts.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 2x - 5y &= 10 \\ 2x - 5(0) &= 10 \\ 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \\ x &= 5 \end{aligned}$$

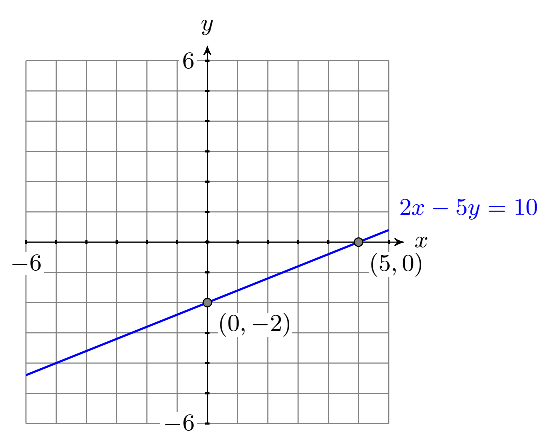
The x-intercept is $(5, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 2x - 5y &= 10 \\ 2(0) - 5y &= 10 \\ -5y &= 10 \\ \frac{-5y}{-5} &= \frac{10}{-5} \\ y &= -2 \end{aligned}$$

The y-intercept is $(0, -2)$.

Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



19. First, find the x-and y-intercepts.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 3x - 2y &= 6 \\ 3x - 2(0) &= 6 \\ 3x &= 6 \\ \frac{3x}{3} &= \frac{6}{3} \\ x &= 2 \end{aligned}$$

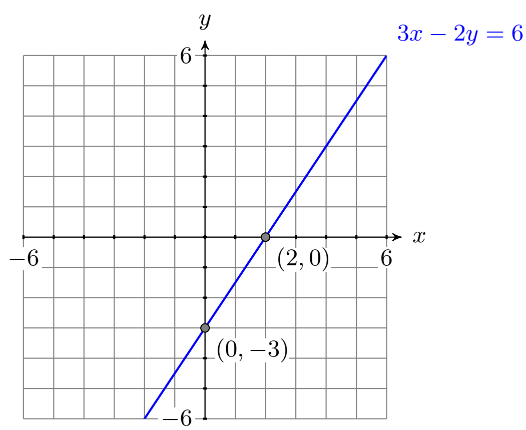
The x-intercept is $(2, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 3x - 2y &= 6 \\ 3(0) - 2y &= 6 \\ -2y &= 6 \\ \frac{-2y}{-2} &= \frac{6}{-2} \\ y &= -3 \end{aligned}$$

The y-intercept is $(0, -3)$.

Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



21. First, find the x-and y-intercepts.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 2x + 3y &= 6 \\ 2x + 3(0) &= 6 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

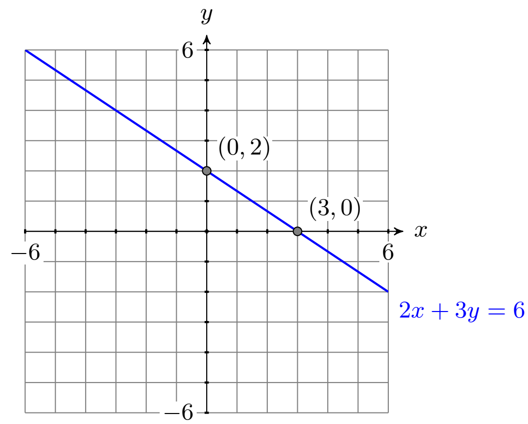
The x-intercept is $(3, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 2x + 3y &= 6 \\ 2(0) + 3y &= 6 \\ 3y &= 6 \\ \frac{3y}{3} &= \frac{6}{3} \\ y &= 2 \end{aligned}$$

The y-intercept is $(0, 2)$.

Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



23. A sketch will help maintain our focus. First, determine the x- and y-intercepts of $4x + 5y = -20$.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 4x + 5y &= -20 \\ 4x + 5(0) &= -20 \\ 4x &= -20 \\ \frac{4x}{4} &= \frac{-20}{4} \\ x &= -5 \end{aligned}$$

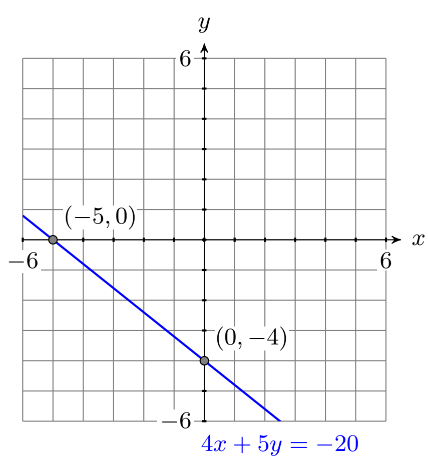
The x-intercept is $(-5, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 4x + 5y &= -20 \\ 4(0) + 5y &= -20 \\ 5y &= -20 \\ \frac{5y}{5} &= \frac{-20}{5} \\ y &= -4 \end{aligned}$$

The y-intercept is $(0, -4)$.

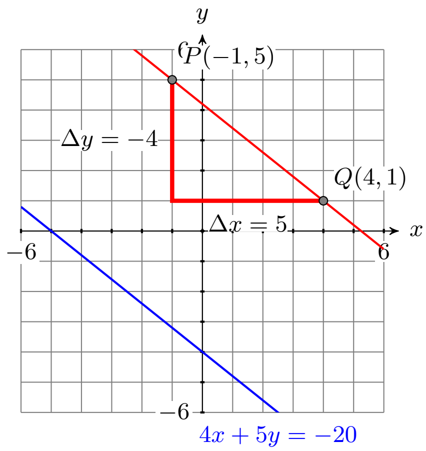
Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



Use the intercepts $(-5, 0)$ and $(0, -4)$ to determine the slope. Pick a direction to subtract and stay consistent.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{0 - (-4)}{-5 - 0} \\ &= -\frac{4}{5} \end{aligned}$$

Because the line $4x + 5y = -20$ has slope $-4/5$, the slope of any parallel line will be the same, namely $-4/5$. So, to plot the parallel line, plot the point $P(-1, 5)$, then move downwards 4 units and right 5 units, arriving at the point $Q(4, 1)$. Draw a line through points P and Q.



To find the equation of the parallel line, use the point-slope form and substitute $-4/5$ for m and $(-1, 5)$ for (x_0, y_0)

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 5 &= -\frac{4}{5}(x - (-1)) \\ y - 5 &= -\frac{4}{5}(x + 1) \end{aligned}$$

We must now put our final answer in standard form.

$$\begin{aligned} y - 5 &= -\frac{4}{5}(x + 1) && \text{Point-slope form.} \\ y - 5 &= -\frac{4}{5}x - \frac{4}{5} && \text{Distribute } -4/5. \end{aligned}$$

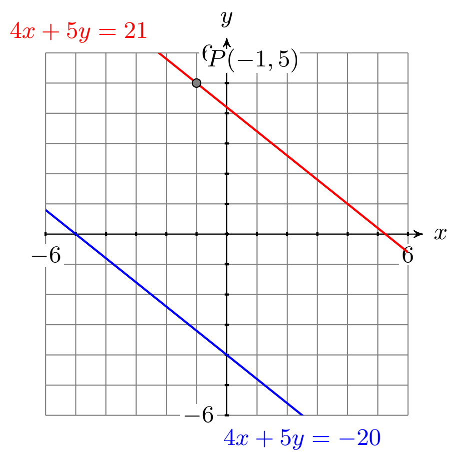
Standard form does not allow fractional coefficients. Clear the fractions by multiplying both sides by the common denominator.

$$\begin{aligned} 5(y - 5) &= \left(-\frac{4}{5}x - \frac{4}{5}\right)5 && \text{Multiply both sides by 5.} \\ 5y - 25 &= -4x - 4 && \text{Simplify.} \end{aligned}$$

We need to put our result in the standard form $Ax + By = C$.

$$\begin{aligned} 5y - 25 + 4x &= -4x - 4 + 4x && \text{Add } 4x \text{ to both sides.} \\ 4x + 5y - 25 &= -4 && \text{Simplify.} \\ 4x + 5y - 25 + 25 &= -4 + 25 && \text{Add 25 to both sides.} \\ 4x + 5y &= 21 && \text{Simplify.} \end{aligned}$$

Label the lines with their equations.



25. A sketch will help maintain our focus. First, determine the x- and y-intercepts of $5x + 2y = 10$.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 5x + 2y &= 10 \\ 5x + 2(0) &= 10 \\ 5x &= 10 \\ \frac{5x}{5} &= \frac{10}{5} \\ x &= 2 \end{aligned}$$

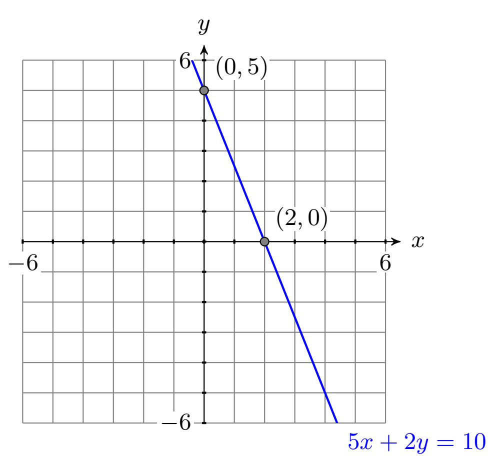
The x-intercept is $(2, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 5x + 2y &= 10 \\ 5(0) + 2y &= 10 \\ 2y &= 10 \\ \frac{2y}{2} &= \frac{10}{2} \\ y &= 5 \end{aligned}$$

The y-intercept is $(0, 5)$.

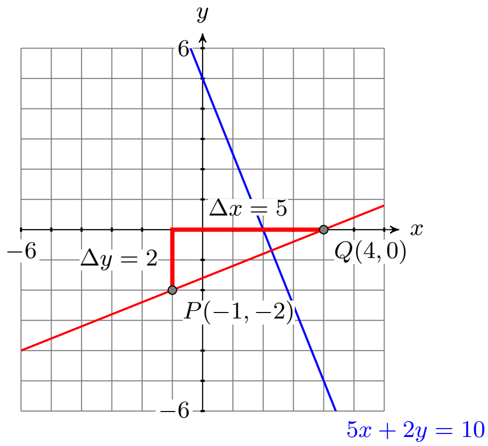
Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



Use the intercepts $(2, 0)$ and $(0, 5)$ to determine the slope. Pick a direction to subtract and stay consistent.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{0 - 5}{2 - 0} \\ &= -\frac{5}{2} \end{aligned}$$

Because the line $5x + 2y = 10$ has slope $-5/2$, the slope of any perpendicular line will be its negative reciprocal, namely $2/5$. So, to plot the perpendicular line, plot the point $P(-1, -2)$, then move upwards 2 units and right 5 units, arriving at the point $Q(4, 0)$. Draw a line through points P and Q.



To find the equation of the perpendicular line, use the point-slope form and substitute $2/5$ for m and $(-1, -2)$ for (x_0, y_0)

$$y - y_0 = m(x - x_0)$$
$$y - (-2) = \frac{2}{5}(x - (-1))$$
$$y + 2 = \frac{2}{5}(x + 1)$$

We must now put our final answer in standard form.

$$y + 2 = \frac{2}{5}(x + 1) \qquad \text{Point-slope form.}$$
$$y + 2 = \frac{2}{5}x + \frac{2}{5} \qquad \text{Distribute } 2/5.$$

Standard form does not allow fractional coefficients. Clear the fractions by multiplying both sides by the common denominator.

$$5(y + 2) = \left(\frac{2}{5}x + \frac{2}{5}\right)5 \qquad \text{Multiply both sides by 5.}$$
$$5y + 10 = 2x + 2 \qquad \text{Simplify.}$$

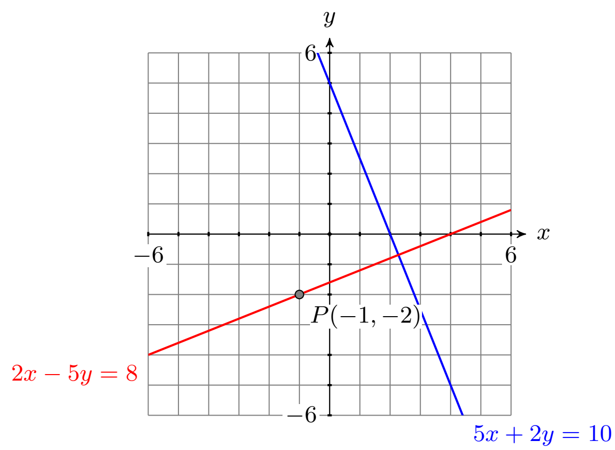
We need to put our result in the standard form $Ax + By = C$.

$$5y + 10 - 2x = 2x + 2 - 2x \qquad \text{Subtract } 2x \text{ from both sides.}$$
$$-2x + 5y + 10 = 2 \qquad \text{Simplify.}$$
$$-2x + 5y + 10 - 10 = 2 - 10 \qquad \text{Subtract 10 from both sides.}$$
$$-2x + 5y = -8 \qquad \text{Simplify.}$$

Standard form $Ax + By = C$ requires that $A \geq 0$.

$$-1(-2x + 5y) = (-8)(-1) \qquad \text{Multiply both sides by } -1.$$
$$2x - 5y = 8 \qquad \text{Distribute } -1 \text{ and simplify.}$$

Label the lines with their equations.



27. A sketch will help maintain our focus. First, determine the x- and y-intercepts of $4x + 3y = -12$.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 4x + 3y &= -12 \\ 4x + 3(0) &= -12 \\ 4x &= -12 \\ \frac{4x}{4} &= \frac{-12}{4} \\ x &= -3 \end{aligned}$$

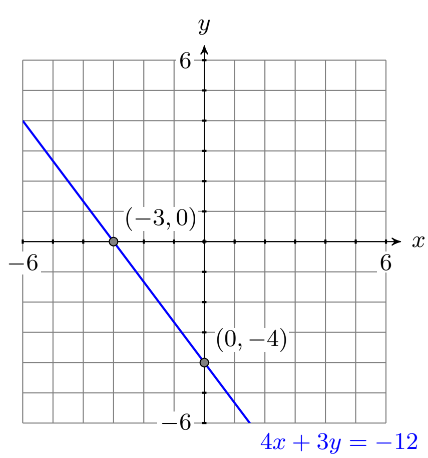
The x-intercept is $(-3, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 4x + 3y &= -12 \\ 4(0) + 3y &= -12 \\ 3y &= -12 \\ \frac{3y}{3} &= \frac{-12}{3} \\ y &= -4 \end{aligned}$$

The y-intercept is $(0, -4)$.

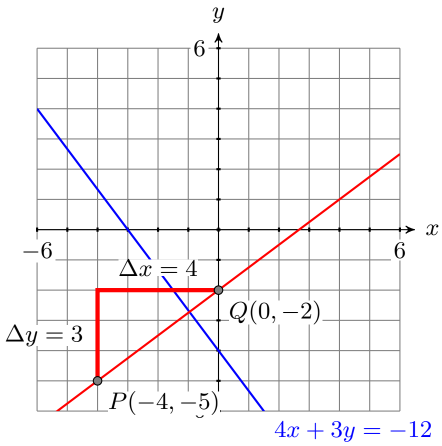
Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



Use the intercepts $(-3, 0)$ and $(0, -4)$ to determine the slope. Pick a direction to subtract and stay consistent.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{0 - (-4)}{-3 - 0} \\ &= -\frac{4}{3} \end{aligned}$$

Because the line $4x+3y=-12$ has slope $-4/3$, the slope of any perpendicular line will be its negative reciprocal, namely $3/4$. So, to plot the perpendicular line, plot the point $P(-4,-5)$, then move upwards 3 units and right 4 units, arriving at the point $Q(0,-2)$. Draw a line through points P and Q.



To find the equation of the perpendicular line, use the point-slope form and substitute $3/4$ for m and $(-4,-5)$ for (x_0, y_0)

$$y - y_0 = m(x - x_0)$$
$$y - (-5) = \frac{3}{4}(x - (-4))$$
$$y + 5 = \frac{3}{4}(x + 4)$$

We must now put our final answer in standard form.

$y + 5 = \frac{3}{4}(x + 4)$	Point-slope form.
$y + 5 = \frac{3}{4}x + 3$	Distribute 3/4.

Standard form does not allow fractional coefficients. Clear the fractions by multiplying both sides by the common denominator.

$4(y + 5) = \left(\frac{3}{4}x + 3\right)4$	Multiply both sides by 4.
$4y + 20 = 3x + 12$	Simplify.

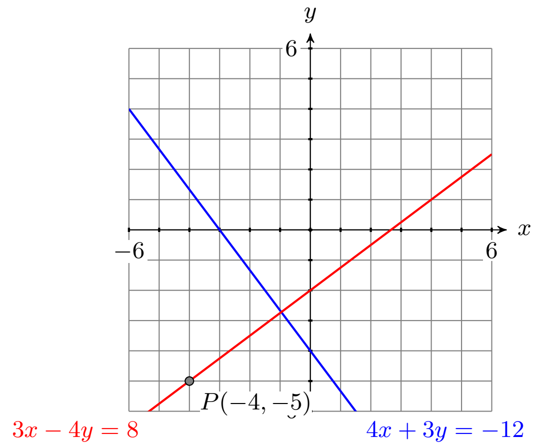
We need to put our result in the standard form $Ax + By = C$.

$4y + 20 - 3x = 3x + 12 - 3x$	Subtract $3x$ from both sides.
$-3x + 4y + 20 = 12$	Simplify.
$-3x + 4y + 20 - 20 = 12 - 20$	Subtract 20 from both sides.
$-3x + 4y = -8$	Simplify.

Standard form $Ax + By = C$ requires that $A \geq 0$.

$-1(-3x + 4y) = (-8)(-1)$	Multiply both sides by -1.
$3x - 4y = 8$	Distribute -1 and simplify.

Label the lines with their equations.



29. A sketch will help maintain our focus. First, determine the x- and y-intercepts of $5x + 4y = 20$.

To find the x-intercept, let $y = 0$.

$$\begin{aligned} 5x + 4y &= 20 \\ 5x + 4(0) &= 20 \\ 5x &= 20 \\ \frac{5x}{5} &= \frac{20}{5} \\ x &= 4 \end{aligned}$$

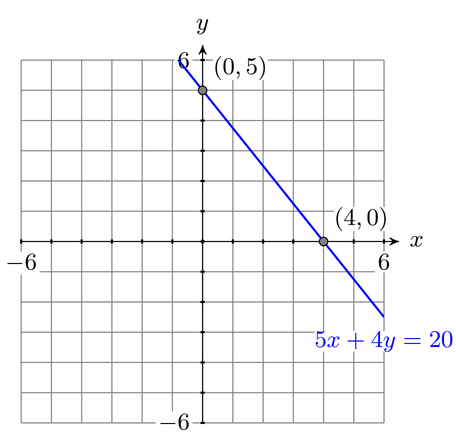
The x-intercept is $(4, 0)$.

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 5x + 4y &= 20 \\ 5(0) + 4y &= 20 \\ 4y &= 20 \\ \frac{4y}{4} &= \frac{20}{4} \\ y &= 5 \end{aligned}$$

The y-intercept is $(0, 5)$.

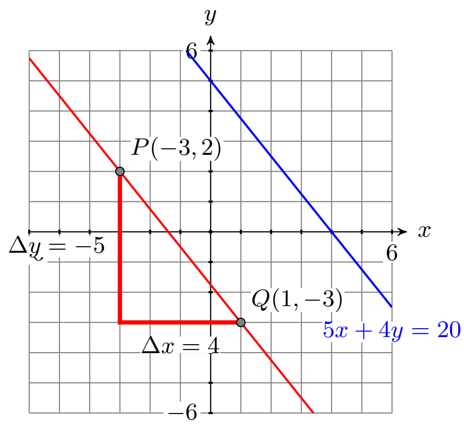
Plot the x- and y-intercepts, label them with their coordinates, then draw the line through them and label the line with its equation.



Use the intercepts $(4, 0)$ and $(0, 5)$ to determine the slope. Pick a direction to subtract and stay consistent.

$$\begin{aligned} m &= \frac{\Delta y}{\Delta x} \\ &= \frac{0 - 5}{4 - 0} \\ &= -\frac{5}{4} \end{aligned}$$

Because the line $5x + 4y = 20$ has slope $-5/4$, the slope of any parallel line will be the same, namely $-5/4$. So, to plot the parallel line, plot the point $P(-3, 2)$, then move downwards 5 units and right 4 units, arriving at the point $Q(1, -3)$. Draw a line through points P and Q.



To find the equation of the parallel line, use the point-slope form and substitute $-5/4$ for m and $(-3, 2)$ for (x_0, y_0)

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y - 2 &= -\frac{5}{4}(x - (-3)) \\ y - 2 &= -\frac{5}{4}(x + 3) \end{aligned}$$

We must now put our final answer in standard form.

$$\begin{aligned} y - 2 &= -\frac{5}{4}(x + 3) && \text{Point-slope form.} \\ y - 2 &= -\frac{5}{4}x - \frac{15}{4} && \text{Distribute } -5/4. \end{aligned}$$

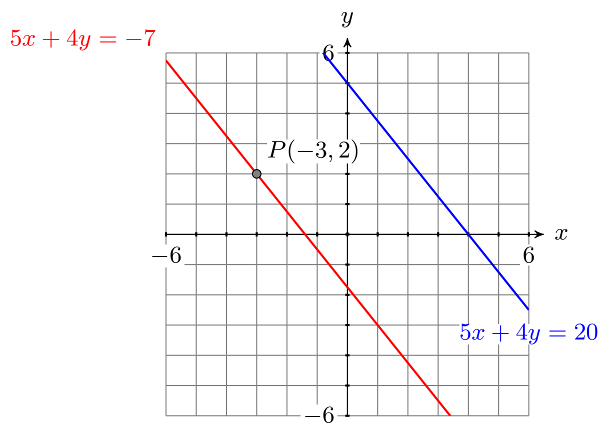
Standard form does not allow fractional coefficients. Clear the fractions by multiplying both sides by the common denominator.

$$\begin{aligned} 4(y - 2) &= \left(-\frac{5}{4}x - \frac{15}{4}\right)4 && \text{Multiply both sides by 4.} \\ 4y - 8 &= -5x - 15 && \text{Simplify.} \end{aligned}$$

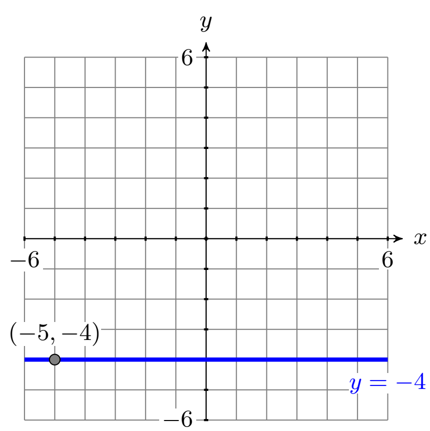
We need to put our result in the standard form $Ax + By = C$.

$$\begin{aligned} 4y - 8 + 5x &= -5x - 15 + 5x && \text{Add } 5x \text{ to both sides.} \\ 5x + 4y - 8 &= -15 && \text{Simplify.} \\ 5x + 4y - 8 + 8 &= -15 + 8 && \text{Add 8 to both sides.} \\ 5x + 4y &= -7 && \text{Simplify.} \end{aligned}$$

Label the lines with their equations.

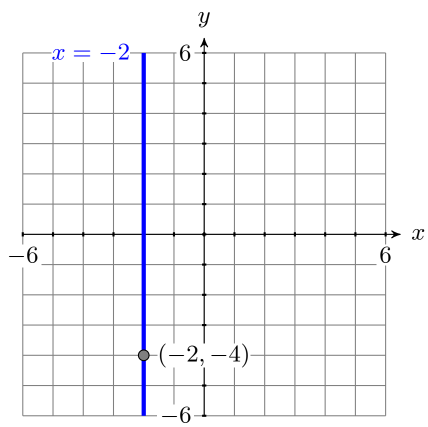


31. Plot the point $P(-5, -4)$ and draw a horizontal line through the point.



Because every point on the line has a y-value equal to -4, the equation of the line is $y = -4$.

33. Plot the point $P(-2, -4)$ and draw a vertical line through the point.



Because every point on the line has a x-value equal to -2, the equation of the line is $x = -2$.

Chapter 4

Systems

4.1 Solving Systems by Graphing

1. First, determine the x- and y-intercepts of $3x - 4y = 24$.

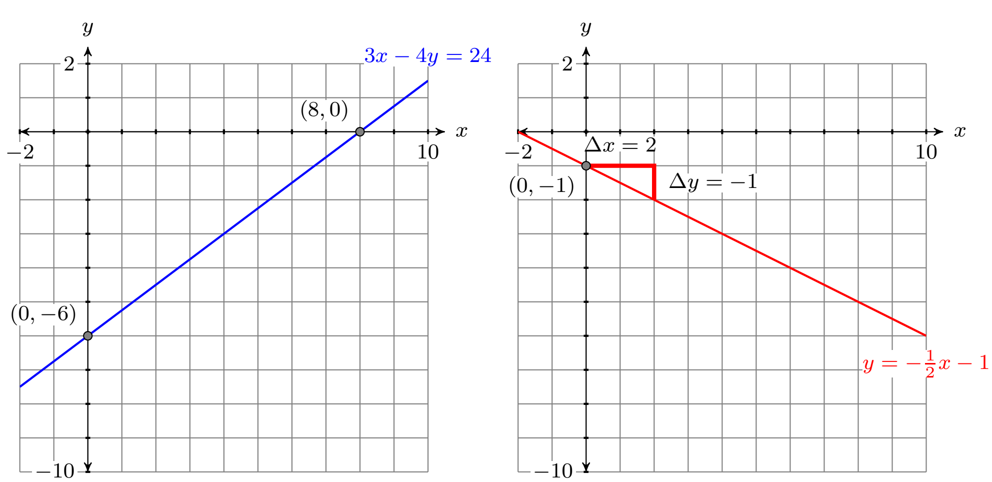
To find the x-intercept, let $y = 0$.

$$\begin{aligned} 3x - 4y &= 24 \\ 3x - 4(0) &= 24 \\ 3x &= 24 \\ x &= 8 \end{aligned}$$

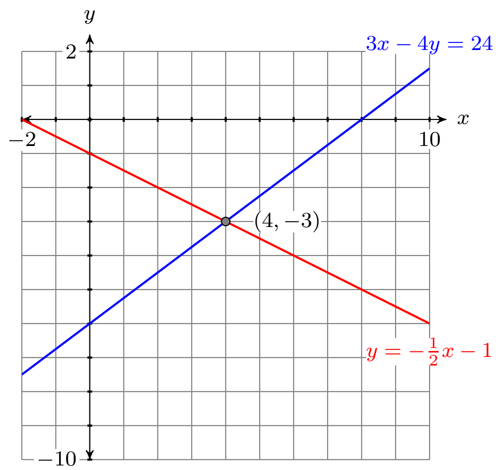
To find the y-intercept, let $x = 0$.

$$\begin{aligned} 3x - 4y &= 24 \\ 3(0) - 4y &= 24 \\ -4y &= 24 \\ y &= -6 \end{aligned}$$

Plot and label the intercepts, then draw the line $3x - 4y = 24$ through them and label it with its equation. Next, the line $y = -\frac{1}{2}x - 1$ has slope $-1/2$ and y-intercept $(0, -1)$. Plot and label $(0, -1)$, then move 2 units to the right and 1 unit down. Label the resulting line with its equation.



Next, place both lines on the same coordinate system, label each line with its equation, then label the point of intersection with its coordinates.



Substitute the point $(x, y) = (4, -3)$ in both equations to see if it checks.

$$\begin{aligned} 3x - 4y &= 24 \\ 3(4) - 4(-3) &= 24 \\ 24 &= 24 \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{2}x - 1 \\ -3 &= -\frac{1}{2}(4) - 1 \\ -3 &= -3 \end{aligned}$$

Hence, the solution $(x, y) = (4, -3)$ checks.

3. First, determine the x- and y-intercepts of $2x + y = 6$.

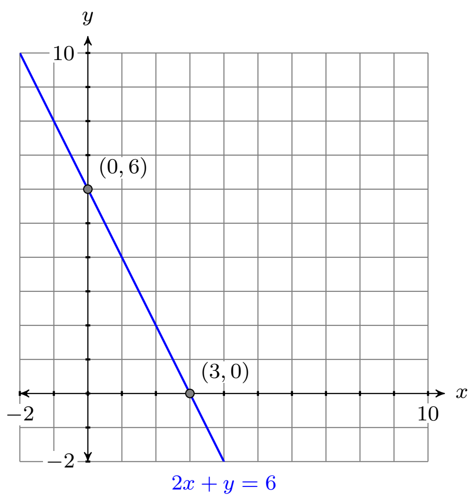
To find the x-intercept, let $y = 0$.

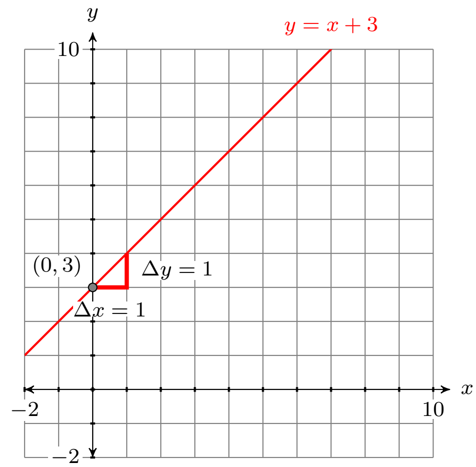
$$\begin{aligned} 2x + y &= 6 \\ 2x + (0) &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

To find the y-intercept, let $x = 0$.

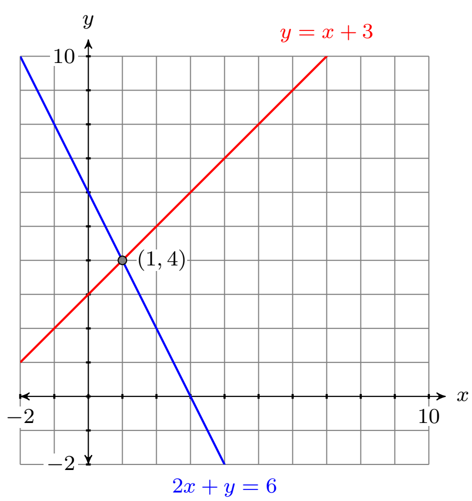
$$\begin{aligned} 2x + y &= 6 \\ 2(0) + y &= 6 \\ y &= 6 \end{aligned}$$

Plot and label the intercepts, then draw the line $2x + y = 6$ through them and label it with its equation. Next, the line $y = x + 3$ has slope 1 and y-intercept $(0, 3)$. Plot and label $(0, 3)$, then move 1 unit to the right and 1 unit up. Label the resulting line with its equation.





Next, place both lines on the same coordinate system, label each line with its equation, then label the point of intersection with its coordinates.



Substitute the point $(x, y) = (1, 4)$ in both equations to see if it checks.

$$\begin{aligned} 2x + y &= 6 \\ 2(1) + (4) &= 6 \\ 6 &= 6 \end{aligned} \qquad\qquad \begin{aligned} y &= x + 3 \\ 4 &= (1) + 3 \\ 4 &= 4 \end{aligned}$$

Hence, the solution $(x, y) = (1, 4)$ checks.

5. First, determine the x- and y-intercepts of $x + 2y = -6$.

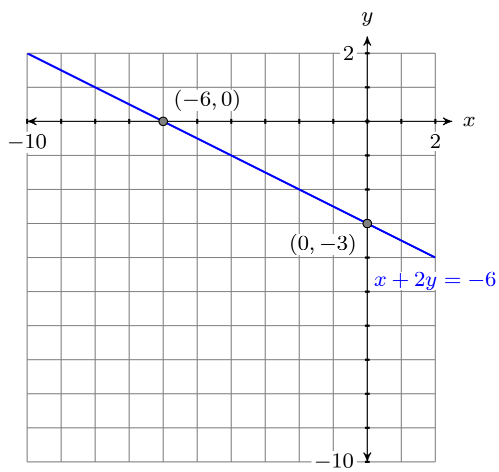
To find the x-intercept, let $y = 0$.

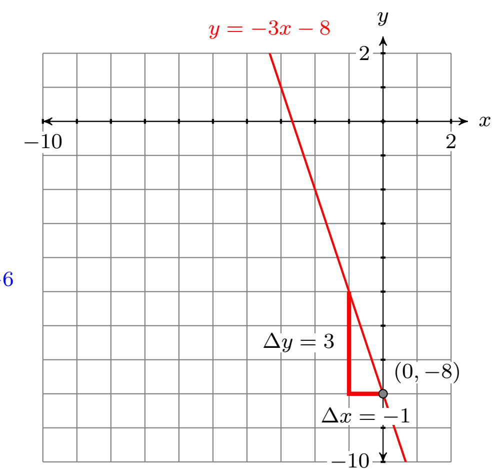
$$\begin{aligned} x + 2y &= -6 \\ x + 2(0) &= -6 \\ x &= -6 \end{aligned}$$

To find the y-intercept, let $x = 0$.

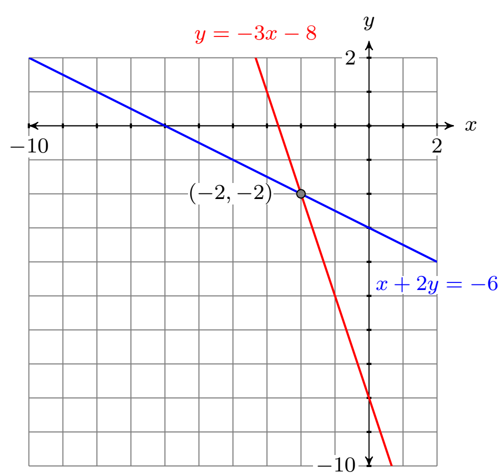
$$\begin{aligned} x + 2y &= -6 \\ (0) + 2y &= -6 \\ 2y &= -6 \\ y &= -3 \end{aligned}$$

Plot and label the intercepts, then draw the line $x+2y = -6$ through them and label it with its equation. Next, the line $y = -3x - 8$ has slope -3 and y-intercept $(0, -8)$. Plot and label $(0, -8)$, then move 1 unit to the left and 3 units up. Label the resulting line with its equation.





Next, place both lines on the same coordinate system, label each line with its equation, then label the point of intersection with its coordinates.



Substitute the point $(x, y) = (-2, -2)$ in both equations to see if it checks.

$$\begin{aligned} x + 2y &= -6 \\ (-2) + 2(-2) &= -6 \\ -6 &= -6 \end{aligned} \qquad\qquad \begin{aligned} y &= -3x - 8 \\ -2 &= -3(-2) - 8 \\ -2 &= -2 \end{aligned}$$

Hence, the solution $(x, y) = (-2, -2)$ checks.

7. First, determine the x- and y-intercepts of $-x - 3y = 3$.

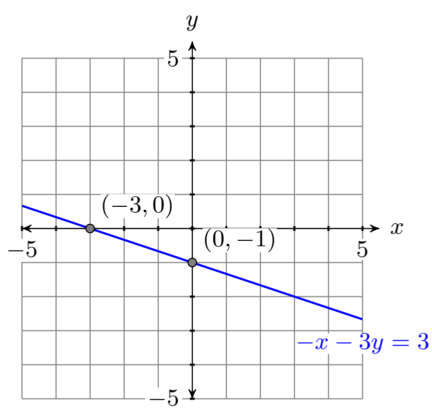
To find the x-intercept, let $y = 0$.

$$\begin{aligned} -x - 3y &= 3 \\ -x - 3(0) &= 3 \\ -x &= 3 \\ x &= -3 \end{aligned}$$

To find the y-intercept, let $x = 0$.

$$\begin{aligned} -x - 3y &= 3 \\ -(0) - 3y &= 3 \\ -3y &= 3 \\ y &= -1 \end{aligned}$$

Plot and label the intercepts, then draw the line $-x - 3y = 3$ through them and label it with its equation.



Secondly, determine the x- and y-intercepts of $x - 4y = -4$.

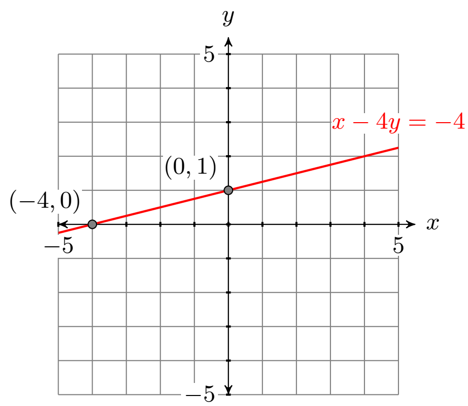
To find the x-intercept, let $y = 0$.

$$\begin{aligned} x - 4y &= -4 \\ x - 4(0) &= -4 \\ x &= -4 \end{aligned}$$

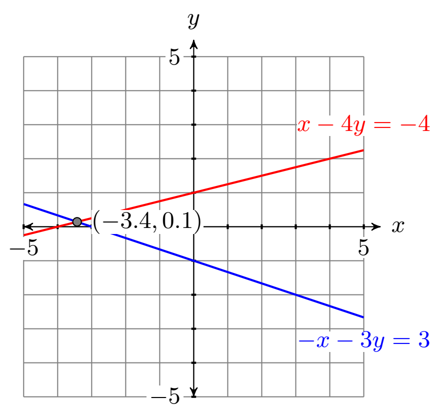
To find the y-intercept, let $x = 0$.

$$\begin{aligned} x - 4y &= -4 \\ (0) - 4y &= -4 \\ -4y &= -4 \\ y &= 1 \end{aligned}$$

Plot and label the intercepts, then draw the line $x - 4y = -4$ through them and label it with its equation.



Finally, plot both lines on the same coordinate system, label each with its equation, then label the point of intersection with its approximate coordinates.



Hence, the solution is approximately $(x, y) \approx (-3.4, 0.1)$.

Check: Remember, our estimate is an approximation so we don't expect the solution to check exactly (though sometimes it might). Substitute the approximation $(x, y) \approx (-3.4, 0.1)$ in both equations to see how close it checks. We use a calculator to perform the arithmetic.

$$\begin{aligned} -x - 3y &= 3 \\ -(-3.4) - 3(0.1) &\approx 3 \\ 3.1 &\approx 3 \end{aligned} \qquad \begin{aligned} x - 4y &= -4 \\ (-3.4) - 4(0.1) &\approx -4 \\ -3.8 &\approx -4 \end{aligned}$$

That's fairly close, suggesting the approximation $(x, y) \approx (-3.4, 0.1)$ is fairly close to the proper solution.

9. First, determine the x- and y-intercepts of $-3x + 3y = -9$.

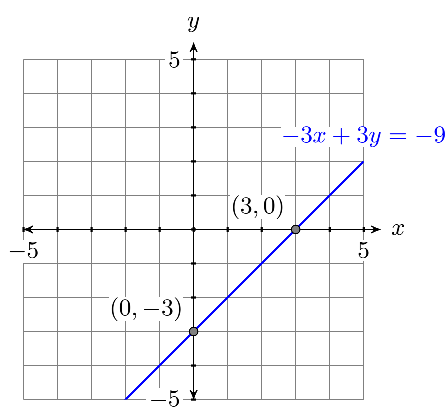
To find the x-intercept, let $y = 0$.

$$\begin{aligned} -3x + 3y &= -9 \\ -3x + 3(0) &= -9 \\ -3x &= -9 \\ x &= 3 \end{aligned}$$

To find the y-intercept, let $x = 0$.

$$\begin{aligned} -3x + 3y &= -9 \\ -3(0) + 3y &= -9 \\ 3y &= -9 \\ y &= -3 \end{aligned}$$

Plot and label the intercepts, then draw the line $-3x + 3y = -9$ through them and label it with its equation.



Secondly, determine the x- and y-intercepts of $-3x + 3y = -12$.

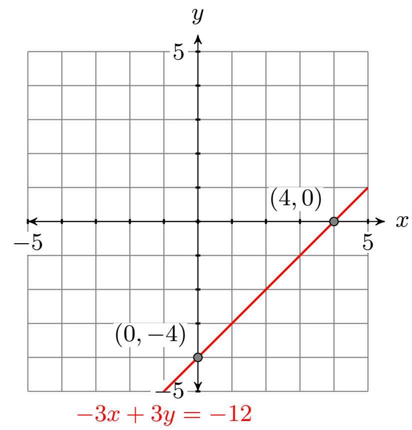
To find the x-intercept, let $y = 0$.

$$\begin{aligned} -3x + 3y &= -12 \\ -3x + 3(0) &= -12 \\ -3x &= -12 \\ x &= 4 \end{aligned}$$

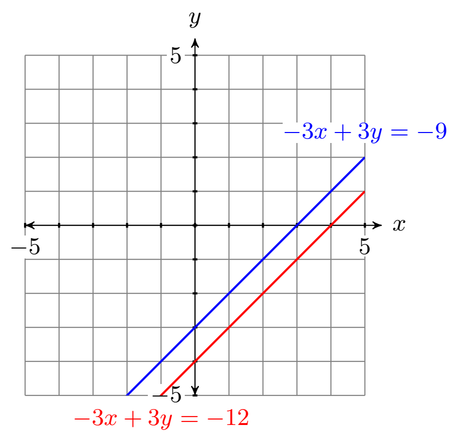
To find the y-intercept, let $x = 0$.

$$\begin{aligned} -3x + 3y &= -12 \\ -3(0) + 3y &= -12 \\ 3y &= -12 \\ y &= -4 \end{aligned}$$

Plot and label the intercepts, then draw the line $-3x + 3y = -12$ through them and label it with its equation.



Finally, plot both lines on the same coordinate system and label each with its equation.



It appears that the lines might be parallel. Let's put each into slope-intercept form to check this supposition.

Solve $-3x + 3y = -9$ for y:

$$\begin{aligned} -3x + 3y &= -9 \\ -3x + 3y + 3x &= -9 + 3x \\ 3y &= -9 + 3x \\ \frac{3y}{3} &= \frac{-9 + 3x}{3} \\ y &= x - 3 \end{aligned}$$

Solve $-3x + 3y = -12$ for y:

$$\begin{aligned} -3x + 3y &= -12 \\ -3x + 3y + 3x &= -12 + 3x \\ 3y &= -12 + 3x \\ \frac{3y}{3} &= \frac{-12 + 3x}{3} \\ y &= x - 4 \end{aligned}$$

Note that both lines have slope 1. However, the first line has a y-intercept at $(0, -3)$, while the second line has a y-intercept at $(0, -4)$. Hence, the lines are distinct and parallel. Therefore, the system has no solutions.

11. First, determine the x- and y-intercepts of $6x - 7y = -42$.

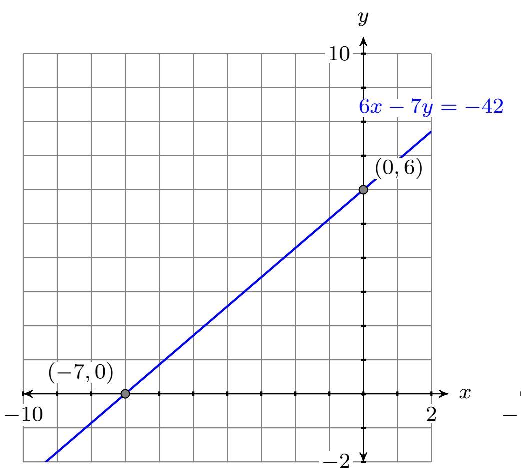
To find the x-intercept, let $y = 0$.

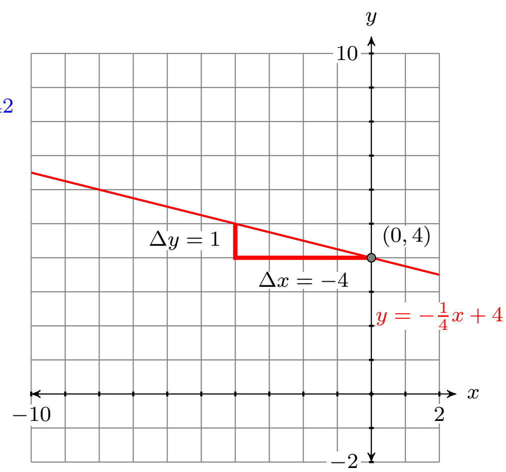
$$\begin{aligned} 6x - 7y &= -42 \\ 6x - 7(0) &= -42 \\ 6x &= -42 \\ x &= -7 \end{aligned}$$

To find the y-intercept, let $x = 0$.

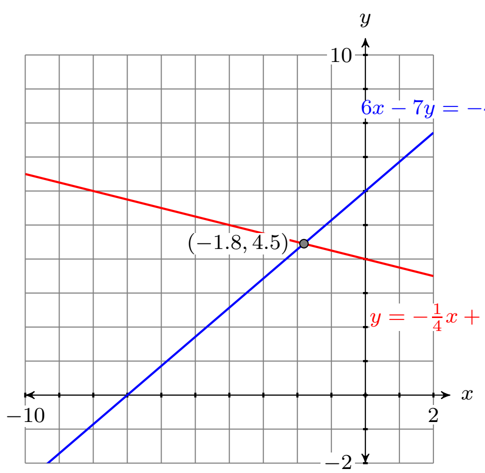
$$\begin{aligned} 6x - 7y &= -42 \\ 6(0) - 7y &= -42 \\ -7y &= -42 \\ y &= 6 \end{aligned}$$

Plot and label the intercepts, then draw the line $6x - 7y = -42$ through them and label it with its equation. Next, the line $y = -\frac{1}{4}x + 4$ has slope $-1/4$ and y-intercept $(0, 4)$. Plot and label $(0, 4)$, then move 4 units to the left and 1 unit up. Label the resulting line with its equation.





Next, place both lines on the same coordinate system, label each line with its equation, then label the point of intersection with its coordinates.



Hence, the solution is approximately $(x, y) \approx (-1.8, 4.5)$.

Check: Remember, our estimate is an approximation so we don't expect the solution to check exactly (though sometimes it might). Substitute the approximation $(x, y) \approx (-1.8, 4.5)$ in both equations to see how close it checks. We use a calculator to perform the arithmetic.

$$\begin{aligned} 6x - 7y &= -42 \\ 6(-1.8) - 7(4.5) &\approx -42 \\ -42.3 &\approx -42 \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{4}x + 4 \\ 4.5 &\approx -\frac{1}{4}(-1.8) + 4 \\ 4.5 &\approx 4.45 \end{aligned}$$

That's fairly close, suggesting the approximation $(x, y) \approx (-1.8, 4.5)$ is fairly close to the proper solution.

13. First, determine the x- and y-intercepts of $6x - 7y = -42$.

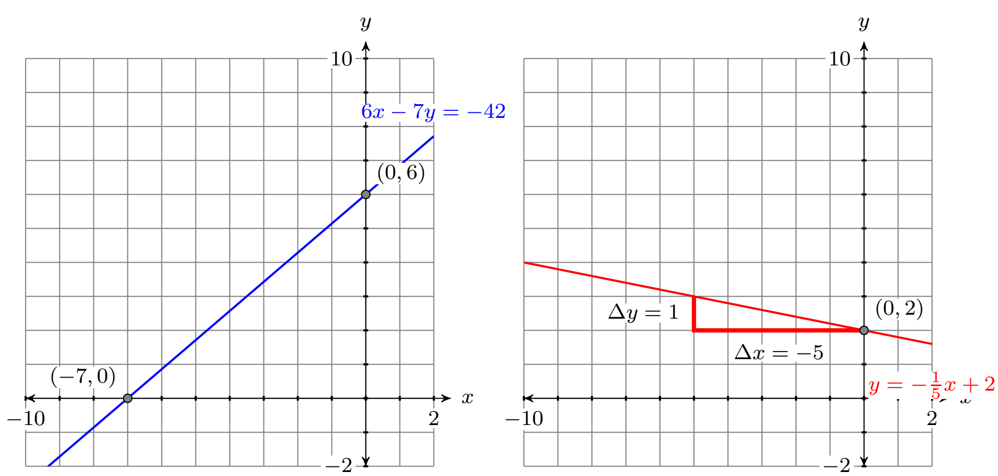
To find the x-intercept, let $y = 0$.

$$\begin{aligned} 6x - 7y &= -42 \\ 6x - 7(0) &= -42 \\ 6x &= -42 \\ x &= -7 \end{aligned}$$

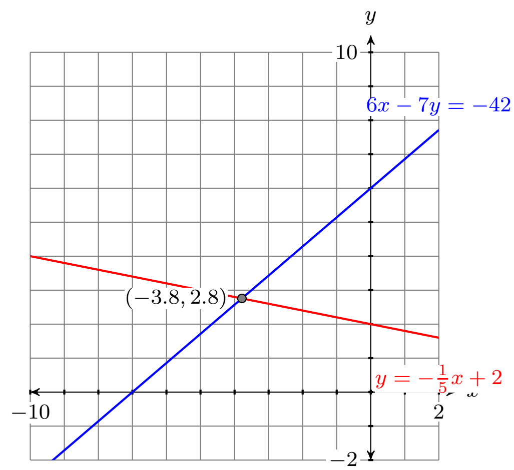
To find the y-intercept, let $x = 0$.

$$\begin{aligned} 6x - 7y &= -42 \\ 6(0) - 7y &= -42 \\ -7y &= -42 \\ y &= 6 \end{aligned}$$

Plot and label the intercepts, then draw the line $6x - 7y = -42$ through them and label it with its equation. Next, the line $y = -\frac{1}{5}x + 2$ has slope $-1/5$ and y-intercept $(0, 2)$. Plot and label $(0, 2)$, then move 5 units to the left and 1 unit up. Label the resulting line with its equation.



Next, place both lines on the same coordinate system, label each line with its equation, then label the point of intersection with its coordinates.



Hence, the solution is approximately $(x, y) \approx (-3.8, 2.8)$.

Check: Remember, our estimate is an approximation so we don't expect the solution to check exactly (though sometimes it might). Substitute the approximation $(x, y) \approx (-3.8, 2.8)$ in both equations to see how close it checks. We use a calculator to perform the arithmetic.

$$\begin{aligned} 6x - 7y &= -42 \\ 6(-3.8) - 7(2.8) &\approx -42 \\ -42.4 &\approx -42 \end{aligned}$$

$$\begin{aligned} y &= -\frac{1}{5}x + 2 \\ 2.8 &\approx -\frac{1}{5}(-3.8) + 2 \\ 2.8 &\approx 2.76 \end{aligned}$$

That's fairly close, suggesting the approximation $(x, y) \approx (-3.8, 2.8)$ is fairly close to the proper solution.

15. First, determine the x- and y-intercepts of $6x + 3y = 12$.

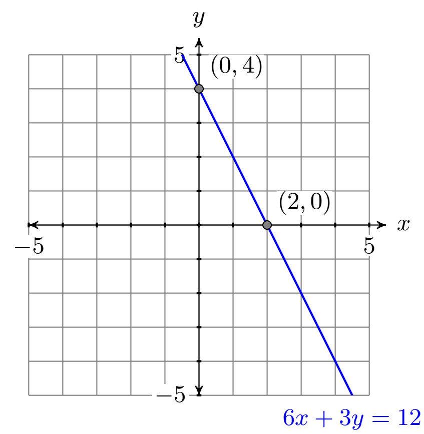
To find the x-intercept, let $y = 0$.

$$\begin{aligned} 6x + 3y &= 12 \\ 6x + 3(0) &= 12 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

To find the y-intercept, let $x = 0$.

$$\begin{aligned} 6x + 3y &= 12 \\ 6(0) + 3y &= 12 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

Plot and label the intercepts, then draw the line $6x + 3y = 12$ through them and label it with its equation.



Secondly, determine the x- and y-intercepts of $-2x - y = 4$.

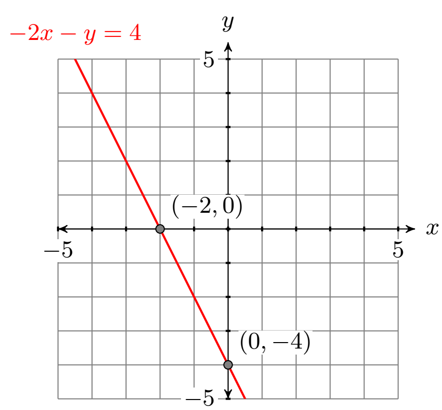
To find the x-intercept, let $y = 0$.

$$\begin{aligned} -2x - y &= 4 \\ -2x - (0) &= 4 \\ -2x &= 4 \\ x &= -2 \end{aligned}$$

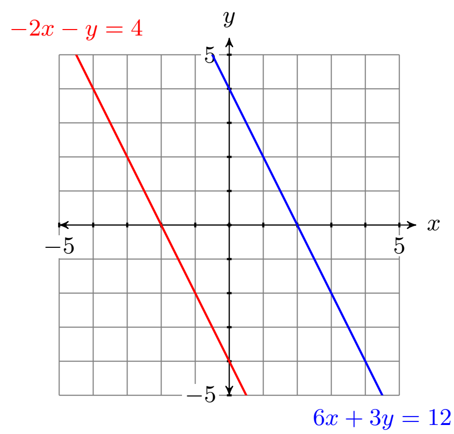
To find the y-intercept, let $x = 0$.

$$\begin{aligned} -2x - y &= 4 \\ -2(0) - y &= 4 \\ -y &= 4 \\ y &= -4 \end{aligned}$$

Plot and label the intercepts, then draw the line $-2x - y = 4$ through them and label it with its equation.



Finally, plot both lines on the same coordinate system and label each with its equation.



It appears that the lines might be parallel. Let's put each into slope-intercept form to check this supposition.

Solve $6x + 3y = 12$ for y:

$$\begin{aligned}
6x + 3y &= 12 \\
6x + 3y - 6x &= 12 - 6x \\
3y &= 12 - 6x \\
\frac{3y}{3} &= \frac{12 - 6x}{3} \\
y &= -2x + 4
\end{aligned}$$

Solve $-2x - y = 4$ for y:

$$\begin{aligned}
-2x - y &= 4 \\
-2x - y + 2x &= 4 + 2x \\
-y &= 4 + 2x \\
\frac{-y}{-1} &= \frac{4 + 2x}{-1} \\
y &= -2x - 4
\end{aligned}$$

Note that both lines have slope -2. However, the first line has a y-intercept at $(0, 4)$, while the second line has a y-intercept at $(0, -4)$. Hence, the lines are distinct and parallel. Therefore, the system has no solutions.

17. First, determine the x- and y-intercepts of $3x + y = 3$.

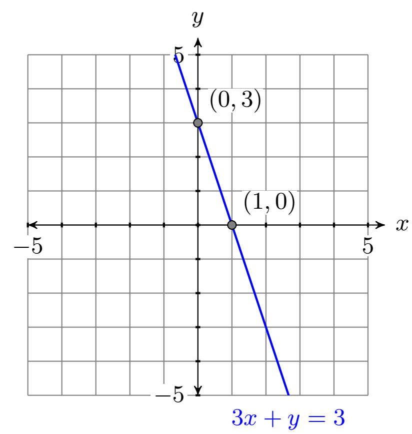
To find the x-intercept, let $y = 0$.

$$\begin{aligned}
3x + y &= 3 \\
3x + (0) &= 3 \\
3x &= 3 \\
x &= 1
\end{aligned}$$

To find the y-intercept, let $x = 0$.

$$\begin{aligned}
3x + y &= 3 \\
3(0) + y &= 3 \\
y &= 3
\end{aligned}$$

Plot and label the intercepts, then draw the line $3x + y = 3$ through them and label it with its equation.



Secondly, determine the x- and y-intercepts of $-2x + 3y = -6$.

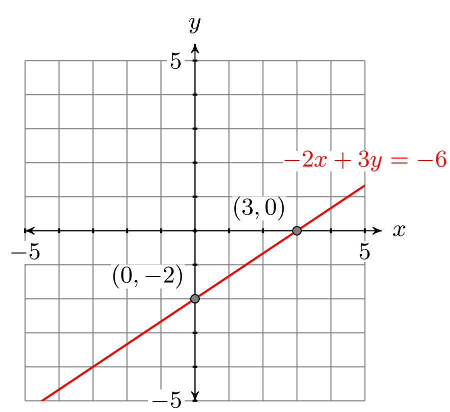
To find the x-intercept, let $y = 0$.

$$\begin{aligned} -2x + 3y &= -6 \\ -2x + 3(0) &= -6 \\ -2x &= -6 \\ x &= 3 \end{aligned}$$

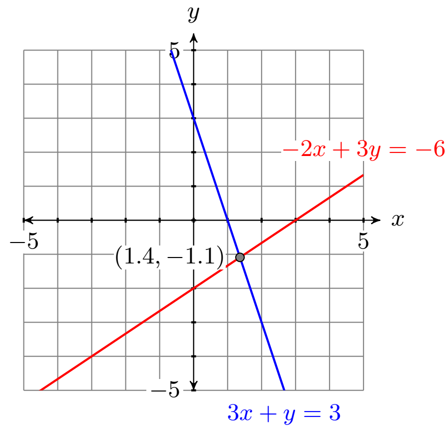
To find the y-intercept, let $x = 0$.

$$\begin{aligned} -2x + 3y &= -6 \\ -2(0) + 3y &= -6 \\ 3y &= -6 \\ y &= -2 \end{aligned}$$

Plot and label the intercepts, then draw the line $-2x + 3y = -6$ through them and label it with its equation.



Finally, plot both lines on the same coordinate system, label each with its equation, then label the point of intersection with its approximate coordinates.



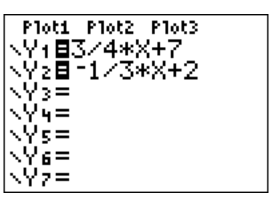
Hence, the solution is approximately $(x, y) \approx (1.4, -1.1)$.

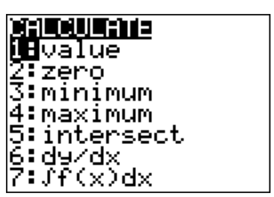
Check: Remember, our estimate is an approximation so we don't expect the solution to check exactly (though sometimes it might). Substitute the approximation $(x, y) \approx (1.4, -1.1)$ in both equations to see how close it checks. We use a calculator to perform the arithmetic.

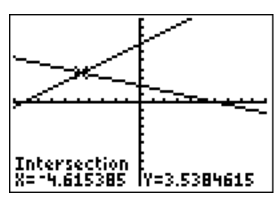
$$\begin{aligned} 3x + y &= 3 \\ 3(1.4) + (-1.1) &\approx 3 \\ 3.1 &\approx 3 \end{aligned} \qquad\qquad \begin{aligned} -2x + 3y &= -6 \\ -2(1.4) + 3(-1.1) &\approx -6 \\ -6.1 &\approx -6 \end{aligned}$$

That's fairly close, suggesting the approximation $(x, y) \approx (1.4, -1.1)$ is fairly close to the proper solution.

19. Enter the equations $y = \frac{3}{4}x + 7$ and $y = -\frac{1}{3}x + 2$ in the **Y=** menu as shown in the first image below. Select 6:ZStandard from the ZOOM menu to sketch the system. Press 2ND CALC to open the Calculate menu (see the second image below), then select 5:intersect. Press ENTER for "First curve," ENTER for "Second curve," and ENTER for "Guess." The result is shown in the third image below.





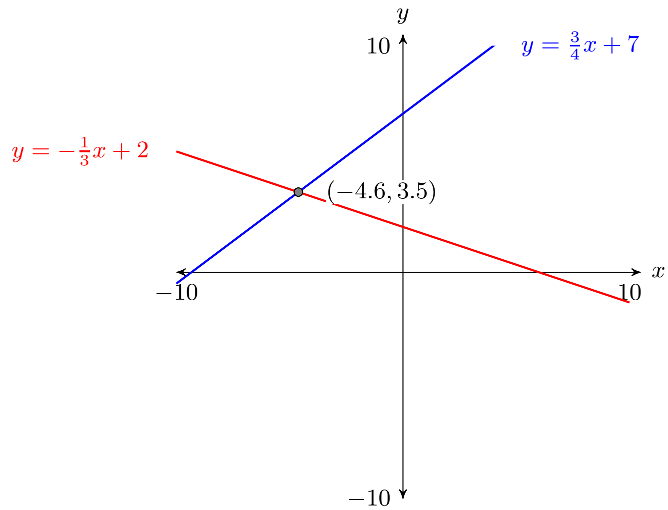


The calculator reports the solution:

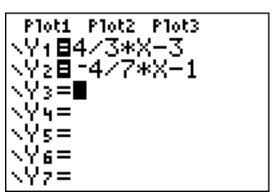
$$(x, y) \approx (-4.615385, 3.5384615)$$

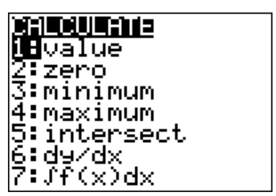
Rounding to the nearest tenth, we get $(x, y) \approx (-4.6, 3.5)$.

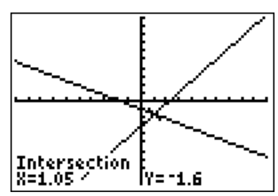
Using the *Calculator Submission Guidelines*, report the solution on your homework paper as follows.



21. Enter the equations $y = \frac{4}{3}x - 3$ and $y = -\frac{4}{7}x - 1$ in the **Y=** menu as shown in the first image below. Select 6:ZStandard from the ZOOM menu to sketch the system. Press 2ND CALC to open the Calculate menu (see the second image below), then select 5:intersect. Press ENTER for "First curve," ENTER for "Second curve," and ENTER for "Guess." The result is shown in the third image below.





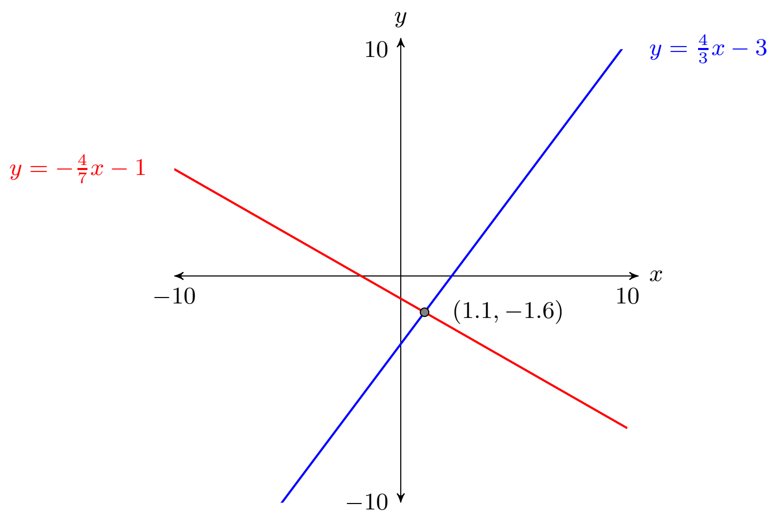


The calculator reports the solution:

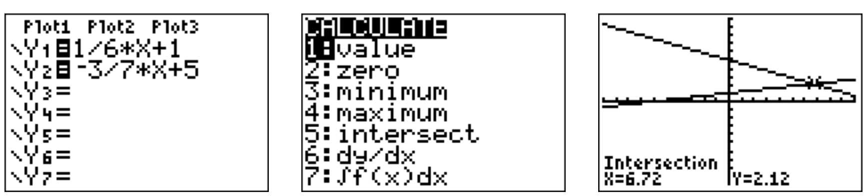
$$(x, y) \approx (1.05, -1.6)$$

Rounding to the nearest tenth, we get $(x, y) \approx (1.1, -1.6)$.

Using the *Calculator Submission Guidelines*, report the solution on your homework paper as follows.



23. Enter the equations $y = \frac{1}{6}x + 1$ and $y = -\frac{3}{7}x + 5$ in the **Y=** menu as shown in the first image below. Select 6:ZStandard from the ZOOM menu to sketch the system. Press 2ND CALC to open the Calculate menu (see the second image below), then select 5:intersect. Press ENTER for "First curve," ENTER for "Second curve," and ENTER for "Guess." The result is shown in the third image below.

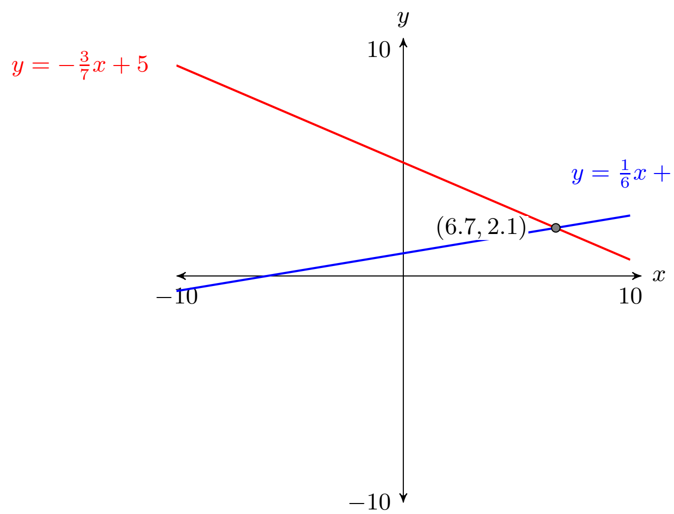


The calculator reports the solution:

$$(x, y) \approx (6.72, 2.12)$$

Rounding to the nearest tenth, we get $(x, y) \approx (6.7, 2.1)$.

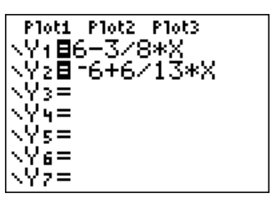
Using the *Calculator Submission Guidelines*, report the solution on your homework paper as follows.

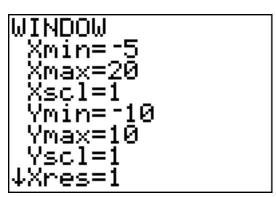


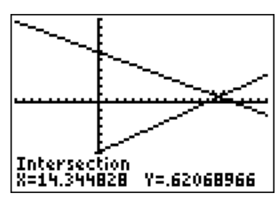
25. First, solve each equation for y, so that we can enter the resulting equations into the **Y=** menu.

$$\begin{aligned} 6x + 16y &= 96 \\ 6x + 16y - 6x &= 96 - 6x \\ 16y &= 96 - 6x \\ \frac{16y}{16} &= \frac{96 - 6x}{16} \\ y &= 6 - \frac{3}{8}x \end{aligned} \qquad \begin{aligned} -6x + 13y &= -78 \\ -6x + 13y + 6x &= -78 + 6x \\ 13y &= -78 + 6x \\ \frac{13y}{13} &= \frac{-78 + 6x}{13} \\ y &= -6 + \frac{6}{13}x \end{aligned}$$

Enter the equations $y = 6 - \frac{3}{8}x$ and $y = -6 + \frac{6}{13}x$ in the **Y=** menu as shown in the first image below. Select 6:ZStandard from the ZOOM menu to sketch the system. Make the adjustments in the second image shown below so that the point of intersection of the two lines is visible in the viewing window. Press 2ND CALC to open the Calculate menu, then select 5:intersect. Press ENTER for "First curve," ENTER for "Second curve," and ENTER for "Guess." The result is shown in the third image below.





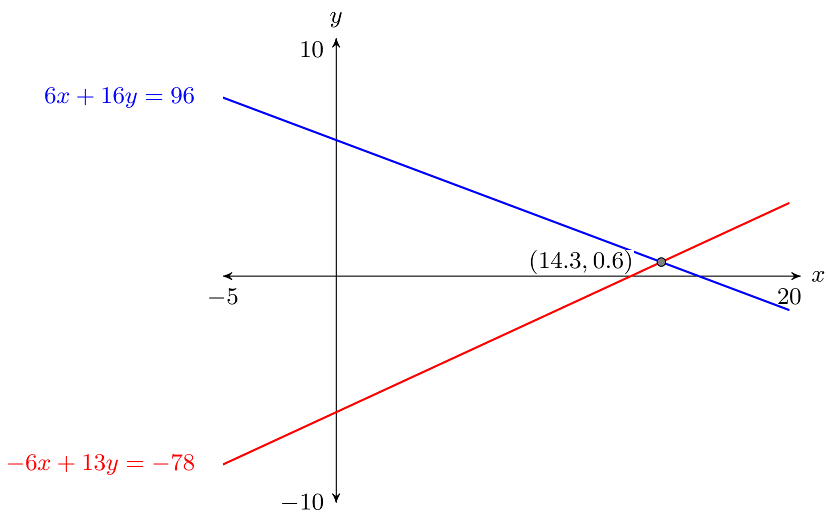


The calculator reports the solution:

$$(x, y) \approx (14.344828, 0.62068966)$$

Rounding to the nearest tenth, we get $(x, y) \approx (14.3, 0.6)$.

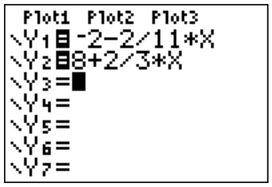
Using the *Calculator Submission Guidelines*, report the solution on your homework paper as follows.

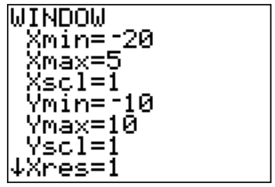


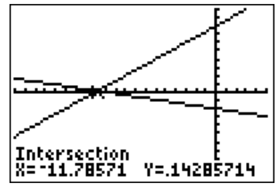
27. First, solve each equation for y, so that we can enter the resulting equations into the **Y=** menu.

$$\begin{aligned} -2x - 11y &= 22 \\ -2x - 11y + 2x &= 22 + 2x \\ -11y &= 22 + 2x \\ \frac{-11y}{-11} &= \frac{22 + 2x}{-11} \\ y &= -2 - \frac{2}{11}x \end{aligned} \qquad \begin{aligned} 8x - 12y &= -96 \\ 8x - 12y - 8x &= -96 - 8x \\ -12y &= -96 - 8x \\ \frac{-12y}{-12} &= \frac{-96 - 8x}{-12} \\ y &= 8 + \frac{2}{3}x \end{aligned}$$

Enter the equations $y = -2 - \frac{2}{11}x$ and $y = 8 + \frac{2}{3}x$ in the **Y=** menu as shown in the first image below. Select 6:ZStandard from the ZOOM menu to sketch the system. Make the adjustments in the second image shown below so that the point of intersection of the two lines is visible in the viewing window. Press 2ND CALC to open the Calculate menu, then select 5:intersect. Press ENTER for “First curve,” ENTER for “Second curve,” and ENTER for “Guess.” The result is shown in the third image below.





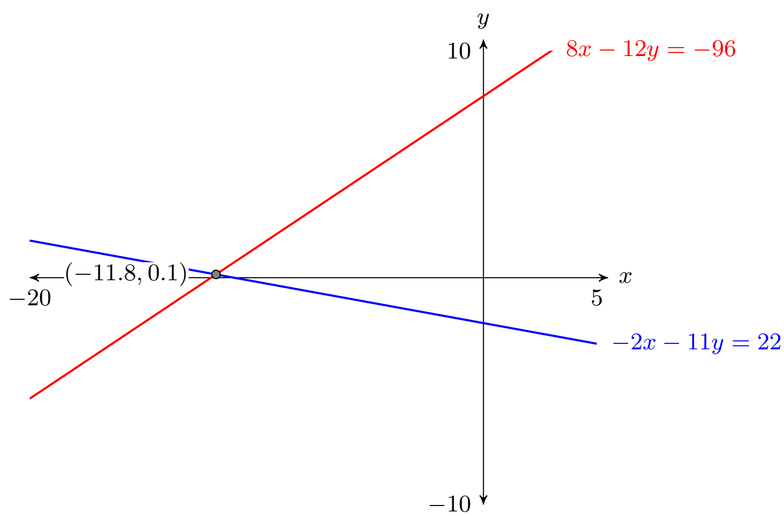


The calculator reports the solution:

$$(x, y) \approx (-11.78571, 0.14285714)$$

Rounding to the nearest tenth, we get $(x, y) \approx (-11.8, 0.1)$.

Using the *Calculator Submission Guidelines*, report the solution on your homework paper as follows.



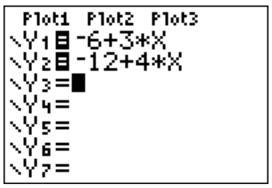
29. First, solve each equation for y, so that we can enter the resulting equations into the **Y=** menu.

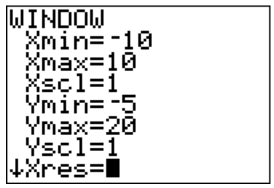
$$\begin{aligned} -6x + 2y &= -12 \\ -6x + 2y + 6x &= -12 + 6x \\ 2y &= -12 + 6x \\ \frac{2y}{2} &= \frac{-12 + 6x}{2} \\ y &= -6 + 3x \end{aligned}$$

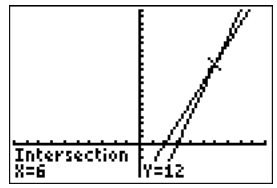
$$\begin{aligned} -12x + 3y &= -36 \\ -12x + 3y + 12x &= -36 + 12x \\ 3y &= -36 + 12x \\ \frac{3y}{3} &= \frac{-36 + 12x}{3} \\ y &= -12 + 4x \end{aligned}$$

Enter the equations $y = -6+3x$ and $y = -12+4x$ in the **Y=** menu as shown in the first image below. Select 6:ZStandard from the ZOOM menu to sketch the system. Make the adjustments in the second image shown below so that

the point of intersection of the two lines is visible in the viewing window. Press 2ND CALC to open the Calculate menu, then select 5:intersect. Press ENTER for “First curve,” ENTER for “Second curve,” and ENTER for “Guess.” The result is shown in the third image below.





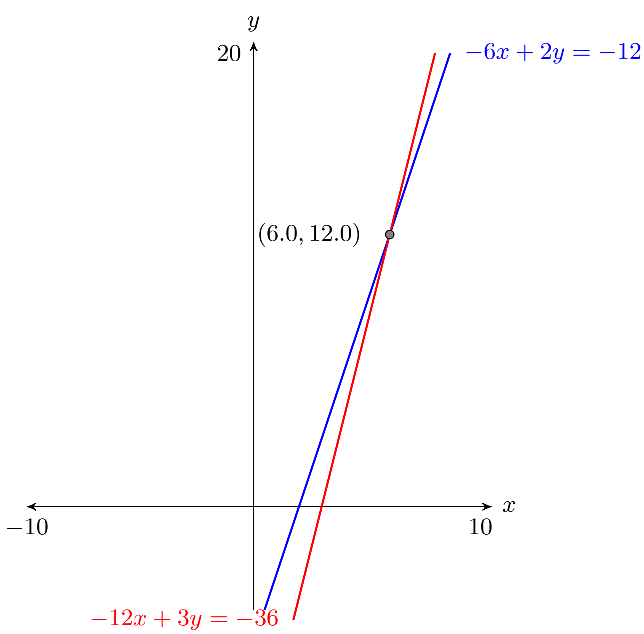


The calculator reports the solution:

$$(x, y) \approx (6, 12)$$

Rounding to the nearest tenth, we get $(x, y) \approx (6.0, 12.0)$.

Using the *Calculator Submission Guidelines*, report the solution on your homework paper as follows.



4.2 Solving Systems by Substitution

1. The second equation, $y = 6 - 2x$, is already solved for y. Substitute $6 - 2x$ for y in the first equation and solve for x.

$$\begin{aligned} -7x + 7y &= 63 && \text{First Equation.} \\ -7x + 7(6 - 2x) &= 63 && \text{Substitute } 6 - 2x \text{ for } y. \\ -7x + 42 - 14x &= 63 && \text{Distribute the 7.} \\ 42 - 21x &= 63 && \text{Combine like terms.} \\ -21x &= 21 && \text{Subtract 42 from both sides.} \\ x &= -1 && \text{Divide both sides by } -21. \end{aligned}$$

Finally, to find the y-value, substitute -1 for x in the equation $y = 6 - 2x$.

$$\begin{aligned} y &= 6 - 2x \\ y &= 6 - 2(-1) && \text{Substitute } -1 \text{ for } x. \\ y &= 6 + 2 && \text{Multiply.} \\ y &= 8 && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-1, 8)$ is the solution of the system.

Check: We now show that the solution satisfies both equations.

Substitute $(x, y) = (-1, 8)$ in the first equation.

$$\begin{aligned} -7x + 7y &= 63 \\ -7(-1) + 7(8) &= 63 \\ 7 + 56 &= 63 \\ 63 &= 63 \end{aligned}$$

Substitute $(x, y) = (-1, 8)$ in the second equation.

$$\begin{aligned} y &= 6 - 2x \\ 8 &= 6 - 2(-1) \\ 8 &= 6 + 2 \\ 8 &= 8 \end{aligned}$$

The last statement in each check, being a true statement, shows that the solution $(x, y) = (-1, 8)$ satisfies both equations and thus is a solution of the system.

3. The first equation, $x = 19 + 7y$, is already solved for x. Substitute $19 + 7y$ for x in the second equation and solve for y.

$$\begin{aligned} 3x - 3y &= 3 && \text{Second Equation.} \\ 3(19 + 7y) - 3y &= 3 && \text{Substitute } 19 + 7y \text{ for } x. \\ 57 + 21y - 3y &= 3 && \text{Distribute the 3.} \\ 57 + 18y &= 3 && \text{Combine like terms.} \\ 18y &= -54 && \text{Subtract 57 from both sides.} \\ y &= -3 && \text{Divide both sides by 18.} \end{aligned}$$

Finally, to find the x-value, substitute -3 for y in the equation $x = 19 + 7y$.

$$\begin{aligned}
x &= 19 + 7y \\
x &= 19 + 7(-3) && \text{Substitute } -3 \text{ for } y. \\
x &= 19 - 21 && \text{Multiply.} \\
x &= -2 && \text{Simplify.}
\end{aligned}$$

Hence, $(x, y) = (-2, -3)$ is the solution of the system.

Check: We now show that the solution satisfies both equations.

Substitute $(x, y) = (-2, -3)$ in the first equation.

$$\begin{aligned}
x &= 19 + 7y \\
-2 &= 19 + 7(-3) \\
-2 &= 19 - 21 \\
-2 &= -2
\end{aligned}$$

Substitute $(x, y) = (-2, -3)$ in the second equation.

$$\begin{aligned}
3x - 3y &= 3 \\
3(-2) - 3(-3) &= 3 \\
-6 + 9 &= 3 \\
3 &= 3
\end{aligned}$$

The last statement in each check, being a true statement, shows that the solution $(x, y) = (-2, -3)$ satisfies both equations and thus is a solution of the system.

5. The first equation, $x = -5 - 2y$, is already solved for x. Substitute $-5 - 2y$ for x in the second equation and solve for y.

$$\begin{aligned}
-2x - 6y &= 18 && \text{Second Equation.} \\
-2(-5 - 2y) - 6y &= 18 && \text{Substitute } -5 - 2y \text{ for } x. \\
10 + 4y - 6y &= 18 && \text{Distribute the } -2. \\
10 - 2y &= 18 && \text{Combine like terms.} \\
-2y &= 8 && \text{Subtract 10 from both sides.} \\
y &= -4 && \text{Divide both sides by } -2.
\end{aligned}$$

Finally, to find the x-value, substitute -4 for y in the equation $x = -5 - 2y$.

$$\begin{aligned}
x &= -5 - 2y \\
x &= -5 - 2(-4) && \text{Substitute } -4 \text{ for } y. \\
x &= -5 + 8 && \text{Multiply.} \\
x &= 3 && \text{Simplify.}
\end{aligned}$$

Hence, $(x, y) = (3, -4)$ is the solution of the system.

Check: We now show that the solution satisfies both equations.

Substitute $(x, y) = (3, -4)$ in the first equation.

$$\begin{aligned} x &= -5 - 2y \\ 3 &= -5 - 2(-4) \\ 3 &= -5 + 8 \\ 3 &= 3 \end{aligned}$$

Substitute $(x, y) = (3, -4)$ in the second equation.

$$\begin{aligned} -2x - 6y &= 18 \\ -2(3) - 6(-4) &= 18 \\ -6 + 24 &= 18 \\ 18 &= 18 \end{aligned}$$

The last statement in each check, being a true statement, shows that the solution $(x, y) = (3, -4)$ satisfies both equations and thus is a solution of the system.

7. The second equation, $y = 15+3x$, is already solved for y. Substitute $15+3x$ for y in the first equation and solve for x.

$$\begin{aligned} 6x - 8y &= 24 && \text{First Equation.} \\ 6x - 8(15 + 3x) &= 24 && \text{Substitute } 15 + 3x \text{ for } y. \\ 6x - 120 - 24x &= 24 && \text{Distribute the } -8. \\ -120 - 18x &= 24 && \text{Combine like terms.} \\ -18x &= 144 && \text{Add 120 to both sides.} \\ x &= -8 && \text{Divide both sides by } -18. \end{aligned}$$

Finally, to find the y-value, substitute -8 for x in the equation $y = 15 + 3x$.

$$\begin{aligned} y &= 15 + 3x \\ y &= 15 + 3(-8) && \text{Substitute } -8 \text{ for } x. \\ y &= 15 - 24 && \text{Multiply.} \\ y &= -9 && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-8, -9)$ is the solution of the system.

Check: We now show that the solution satisfies both equations.

Substitute $(x, y) = (-8, -9)$ in the first equation.

$$\begin{aligned} 6x - 8y &= 24 \\ 6(-8) - 8(-9) &= 24 \\ -48 + 72 &= 24 \\ 24 &= 24 \end{aligned}$$

Substitute $(x, y) = (-8, -9)$ in the second equation.

$$\begin{aligned} y &= 15 + 3x \\ -9 &= 15 + 3(-8) \\ -9 &= 15 - 24 \\ -9 &= -9 \end{aligned}$$

The last statement in each check, being a true statement, shows that the solution $(x, y) = (-8, -9)$ satisfies both equations and thus is a solution of the system.

9. The first step is to solve either equation for either variable. This means that we could solve the first equation for x or y, but it also means that we could solve the second equation for x or y. Of these four possible choices, solving the first equation for x seems the easiest way to start.

$$\begin{aligned} -x + 9y &= 46 && \text{First Equation.} \\ -x &= 46 - 9y && \text{Subtract } 9y \text{ from both sides.} \\ x &= -46 + 9y && \text{Multiply both sides by } -1. \end{aligned}$$

Next, substitute $-46 + 9y$ for x in the second equation and solve for x.

$$\begin{aligned} 7x - 4y &= -27 && \text{Second Equation.} \\ 7(-46 + 9y) - 4y &= -27 && \text{Substitute } -46 + 9y \text{ for } x. \\ -322 + 63y - 4y &= -27 && \text{Distribute the 7.} \\ -322 + 59y &= -27 && \text{Combine like terms.} \\ 59y &= 295 && \text{Add 322 to both sides.} \\ y &= 5 && \text{Divide both sides by 59.} \end{aligned}$$

Finally, to find the x-value, substitute 5 for y in the equation $x = -46 + 9y$.

$$\begin{aligned} x &= -46 + 9y \\ x &= -46 + 9(5) && \text{Substitute 5 for } y. \\ x &= -46 + 45 && \text{Multiply.} \\ x &= -1 && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-1, 5)$ is the solution of the system.

11. The first step is to solve either equation for either variable. This means that we could solve the first equation for x or y, but it also means that we could solve the second equation for x or y. Of these four possible choices, solving the first equation for x seems the easiest way to start.

$$\begin{aligned} -x + 4y &= 22 && \text{First Equation.} \\ -x &= 22 - 4y && \text{Subtract } 4y \text{ from both sides.} \\ x &= -22 + 4y && \text{Multiply both sides by } -1. \end{aligned}$$

Next, substitute $-22 + 4y$ for x in the second equation and solve for x.

$$\begin{aligned} 8x + 7y &= -20 && \text{Second Equation.} \\ 8(-22 + 4y) + 7y &= -20 && \text{Substitute } -22 + 4y \text{ for } x. \\ -176 + 32y + 7y &= -20 && \text{Distribute the 8.} \\ -176 + 39y &= -20 && \text{Combine like terms.} \\ 39y &= 156 && \text{Add 176 to both sides.} \\ y &= 4 && \text{Divide both sides by 39.} \end{aligned}$$

Finally, to find the x-value, substitute 4 for y in the equation $x = -22 + 4y$.

$$\begin{aligned} x &= -22 + 4y && \\ x &= -22 + 4(4) && \text{Substitute 4 for } y. \\ x &= -22 + 16 && \text{Multiply.} \\ x &= -6 && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-6, 4)$ is the solution of the system.

13. The first step is to solve either equation for either variable. This means that we could solve the first equation for x or y, but it also means that we could solve the second equation for x or y. Of these four possible choices, solving the first equation for x seems the easiest way to start.

$$\begin{aligned} x + 2y &= -4 && \text{First Equation.} \\ x &= -4 - 2y && \text{Subtract } 2y \text{ from both sides.} \end{aligned}$$

Next, substitute $-4 - 2y$ for x in the second equation and solve for y.

$$\begin{aligned} 6x - 4y &= -56 && \text{Second Equation.} \\ 6(-4 - 2y) - 4y &= -56 && \text{Substitute } -4 - 2y \text{ for } x. \\ -24 - 12y - 4y &= -56 && \text{Distribute the 6.} \\ -24 - 16y &= -56 && \text{Combine like terms.} \\ -16y &= -32 && \text{Add 24 to both sides.} \\ y &= 2 && \text{Divide both sides by } -16. \end{aligned}$$

Finally, to find the x-value, substitute 2 for y in the equation $x = -4 - 2y$.

$$\begin{aligned} x &= -4 - 2y && \\ x &= -4 - 2(2) && \text{Substitute 2 for } y. \\ x &= -4 - 4 && \text{Multiply.} \\ x &= -8 && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-8, 2)$ is the solution of the system.

15. The first step is to solve either equation for either variable. This means that we could solve the first equation for x or y, but it also means that we could solve the second equation for x or y. Of these four possible choices, solving the first equation for x seems the easiest way to start.

$$\begin{aligned} x + 6y &= -49 && \text{First Equation.} \\ x &= -49 - 6y && \text{Subtract } 6y \text{ from both sides.} \end{aligned}$$

Next, substitute $-49-6y$ for x in the second equation and solve for y.

$$\begin{aligned}
-3x+4y &= -7 && \text{Second Equation.}\\
-3(-49-6y)+4y &= -7 && \text{Substitute } -49-6y \text{ for } x.\\
147+18y+4y &= -7 && \text{Distribute the } -3.\\
147+22y &= -7 && \text{Combine like terms.}\\
22y &= -154 && \text{Subtract 147 from both sides.}\\
y &= -7 && \text{Divide both sides by 22.}
\end{aligned}$$

Finally, to find the x-value, substitute -7 for y in the equation $x=-49-6y$.

$$\begin{aligned}
x &= -49-6y\\
x &= -49-6(-7) && \text{Substitute } -7 \text{ for } y.\\
x &= -49+42 && \text{Multiply.}\\
x &= -7 && \text{Simplify.}
\end{aligned}$$

Hence, $(x,y)=(-7,-7)$ is the solution of the system.

17.

19. The first equation, $x=-2y-4$, is already solved for x, so let's substitute $-2y-4$ for x in the second equation.

$$\begin{aligned}
-4x-8y &= -6 && \text{Second Equation.}\\
-4(-2y-4)-8y &= -6 && \text{Substitute } -2y-4 \text{ for } x.\\
8y+16-8y &= -6 && \text{Distribute the } -4.\\
16 &= -6 && \text{Combine like terms.}
\end{aligned}$$

The resulting statement, $16=-6$, is false. This should give us a clue that there are no solutions. Perhaps we are dealing with parallel lines? Let's put both equations in slope-intercept form so that we can compare them.

Solve $x=-2y-4$ for y:

$$\begin{aligned}
x &= -2y-4\\
x+4 &= -2y\\
\frac{x+4}{-2} &= y\\
y &= -\frac{1}{2}x-2
\end{aligned}$$

Solve $-4x-8y=-6$ for y:

$$\begin{aligned}
-4x-8y &= -6\\
-8y &= 4x-6\\
y &= \frac{4x-6}{-8}\\
y &= -\frac{1}{2}x+\frac{3}{4}
\end{aligned}$$

Hence, the lines have the same slope, $-1/2$, but different y-intercepts (one has y-intercept $(0,-2)$, the other has y-intercept $(0,3/4)$. Hence, these are two distinct parallel lines and the system has no solution.

21. The first step is to solve either equation for either variable. This means that we could solve the first equation for x or y, but it also means that we could solve the second equation for x or y. Of these four possible choices, solving the second equation for y seems the easiest way to start.

$$\begin{aligned} -7x + y &= 19 && \text{Second Equation.} \\ y &= 19 + 7x && \text{Add } 7x \text{ to both sides.} \end{aligned}$$

Next, substitute $19 + 7x$ for y in the first equation and solve for x.

$$\begin{aligned} -2x - 2y &= 26 && \text{First Equation.} \\ -2x - 2(19 + 7x) &= 26 && \text{Substitute } y = 19 + 7x \text{ for } y. \\ -2x - 38 - 14x &= 26 && \text{Distribute the } -2. \\ -38 - 16x &= 26 && \text{Combine like terms.} \\ -16x &= 64 && \text{Add 38 to both sides.} \\ x &= -4 && \text{Divide both sides by } -16. \end{aligned}$$

Finally, to find the y-value, substitute -4 for x in the equation $y = 19 + 7x$.

$$\begin{aligned} y &= 19 + 7x && \\ y &= 19 + 7(-4) && \text{Substitute } -4 \text{ for } x. \\ y &= 19 - 28 && \text{Multiply.} \\ y &= -9 && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-4, -9)$ is the solution of the system.

23. The first step is to solve either equation for either variable. This means that we could solve the first equation for x or y, but it also means that we could solve the second equation for x or y. Of these four possible choices, solving the second equation for y seems the easiest way to start.

$$\begin{aligned} -3x + y &= 22 && \text{Second Equation.} \\ y &= 22 + 3x && \text{Add } 3x \text{ to both sides.} \end{aligned}$$

Next, substitute $22 + 3x$ for y in the first equation and solve for x.

$$\begin{aligned} 3x - 4y &= -43 && \text{First Equation.} \\ 3x - 4(22 + 3x) &= -43 && \text{Substitute } y = 22 + 3x \text{ for } y. \\ 3x - 88 - 12x &= -43 && \text{Distribute the } -4. \\ -88 - 9x &= -43 && \text{Combine like terms.} \\ -9x &= 45 && \text{Add 88 to both sides.} \\ x &= -5 && \text{Divide both sides by } -9. \end{aligned}$$

Finally, to find the y-value, substitute -5 for x in the equation $y = 22 + 3x$.

$$\begin{aligned}
y &= 22 + 3x \\
y &= 22 + 3(-5) && \text{Substitute } -5 \text{ for } x. \\
y &= 22 - 15 && \text{Multiply.} \\
y &= 7 && \text{Simplify.}
\end{aligned}$$

Hence, $(x, y) = (-5, 7)$ is the solution of the system.

25.

27. The second equation, $y = -\frac{8}{7}x+9$, is already solved for y, so let's substitute $-\frac{8}{7}x + 9$ for y in the first equation.

$$\begin{aligned}
-8x - 7y &= 2 && \text{First Equation.} \\
-8x - 7\left(-\frac{8}{7}x + 9\right) &= 2 && \text{Substitute } -\frac{8}{7}x + 9 \text{ for } y. \\
-8x + 8x - 63 &= 2 && \text{Distribute the } -7. \\
-63 &= 2 && \text{Combine like terms.}
\end{aligned}$$

The resulting statement, $-63 = 2$, is false. This should give us a clue that there are no solutions. Perhaps we are dealing with parallel lines? The second equation is already solved for y so let's solve the first equation for y to determine the situation.

$$\begin{aligned}
-8x - 7y &= 2 && \text{First Equation.} \\
-7y &= 8x + 2 && \text{Add } 8x \text{ to both sides.} \\
y &= \frac{8x + 2}{-7} && \text{Divide both sides by } -7. \\
y &= -\frac{8}{7}x - \frac{2}{7} && \text{Divide both terms by } -7.
\end{aligned}$$

Thus, our system is equivalent to the following system of equations.

$$\begin{aligned}
y &= -\frac{8}{7}x - \frac{2}{7} \\
y &= -\frac{8}{7}x + 9
\end{aligned}$$

These lines have the same slope, $-8/7$, but different y-intercepts (one has y-intercept $(0, -2/7)$, the other has y-intercept $(0, 9)$. Hence, these are two distinct parallel lines and the system has no solution.

29. We begin by solving the first equation for x.

$$\begin{aligned} 3x - 5y &= 3 && \text{First equation.} \\ 3x &= 3 + 5y && \text{Add } 5y \text{ to both sides} \\ x &= \frac{3+5y}{3} && \text{Divide both sides by 3.} \\ x &= 1 + \frac{5}{3}y && \text{Divide both terms by 3.} \end{aligned}$$

Next, substitute $1 + \frac{5}{3}y$ for x in the second equation.

$$\begin{aligned} 5x - 7y &= 2 && \text{Second equation.} \\ 5\left(1 + \frac{5}{3}y\right) - 7y &= 2 && \text{Substitute } 1 + \frac{5}{3}y \text{ for } x. \\ 5 + \frac{25}{3}y - 7y &= 2 && \text{Distribute the 5.} \\ 15 + 25y - 21y &= 6 && \text{Multiply both sides by 3.} \\ 15 + 4y &= 6 && \text{Combine like terms.} \\ 4y &= -9 && \text{Subtract 15 from both sides.} \\ y &= -\frac{9}{4} && \text{Divide both sides by 4.} \end{aligned}$$

Finally, substitute $-9/4$ for y in $x = 1 + \frac{5}{3}y$.

$$\begin{aligned} x &= 1 + \frac{5}{3}y \\ x &= 1 + \frac{5}{3}\left(-\frac{9}{4}\right) && \text{Substitute } -9/4 \text{ for } y. \\ x &= 1 - \frac{15}{4} && \text{Multiply.} \\ x &= \frac{4}{4} - \frac{15}{4} && \text{Equivalent fractions with.} \\ & && \text{a common denominator.} \\ x &= -\frac{11}{4} && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (-11/4, -9/4)$ is the solution of the system.

Check: First, store $-11/4$ in X with the following keystrokes. The result is shown in the first image below.

(-) 1 1 ÷ 4 STO▷ X, T, θ, n ENTER

Store $-9/4$ in Y with the following keystrokes. The result is shown in the first image below.

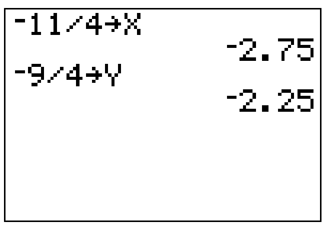
(-) 9 ÷ 4 STO▷ ALPHA 1 ENTER

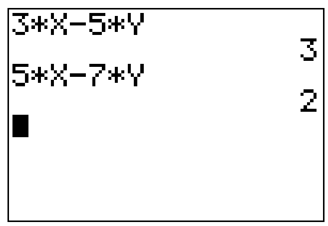
Clear the calculator screen by pressing the `CLEAR` button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

3 × X, T, θ, n − 5 × ALPHA 1 ENTER

Enter the left-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

5 × X, T, θ, n − 7 × ALPHA 1 ENTER





The result in the second image shows that $3x - 5y = 3$ and $5x - 7y = 2$ for $x = -11/4$ and $y = -9/4$. The solution checks.

31. We begin by solving the first equation for x.

$$\begin{aligned}
4x + 3y &= 8 && \text{First equation.}\\
4x &= 8 - 3y && \text{Subtract } 3y \text{ from both sides}\\
x &= \frac{8 - 3y}{4} && \text{Divide both sides by 4.}\\
x &= 2 - \frac{3}{4}y && \text{Divide both terms by 4.}
\end{aligned}$$

Next, substitute $2 - \frac{3}{4}y$ for x in the second equation.

$$\begin{aligned}
3x + 4y &= 2 && \text{Second equation.}\\
3\left(2 - \frac{3}{4}y\right) + 4y &= 2 && \text{Substitute } 2 - \frac{3}{4}y \text{ for } x.\\
6 - \frac{9}{4}y + 4y &= 2 && \text{Distribute the 3.}\\
24 - 9y + 16y &= 8 && \text{Multiply both sides by 4.}\\
24 + 7y &= 8 && \text{Combine like terms.}\\
7y &= -16 && \text{Subtract 24 from both sides.}\\
y &= -\frac{16}{7} && \text{Divide both sides by 7.}
\end{aligned}$$

Finally, substitute $-16/7$ for y in $x = 2 - \dfrac{3}{4}y$.

$$
\begin{aligned}
x &= 2 - \frac{3}{4}y \\
x &= 2 - \frac{3}{4}\left(-\frac{16}{7}\right) && \text{Substitute } -16/7 \text{ for } y. \\
x &= 2 + \frac{12}{7} && \text{Multiply.} \\
x &= \frac{14}{7} + \frac{12}{7} && \text{Equivalent fractions with.} \\
& && \text{a common denominator.} \\
x &= \frac{26}{7} && \text{Simplify.}
\end{aligned}
$$

Hence, $(x, y) = (26/7, -16/7)$ is the solution of the system.

Check: First, store 26/7 in X with the following keystrokes. The result is shown in the first image below.

2 6 ÷ 7 STO▷ X, T, θ, n ENTER

Store $-16/7$ in Y with the following keystrokes. The result is shown in the first image below.

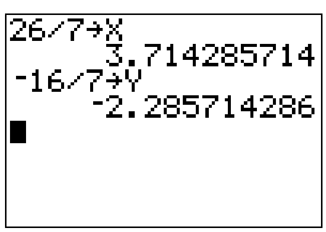
(-) 1 6 ÷ 7 STO▷ ALPHA 1 ENTER

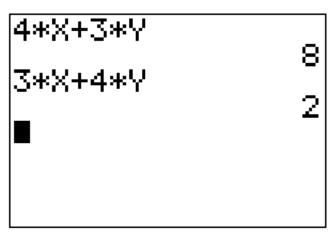
Clear the calculator screen by pressing the CLEAR button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

4 × X, T, θ, n + 3 × ALPHA 1 ENTER

Enter the left-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

3 × X, T, θ, n + 4 × ALPHA 1 ENTER





The result in the second image shows that $4x + 3y = 8$ and $3x + 4y = 2$ for $x = 26/7$ and $y = -16/7$. The solution checks.

33. We begin by solving the first equation for x.

$$\begin{aligned} 3x + 8y &= 6 && \text{First equation.} \\ 3x &= 6 - 8y && \text{Subtract } 8y \text{ from both sides} \\ x &= \frac{6-8y}{3} && \text{Divide both sides by 3.} \\ x &= 2 - \frac{8}{3}y && \text{Divide both terms by 3.} \end{aligned}$$

Next, substitute $2 - \frac{8}{3}y$ for x in the second equation.

$$\begin{aligned} 2x + 7y &= -2 && \text{Second equation.} \\ 2\left(2 - \frac{8}{3}y\right) + 7y &= -2 && \text{Substitute } 2 - \frac{8}{3}y \text{ for } x. \\ 4 - \frac{16}{3}y + 7y &= -2 && \text{Distribute the 2.} \\ 12 - 16y + 21y &= -6 && \text{Multiply both sides by 3.} \\ 12 + 5y &= -6 && \text{Combine like terms.} \\ 5y &= -18 && \text{Subtract 12 from both sides.} \\ y &= -\frac{18}{5} && \text{Divide both sides by 5.} \end{aligned}$$

Finally, substitute $-18/5$ for y in $x = 2 - \frac{8}{3}y$.

$$\begin{aligned} x &= 2 - \frac{8}{3}y \\ x &= 2 - \frac{8}{3}\left(-\frac{18}{5}\right) && \text{Substitute } -18/5 \text{ for } y. \\ x &= 2 + \frac{48}{5} && \text{Multiply.} \\ x &= \frac{10}{5} + \frac{48}{5} && \text{Equivalent fractions with.} \\ & && \text{a common denominator.} \\ x &= \frac{58}{5} && \text{Simplify.} \end{aligned}$$

Hence, $(x, y) = (58/5, -18/5)$ is the solution of the system.

Check: First, store 58/5 in X with the following keystrokes. The result is shown in the first image below.

5 8 ÷ 5 STO▷ X, T, θ, n ENTER

Store $-18/5$ in Y with the following keystrokes. The result is shown in the first image below.

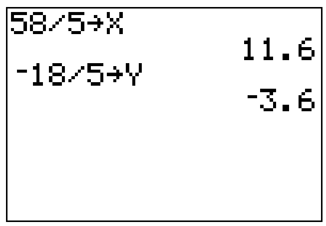
(-) 1 8 ÷ 5 STO▷ ALPHA 1 ENTER

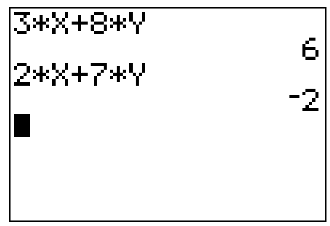
Clear the calculator screen by pressing the `CLEAR` button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

3 × X, T, θ, n + 8 × ALPHA 1 ENTER

Enter the left-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

2 × X, T, θ, n + 7 × ALPHA 1 ENTER





The result in the second image shows that $3x + 8y = 6$ and $2x + 7y = -2$ for $x = 58/5$ and $y = -18/5$. The solution checks.

35. We begin by solving the first equation for x.

$$
\begin{aligned}
4x + 5y &= 4 && \text{First equation.} \\
4x &= 4 - 5y && \text{Subtract } 5y \text{ from both sides} \\
x &= \frac{4 - 5y}{4} && \text{Divide both sides by 4.} \\
x &= 1 - \frac{5}{4}y && \text{Divide both terms by 4.}
\end{aligned}
$$

Next, substitute $1 - \frac{5}{4}y$ for x in the second equation.

$$
\begin{aligned}
-3x - 2y &= 1 && \text{Second equation.} \\
-3\left(1 - \frac{5}{4}y\right) - 2y &= 1 && \text{Substitute } 1 - \frac{5}{4}y \text{ for } x. \\
-3 + \frac{15}{4}y - 2y &= 1 && \text{Distribute the } -3. \\
-12 + 15y - 8y &= 4 && \text{Multiply both sides by 4.} \\
-12 + 7y &= 4 && \text{Combine like terms.} \\
7y &= 16 && \text{Add 12 to both sides.} \\
y &= \frac{16}{7} && \text{Divide both sides by 7.}
\end{aligned}
$$

Finally, substitute 16/7 for y in $x = 1 - \frac{5}{4}y$.

$$
\begin{aligned}
x &= 1 - \frac{5}{4}y \\
x &= 1 - \frac{5}{4}\left(\frac{16}{7}\right) && \text{Substitute 16/7 for } y. \\
x &= 1 - \frac{20}{7} && \text{Multiply.} \\
x &= \frac{7}{7} - \frac{20}{7} && \text{Equivalent fractions with.} \\
& && \text{a common denominator.} \\
x &= -\frac{13}{7} && \text{Simplify.}
\end{aligned}
$$

Hence, $(x, y) = (-13/7, 16/7)$ is the solution of the system.

Check: First, store $-13/7$ in X with the following keystrokes. The result is shown in the first image below.

(-) 1 3 ÷ 7 STO▷ X, T, θ, n ENTER

Store 16/7 in Y with the following keystrokes. The result is shown in the first image below.

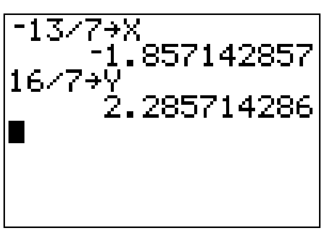
1 6 ÷ 7 STO▷ ALPHA 1 ENTER

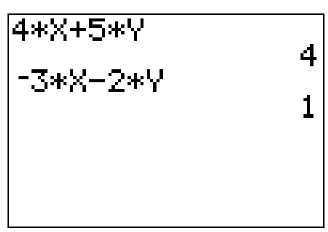
Clear the calculator screen by pressing the `CLEAR` button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

4 × X, T, θ, n + 5 × ALPHA 1 ENTER

Enter the left-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

(-) 3 × X, T, θ, n − 2 × ALPHA 1 ENTER





The result in the second image shows that $4x + 5y = 4$ and $-3x - 2y = 1$ for $x = -13/7$ and $y = 16/7$. The solution checks.

37. The second equation, $y = \frac{3}{2}x - 8$, is already solved for y, so let's substitute $\frac{3}{2}x - 8$ for y in the first equation.

$$\begin{aligned} -9x + 6y &= 9 && \text{First Equation.} \\ -9x + 6\left(\frac{3}{2}x - 8\right) &= 9 && \text{Substitute } \frac{3}{2}x - 8 \text{ for } y. \\ -9x + 9x - 48 &= 9 && \text{Distribute the 6.} \\ -48 &= 9 && \text{Combine like terms.} \end{aligned}$$

The resulting statement, $-48 = 9$, is false. This should give us a clue that there are no solutions. Perhaps we are dealing with parallel lines? The second equation is already solved for y so let's solve the first equation for y to determine the situation.

$$\begin{aligned} -9x + 6y &= 9 && \text{First Equation.} \\ 6y &= 9x + 9 && \text{Add } 9x \text{ to both sides.} \\ y &= \frac{9x + 9}{6} && \text{Divide both sides by 6.} \\ y &= \frac{3}{2}x + \frac{3}{2} && \text{Divide both terms by 6.} \end{aligned}$$

Thus, our system is equivalent to the following system of equations.

$$\begin{aligned} y &= \frac{3}{2}x + \frac{3}{2} \\ y &= \frac{3}{2}x - 8 \end{aligned}$$

These lines have the same slope, $3/2$, but different y-intercepts (one has y-intercept $(0, 3/2)$, the other has y-intercept $(0, -8)$. Hence, these are two distinct parallel lines and the system has no solution.

39. The first equation, $y = -2x - 16$, is already solved for y, so let's substitute $-2x - 16$ for y in the second equation.

$$\begin{aligned} -14x - 7y &= 112 && \text{Second Equation.} \\ -14x - 7(-2x - 16) &= 112 && \text{Substitute } -2x - 16 \text{ for } y. \\ -14x + 14x + 112 &= 112 && \text{Distribute the } -7. \\ 112 &= 112 && \text{Combine like terms.} \end{aligned}$$

The resulting statement, $112 = 112$, is a true statement. Perhaps this is an indication that we are dealing with the same line? Since the first equation is already in slope-intercept form, let's put the second equation into slope-

intercept form so that we can compare them.

$$\begin{aligned}
-14x - 7y &= 112 && \text{Second Equation.}\\
-7y &= 14x + 112 && \text{Add } 14x \text{ to both sides.}\\
y &= \frac{14x+112}{-7} && \text{Divide both sides by } -7.\\
y &= -2x - 16 && \text{Divide both terms by } -7.
\end{aligned}$$

Thus, our system is equivalent to the following system of equations.

$$\begin{aligned}
y &= -2x - 16\\
y &= -2x - 16
\end{aligned}$$

These two lines have the same slope and the same y-intercept and they are exactly the same lines. Thus, there are an infinite number of solutions. Indeed, any point on either line is a solution.

41. The first equation, $x = 16 - 5y$, is already solved for x. Substitute $16 - 5y$ for x in the second equation and solve for y.

$$\begin{aligned}
-4x + 2y &= 24 && \text{Second Equation.}\\
-4(16 - 5y) + 2y &= 24 && \text{Substitute } 16 - 5y \text{ for } x.\\
-64 + 20y + 2y &= 24 && \text{Distribute the } -4.\\
-64 + 22y &= 24 && \text{Combine like terms.}\\
22y &= 88 && \text{Add 64 to both sides.}\\
y &= 4 && \text{Divide both sides by 22.}
\end{aligned}$$

Finally, to find the x-value, substitute 4 for y in the equation
$x = 16 - 5y$.

$$\begin{aligned}
x &= 16 - 5y\\
x &= 16 - 5(4) && \text{Substitute 4 for } y.\\
x &= 16 - 20 && \text{Multiply.}\\
x &= -4 && \text{Simplify.}
\end{aligned}$$

Hence, $(x, y) = (-4, 4)$ is the solution of the system.

43. The first equation, $x = 7y + 18$, is already solved for x, so let's substitute $7y + 18$ for x in the second equation.

$$\begin{aligned}
9x - 63y &= 162 && \text{Second Equation.}\\
9(7y + 18) - 63y &= 162 && \text{Substitute } 7y + 18 \text{ for } x.\\
63y + 162 - 63y &= 162 && \text{Distribute the 9.}\\
162 &= 162 && \text{Combine like terms.}
\end{aligned}$$

The resulting statement, $162 = 162$, is a true statement. Perhaps this is an indication that we are dealing with the same line? Let's put the first and second equations into slope-intercept form so that we can compare them.

Solve $x = 7y + 18$ for y:

$$\begin{aligned} x &= 7y + 18 \\ x - 18 &= 7y \\ \frac{x-18}{7} &= y \\ y &= \frac{1}{7}x - \frac{18}{7} \end{aligned}$$

Solve $9x - 63y = 162$ for y:

$$\begin{aligned} 9x - 63y &= 162 \\ -63y &= -9x + 162 \\ y &= \frac{-9x+162}{-63} \\ y &= \frac{1}{7}x - \frac{18}{7} \end{aligned}$$

Thus, our system is equivalent to the following system of equations.

$$\begin{aligned} y &= \frac{1}{7}x - \frac{18}{7} \\ y &= \frac{1}{7}x - \frac{18}{7} \end{aligned}$$

These two lines have the same slope and the same y-intercept and they are exactly the same lines. Thus, there are an infinite number of solutions. Indeed, any point on either line is a solution.

45. The first equation, $x = -2y + 3$, is already solved for x, so let's substitute $-2y + 3$ for x in the second equation.

$$\begin{aligned} 4x + 8y &= 4 && \text{Second Equation.} \\ 4(-2y+3) + 8y &= 4 && \text{Substitute } -2y+3 \text{ for } x. \\ -8y + 12 + 8y &= 4 && \text{Distribute the 4.} \\ 12 &= 4 && \text{Combine like terms.} \end{aligned}$$

The resulting statement, $12 = 4$, is false. This should give us a clue that there are no solutions. Perhaps we are dealing with parallel lines? Let's put both equations in slope-intercept form so that we can compare them.

Solve $x = -2y + 3$ for y:

$$\begin{aligned} x &= -2y + 3 \\ x - 3 &= -2y \\ \frac{x-3}{-2} &= y \\ y &= -\frac{1}{2}x + \frac{3}{2} \end{aligned}$$

Solve $4x + 8y = 4$ for y:

$$\begin{aligned} 4x + 8y &= 4 \\ 8y &= -4x + 4 \\ y &= \frac{-4x+4}{8} \\ y &= -\frac{1}{2}x + \frac{1}{2} \end{aligned}$$

Hence, the lines have the same slope, $-1/2$, but different y-intercepts (one has y-intercept $(0, 3/2)$, the other has y-intercept $(0, 1/2)$. Hence, these are two distinct parallel lines and the system has no solution.

47. The second equation, $y = -3 - 2x$, is already solved for y. Substitute $-3 - 2x$ for y in the first equation and solve for x.

$$\begin{aligned}
-9x + 4y &= 73 && \text{First Equation.}\\
-9x + 4(-3 - 2x) &= 73 && \text{Substitute } -3 - 2x \text{ for } y.\\
-9x - 12 - 8x &= 73 && \text{Distribute the 4.}\\
-12 - 17x &= 73 && \text{Combine like terms.}\\
-17x &= 85 && \text{Add 12 to both sides.}\\
x &= -5 && \text{Divide both sides by } -17.
\end{aligned}$$

Finally, to find the y-value, substitute -5 for x in the equation $y = -3 - 2x$.

$$\begin{aligned}
y &= -3 - 2x\\
y &= -3 - 2(-5) && \text{Substitute } -5 \text{ for } x.\\
y &= -3 + 10 && \text{Multiply.}\\
y &= 7 && \text{Simplify.}
\end{aligned}$$

Hence, $(x, y) = (-5, 7)$ is the solution of the system.

4.3 Solving Systems by Elimination

1. Start with the given system.

$$\begin{array}{rcrcr}
x & + & 4y & = & 0\\
9x & - & 7y & = & -43
\end{array}$$

We'll concentrate on eliminating x. Multiply the first equation by -9, then add the results.

$$\begin{array}{rcrcr}
-9x & - & 36y & = & 0\\
9x & - & 7y & = & -43\\
\hline
 & - & 43y & = & -43
\end{array}$$

Divide both sides by -43.

$$\begin{aligned}
y &= \frac{-43}{-43} && \text{Divide both sides by } -43.\\
y &= 1 && \text{Simplify.}
\end{aligned}$$

Take the answer $y = 1$ and substitute 1 for y in the first equation.

$$\begin{aligned}
x + 4y &= 0 && \text{First equation.}\\
x + 4(1) &= 0 && \text{Substitute 1 for } y.\\
x + 4 &= 0 && \text{Multiply.}\\
x &= -4 && \text{Subtract 4 from both sides.}
\end{aligned}$$

Hence, the solution is $(x, y) = (-4, 1)$.

Check: We must show that the solution $(x, y) = (-4, 1)$ satisfies both equations.

$$\begin{aligned} x + 4y &= 0 \\ (-4) + 4(1) &= 0 \\ -4 + 4 &= 0 \\ 0 &= 0 \end{aligned} \qquad\qquad \begin{aligned} 9x - 7y &= -43 \\ 9(-4) - 7(1) &= -43 \\ -36 - 7 &= -43 \\ -43 &= -43 \end{aligned}$$

Because each of the last two statements are true, this guarantees that $(x, y) = (-4, 1)$ is a solution of the system.

3. Start with the given system.

$$\begin{array}{rcrcr} 6x & + & y & = & 8 \\ 4x & + & 2y & = & 0 \end{array}$$

We'll concentrate on eliminating y. Multiply the first equation by -2, then add the results.

$$\begin{array}{rcrcr} -12x & - & 2y & = & -16 \\ 4x & + & 2y & = & 0 \\ \hline -8x & & & = & -16 \end{array}$$

Divide both sides by -8.

$$\begin{aligned} x &= \frac{-16}{-8} && \text{Divide both sides by } -8. \\ x &= 2 && \text{Simplify.} \end{aligned}$$

Take the answer $x = 2$ and substitute 2 for x in the first equation.

$$\begin{aligned} 6x + y &= 8 && \text{First equation.} \\ 6(2) + y &= 8 && \text{Substitute 2 for } x. \\ 12 + y &= 8 && \text{Multiply.} \\ y &= -4 && \text{Subtract 12 from both sides.} \end{aligned}$$

Hence, the solution is $(x, y) = (2, -4)$.

Check: We must show that the solution $(x, y) = (2, -4)$ satisfies both equations.

$$\begin{aligned} 6x + y &= 8 \\ 6(2) + (-4) &= 8 \\ 12 - 4 &= 8 \\ 8 &= 8 \end{aligned} \qquad\qquad \begin{aligned} 4x + 2y &= 0 \\ 4(2) + 2(-4) &= 0 \\ 8 - 8 &= 0 \\ 0 &= 0 \end{aligned}$$

Because each of the last two statements are true, this guarantees that $(x, y) = (2, -4)$ is a solution of the system.

5. Start with the given system.

$$\begin{array}{rcrcr} -8x & + & y & = & -56 \\ 4x & + & 3y & = & 56 \end{array}$$

We'll concentrate on eliminating y. Multiply the first equation by -3, then add the results.

$$\begin{array}{rcrcr} 24x & - & 3y & = & 168 \\ 4x & + & 3y & = & 56 \\ \hline 28x & & & = & 224 \end{array}$$

Divide both sides by 28.

$$\begin{aligned} x &= \frac{224}{28} && \text{Divide both sides by 28.} \\ x &= 8 && \text{Simplify.} \end{aligned}$$

Take the answer $x = 8$ and substitute 8 for x in the first equation.

$$\begin{aligned} -8x + y &= -56 && \text{First equation.} \\ -8(8) + y &= -56 && \text{Substitute 8 for } x. \\ -64 + y &= -56 && \text{Multiply.} \\ y &= 8 && \text{Add 64 to both sides.} \end{aligned}$$

Hence, the solution is $(x, y) = (8, 8)$.

Check: We must show that the solution $(x, y) = (8, 8)$ satisfies both equations.

$$\begin{aligned} -8x + y &= -56 \\ -8(8) + (8) &= -56 \\ -64 + 8 &= -56 \\ -56 &= -56 \end{aligned} \qquad \begin{aligned} 4x + 3y &= 56 \\ 4(8) + 3(8) &= 56 \\ 32 + 24 &= 56 \\ 56 &= 56 \end{aligned}$$

Because each of the last two statements are true, this guarantees that $(x, y) = (8, 8)$ is a solution of the system.

7. Start with the given system.

$$\begin{array}{rcrcr} x & + & 8y & = & 41 \\ -5x & - & 9y & = & -50 \end{array}$$

We'll concentrate on eliminating x. Multiply the first equation by 5, then add the results.

$$\begin{array}{rcrcr} 5x & + & 40y & = & 205 \\ -5x & - & 9y & = & -50 \\ \hline & & 31y & = & 155 \end{array}$$

Divide both sides by 31.

$$\begin{aligned} y &= \frac{155}{31} && \text{Divide both sides by 31.} \\ y &= 5 && \text{Simplify.} \end{aligned}$$

Take the answer $y = 5$ and substitute 5 for y in the first equation.

$$\begin{aligned} x + 8y &= 41 && \text{First equation.} \\ x + 8(5) &= 41 && \text{Substitute 5 for } y. \\ x + 40 &= 41 && \text{Multiply.} \\ x &= 1 && \text{Subtract 40 from both sides.} \end{aligned}$$

Hence, the solution is $(x, y) = (1, 5)$.

Check: We must show that the solution $(x, y) = (1, 5)$ satisfies both equations.

$$\begin{aligned} x + 8y &= 41 \\ (1) + 8(5) &= 41 \\ 1 + 40 &= 41 \\ 41 &= 41 \end{aligned} \qquad \begin{aligned} -5x - 9y &= -50 \\ -5(1) - 9(5) &= -50 \\ -5 - 45 &= -50 \\ -50 &= -50 \end{aligned}$$

Because each of the last two statements are true, this guarantees that $(x, y) = (1, 5)$ is a solution of the system.

9. Start with the given system.

$$\begin{array}{rcrcr} -12x & + & 9y & = & 0 \\ -6x & - & 4y & = & -34 \end{array}$$

We'll concentrate on eliminating x. Multiply the second equation by -2, then add the results.

$$\begin{array}{rcrcr} -12x & + & 9y & = & 0 \\ 12x & + & 8y & = & 68 \\ \hline & & 17y & = & 68 \end{array}$$

Divide both sides by 17.

$$\begin{aligned} 17y &= 68 \\ y &= \frac{68}{17} && \text{Divide both sides by 17.} \\ y &= 4 && \text{Simplify.} \end{aligned}$$

Take the answer $y = 4$ and substitute 4 for y in the first equation.

$$\begin{aligned} -12x + 9y &= 0 && \text{First equation.} \\ -12x + 9(4) &= 0 && \text{Substitute 4 for } y. \\ -12x + 36 &= 0 && \text{Multiply.} \\ -12x &= -36 && \text{Subtract 36 from both sides.} \\ x &= 3 && \text{Divide both sides by } -12 \end{aligned}$$

Hence, the solution is $(x, y) = (3, 4)$.

11. Start with the given system.

$$\begin{array}{rcrcr} 27x & - & 6y & = & -96 \\ -3x & - & 5y & = & 22 \end{array}$$

We'll concentrate on eliminating x. Multiply the second equation by 9, then add the results.

$$\begin{array}{rcrcr} 27x & - & 6y & = & -96 \\ -27x & - & 45y & = & 198 \\ \hline & - & 51y & = & 102 \end{array}$$

Divide both sides by -51.

$$\begin{aligned} -51y &= 102 \\ y &= \frac{102}{-51} && \text{Divide both sides by } -51. \\ y &= -2 && \text{Simplify.} \end{aligned}$$

Take the answer $y = -2$ and substitute -2 for y in the first equation.

$$\begin{aligned} 27x - 6y &= -96 && \text{First equation.} \\ 27x - 6(-2) &= -96 && \text{Substitute } -2 \text{ for } y. \\ 27x + 12 &= -96 && \text{Multiply.} \\ 27x &= -108 && \text{Subtract 12 from both sides.} \\ x &= -4 && \text{Divide both sides by 27} \end{aligned}$$

Hence, the solution is $(x, y) = (-4, -2)$.

13. Start with the given system.

$$\begin{array}{rcrcr} 2x & - & 6y & = & 28 \\ -3x & + & 18y & = & -60 \end{array}$$

We'll concentrate on eliminating y. Multiply the first equation by 3, then add the results.

$$\begin{array}{rcrcr} 6x & - & 18y & = & 84 \\ -3x & + & 18y & = & -60 \\ \hline 3x & & & = & 24 \end{array}$$

Divide both sides by 3.

$$\begin{aligned} 3x &= 24 \\ x &= \frac{24}{3} && \text{Divide both sides by 3.} \\ x &= 8 && \text{Simplify.} \end{aligned}$$

Take the answer $x = 8$ and substitute 8 for x in the first equation.

$$\begin{aligned} 2x - 6y &= 28 && \text{First equation.} \\ 2(8) - 6y &= 28 && \text{Substitute 8 for } x. \\ 16 - 6y &= 28 && \text{Multiply.} \\ -6y &= 12 && \text{Subtract 16 from both sides.} \\ y &= -2 && \text{Divide both sides by } -6 \end{aligned}$$

Hence, the solution is $(x, y) = (8, -2)$.

15. Start with the given system.

$$\begin{array}{rcrcr} -32x & + & 7y & = & -238 \\ 8x & - & 4y & = & 64 \end{array}$$

We'll concentrate on eliminating x. Multiply the second equation by 4, then add the results.

$$\begin{array}{rcrcr} -32x & + & 7y & = & -238 \\ 32x & - & 16y & = & 256 \\ \hline & - & 9y & = & 18 \end{array}$$

Divide both sides by -9.

$$\begin{aligned} -9y &= 18 \\ y &= \frac{18}{-9} && \text{Divide both sides by } -9. \\ y &= -2 && \text{Simplify.} \end{aligned}$$

Take the answer $y = -2$ and substitute -2 for y in the first equation.

$$\begin{aligned}
-32x + 7y &= -238 && \text{First equation.}\\
-32x + 7(-2) &= -238 && \text{Substitute } -2 \text{ for } y.\\
-32x - 14 &= -238 && \text{Multiply.}\\
-32x &= -224 && \text{Add 14 to both sides.}\\
x &= 7 && \text{Divide both sides by } -32
\end{aligned}$$

Hence, the solution is $(x, y) = (7, -2)$.

17. Start with the given system.

$$\begin{array}{rcrcr}
3x & - & 7y & = & -75\\
-2x & - & 2y & = & -10
\end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by 2, the second equation by 3, then add the results.

$$\begin{array}{rcrcr}
6x & - & 14y & = & -150\\
-6x & - & 6y & = & -30\\
\hline
 & - & 20y & = & -180
\end{array}$$

Divide both sides by -20.

$$\begin{aligned}
-20y &= -180\\
y &= \frac{-180}{-20} && \text{Divide both sides by } -20.\\
y &= 9 && \text{Simplify.}
\end{aligned}$$

Take the answer $y = 9$ and substitute 9 for y in the first equation (you could also make the substitution in the second equation).

$$\begin{aligned}
3x - 7y &= -75 && \text{First equation.}\\
3x - 7(9) &= -75 && \text{Substitute 9 for } y.\\
3x - 63 &= -75 && \text{Multiply.}\\
3x &= -12 && \text{Add 63 to both sides.}\\
x &= \frac{-12}{3} && \text{Divide both sides by 3.}\\
x &= -4 && \text{Simplify.}
\end{aligned}$$

Hence, the solution is $(x, y) = (-4, 9)$.

19. Start with the given system.

$$\begin{array}{rcrcr} 9x & - & 9y & = & -63 \\ 2x & - & 6y & = & -34 \end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by -2, the second equation by 9, then add the results.

$$\begin{array}{rcrcr} -18x & + & 18y & = & 126 \\ 18x & - & 54y & = & -306 \\ \hline & - & 36y & = & -180 \end{array}$$

Divide both sides by -36.

$$\begin{aligned} -36y &= -180 \\ y &= \frac{-180}{-36} && \text{Divide both sides by } -36. \\ y &= 5 && \text{Simplify.} \end{aligned}$$

Take the answer $y = 5$ and substitute 5 for y in the first equation (you could also make the substitution in the second equation).

$$\begin{aligned} 9x - 9y &= -63 && \text{First equation.} \\ 9x - 9(5) &= -63 && \text{Substitute 5 for } y. \\ 9x - 45 &= -63 && \text{Multiply.} \\ 9x &= -18 && \text{Add 45 to both sides.} \\ x &= \frac{-18}{9} && \text{Divide both sides by 9.} \\ x &= -2 && \text{Simplify.} \end{aligned}$$

Hence, the solution is $(x, y) = (-2, 5)$.

21. Start with the given system.

$$\begin{array}{rcrcr} -9x & - & 2y & = & 28 \\ 5x & - & 3y & = & -32 \end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by -5, the second equation by -9, then add the results.

$$\begin{array}{rcrcr} 45x & + & 10y & = & -140 \\ -45x & + & 27y & = & 288 \\ \hline & & 37y & = & 148 \end{array}$$

Divide both sides by 37.

$$\begin{aligned} 37y &= 148 \\ y &= \frac{148}{37} && \text{Divide both sides by 37.} \\ y &= 4 && \text{Simplify.} \end{aligned}$$

Take the answer $y = 4$ and substitute 4 for y in the first equation (you could also make the substitution in the second equation).

$$\begin{aligned} -9x - 2y &= 28 && \text{First equation.} \\ -9x - 2(4) &= 28 && \text{Substitute 4 for } y. \\ -9x - 8 &= 28 && \text{Multiply.} \\ -9x &= 36 && \text{Add 8 to both sides.} \\ x &= \frac{36}{-9} && \text{Divide both sides by } -9. \\ x &= -4 && \text{Simplify.} \end{aligned}$$

Hence, the solution is $(x, y) = (-4, 4)$.

23. Start with the given system.

$$\begin{array}{rcrcr} -3x & - & 5y & = & -34 \\ 7x & + & 7y & = & 56 \end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by -7, the second equation by -3, then add the results.

$$\begin{array}{rcrcr} 21x & + & 35y & = & 238 \\ -21x & - & 21y & = & -168 \\ \hline & & 14y & = & 70 \end{array}$$

Divide both sides by 14.

$$\begin{aligned} 14y &= 70 \\ y &= \frac{70}{14} && \text{Divide both sides by 14.} \\ y &= 5 && \text{Simplify.} \end{aligned}$$

Take the answer $y = 5$ and substitute 5 for y in the first equation (you could also make the substitution in the second equation).

$$\begin{aligned}
-3x - 5y &= -34 && \text{First equation.}\\
-3x - 5(5) &= -34 && \text{Substitute 5 for } y.\\
-3x - 25 &= -34 && \text{Multiply.}\\
-3x &= -9 && \text{Add 25 to both sides.}\\
x &= \frac{-9}{-3} && \text{Divide both sides by } -3.\\
x &= 3 && \text{Simplify.}
\end{aligned}$$

Hence, the solution is $(x, y) = (3, 5)$.

25. Start with the given system.

$$\begin{array}{rcrcr}
2x & - & 7y & = & -2\\
7x & + & 6y & = & 3
\end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by -7, the second equation by 2, then add the results.

$$\begin{array}{rcrcr}
-14x & + & 49y & = & 14\\
14x & + & 12y & = & 6\\
\hline
 & & 61y & = & 20
\end{array}$$

Divide both sides by 61 to get $y = 20/61$. Next, we could substitute 20/61 for y in either equation and solve to find x. However, in this case it is probably easier to perform elimination again. Start with the given system again.

$$\begin{array}{rcrcr}
2x & - & 7y & = & -2\\
7x & + & 6y & = & 3
\end{array}$$

This time we concentrate on eliminating the variable y. Multiply the first equation by -6, the second equation by -7, then add the results.

$$\begin{array}{rcrcr}
-12x & + & 42y & = & 12\\
-49x & - & 42y & = & -21\\
\hline
-61x & & & = & -9
\end{array}$$

Divide both sides by -61 to get $x = 9/61$. Hence, the solution is $(x, y) = (9/61, 20/61)$.

Check: First, store 9/61 in X with the following keystrokes. The result is shown in the first image below.

9 ÷ 6 1 STO▷ X, T, θ, n ENTER

Store 20/61 in Y with the following keystrokes. The result is shown in the first image below.

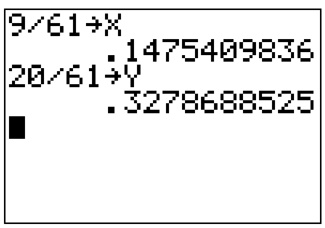
2 0 ÷ 6 1 STO▷ ALPHA 1 ENTER

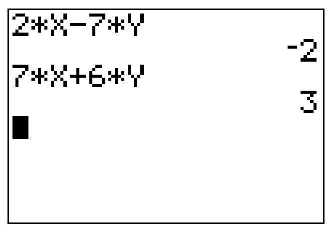
Clear the calculator screen by pressing the CLEAR button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

2 × X, T, θ, n − 7 × ALPHA 1 ENTER

Enter the right-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

7 × X, T, θ, n + 6 × ALPHA 1 ENTER





The result in the second image shows that $2x - 7y = -2$ and $7x + 6y = 3$ for $x = 9/61$ and $y = 20/61$. The solution checks.

27. Start with the given system.

$$\begin{array}{rcrcr} 2x & + & 3y & = & -2 \\ -5x & + & 5y & = & 2 \end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by 5, the second equation by 2, then add the results.

$$\begin{array}{rcrcr} 10x & + & 15y & = & -10 \\ -10x & + & 10y & = & 4 \\ \hline & & 25y & = & -6 \end{array}$$

Divide both sides by 25 to get $y = -6/25$. Next, we could substitute $-6/25$ for y in either equation and solve to find x. However, in this case it is probably easier to perform elimination again. Start with the given system again.

$$\begin{array}{rcrcr} 2x & + & 3y & = & -2 \\ -5x & + & 5y & = & 2 \end{array}$$

This time we concentrate on eliminating the variable y. Multiply the first equation by -5, the second equation by 3, then add the results.

$$\begin{array}{rcrcr} -10x & - & 15y & = & 10 \\ -15x & + & 15y & = & 6 \\ \hline -25x & & & = & 16 \end{array}$$

Divide both sides by -25 to get $x = -16/25$. Hence, the solution is $(x, y) = (-16/25, -6/25)$.

Check: First, store $-16/25$ in X with the following keystrokes. The result is shown in the first image below.

(-) 1 6 ÷ 2 5 STO▷ X, T, θ, n ENTER

Store $-6/25$ in Y with the following keystrokes. The result is shown in the first image below.

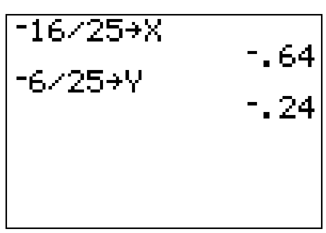
(-) 6 ÷ 2 5 STO▷ ALPHA 1 ENTER

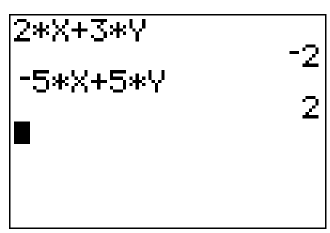
Clear the calculator screen by pressing the `CLEAR` button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

2 × X, T, θ, n + 3 × ALPHA 1 ENTER

Enter the right-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

(-) 5 × X, T, θ, n + 5 × ALPHA 1 ENTER





The result in the second image shows that $2x + 3y = -2$ and $-5x + 5y = 2$ for $x = -16/25$ and $y = -6/25$. The solution checks.

29. Start with the given system.

$$\begin{array}{rcrcr} 9x & + & 4y & = & -4 \\ -7x & - & 9y & = & 3 \end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by 7, the second equation by 9, then add the results.

$$\begin{array}{rcrcr} 63x & + & 28y & = & -28 \\ -63x & - & 81y & = & 27 \\ \hline & - & 53y & = & -1 \end{array}$$

Divide both sides by -53 to get $y = 1/53$. Next, we could substitute $1/53$ for y in either equation and solve to find x. However, in this case it is probably easier to perform elimination again. Start with the given system again.

$$\begin{array}{rcrcr} 9x & + & 4y & = & -4 \\ -7x & - & 9y & = & 3 \end{array}$$

This time we concentrate on eliminating the variable y. Multiply the first equation by 9, the second equation by 4, then add the results.

$$\begin{array}{rcrcr} 81x & + & 36y & = & -36 \\ -28x & - & 36y & = & 12 \\ \hline 53x & & & = & -24 \end{array}$$

Divide both sides by 53 to get $x = -24/53$. Hence, the solution is $(x, y) = (-24/53, 1/53)$.

Check: First, store $-24/53$ in X with the following keystrokes. The result is shown in the first image below.

(-) 2 4 ÷ 5 3 STO▷ X, T, θ, n ENTER

Store 1/53 in Y with the following keystrokes. The result is shown in the first image below.

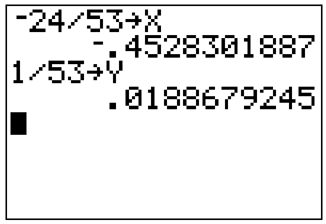
1 ÷ 5 3 STO▷ ALPHA 1 ENTER

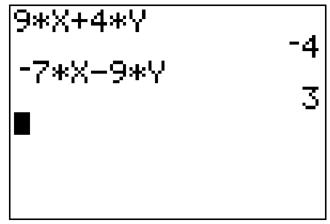
Clear the calculator screen by pressing the `CLEAR` button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

9 × X, T, θ, n + 4 × ALPHA 1 ENTER

Enter the right-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

(-) 7 × X, T, θ, n − 9 × ALPHA 1 ENTER





The result in the second image shows that $9x+4y=-4$ and $-7x-9y=3$ for $x=-24/53$ and $y=1/53$. The solution checks.

31. Start with the given system.

$$\begin{array}{rcrcr} 2x & + & 2y & = & 4 \\ 3x & - & 5y & = & 3 \end{array}$$

We'll first concentrate on eliminating the variable x. Multiply the first equation by -3, the second equation by 2, then add the results.

$$\begin{array}{rcrcr} -6x & - & 6y & = & -12 \\ 6x & - & 10y & = & 6 \\ \hline & - & 16y & = & -6 \end{array}$$

Divide both sides by -16 to get $y=3/8$. Next, we could substitute 3/8 for y in either equation and solve to find x. However, in this case it is probably easier to perform elimination again. Start with the given system again.

$$\begin{array}{rcrcr} 2x & + & 2y & = & 4 \\ 3x & - & 5y & = & 3 \end{array}$$

This time we concentrate on eliminating the variable y. Multiply the first equation by 5, the second equation by 2, then add the results.

$$\begin{array}{rcrcr} 10x & + & 10y & = & 20 \\ 6x & - & 10y & = & 6 \\ \hline 16x & & & = & 26 \end{array}$$

Divide both sides by 16 to get $x=13/8$. Hence, the solution is $(x,y)=(13/8, 3/8)$.

Check: First, store 13/8 in X with the following keystrokes. The result is shown in the first image below.

1 3 ÷ 8 STO▷ X, T, θ, n ENTER

Store 3/8 in Y with the following keystrokes. The result is shown in the first image below.

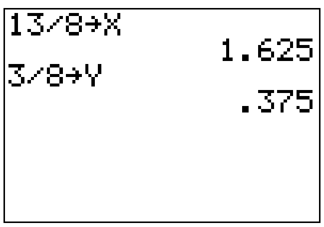
3 ÷ 8 STO▷ ALPHA 1 ENTER

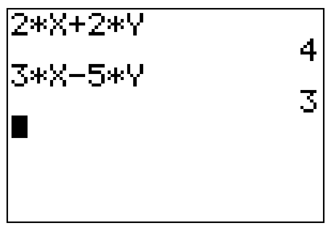
Clear the calculator screen by pressing the `CLEAR` button, then enter the left-hand side of the first equation with the following keystrokes. The result is shown in the second image below.

2 × X, T, θ, n + 2 × ALPHA 1 ENTER

Enter the right-hand side of the second equation with the following keystrokes. The result is shown in the second image below.

3 × X, T, θ, n − 5 × ALPHA 1 ENTER





The result in the second image shows that $2x + 2y = 4$ and $3x - 5y = 3$ for $x = 13/8$ and $y = 3/8$. The solution checks.

33. Start with the given system.

$$\begin{array}{rcrcr} x & + & 7y & = & -32 \\ -8x & - & 56y & = & 256 \end{array}$$

We'll concentrate on eliminating x. Multiply the first equation by 8, then add the results.

$$\begin{array}{rcrcr} 8x & + & 56y & = & -256 \\ -8x & - & 56y & = & 256 \\ \hline & & 0 & = & 0 \end{array}$$

Note that this last statement, $0 = 0$, is true. Hence, the system has an infinite number of solutions.

35. Start with the given system.

$$\begin{array}{rcrcr} 16x & - & 16y & = & -256 \\ -8x & + & 8y & = & 128 \end{array}$$

We'll concentrate on eliminating x. Multiply the second equation by 2, then add the results.

$$\begin{array}{rcrcr} 16x & - & 16y & = & -256 \\ -16x & + & 16y & = & 256 \\ \hline & & 0 & = & 0 \end{array}$$

Note that this last statement, $0 = 0$, is true. Hence, the system has an infinite number of solutions.

37. Start with the given system.

$$\begin{array}{rcrcr} x & - & 4y & = & -37 \\ 2x & - & 8y & = & 54 \end{array}$$

We'll concentrate on eliminating x. Multiply the first equation by -2, then add the results.

$$\begin{array}{rcrcr} -2x & + & 8y & = & 74 \\ 2x & - & 8y & = & 54 \\ \hline & & 0 & = & 128 \end{array}$$

Note that this last statement, $0 = 128$, is false. Hence, the system has no solution.

39. Start with the given system.

$$\begin{array}{rcrcr} x & + & 9y & = & 73 \\ -4x & - & 5y & = & -44 \end{array}$$

We'll concentrate on eliminating x. Multiply the first equation by 4, then add the results.

$$\begin{array}{rcrcr} 4x & + & 36y & = & 292 \\ -4x & - & 5y & = & -44 \\ \hline & & 31y & = & 248 \end{array}$$

Divide both sides by 31.

$$\begin{aligned} 31y &= 248 \\ y &= \frac{248}{31} && \text{Divide both sides by 31.} \\ y &= 8 && \text{Simplify.} \end{aligned}$$

Take the answer $y = 8$ and substitute 8 for y in the first equation.

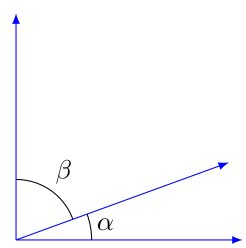
$$\begin{aligned} x + 9y &= 73 && \text{First equation.} \\ x + 9(8) &= 73 && \text{Substitute 8 for } y. \\ x + 72 &= 73 && \text{Multiply.} \\ x &= 1 && \text{Subtract 72 from both sides.} \end{aligned}$$

Hence, the solution is $(x, y) = (1, 8)$.

4.4 Applications of Linear Systems

1. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Our variable dictionary will take the form of a diagram, naming the two complementary angles α and β.



2. *Set up a System of Equations.* The "second angle is 42 degrees larger than 3 times the first angle" becomes

$$\beta = 42 + 3\alpha$$

Secondly, the angles are complementary, meaning that the sum of the angles is 90°.

$$\alpha + \beta = 90$$

Thus, we have a system of two equations in two unknowns α and β.

3. *Solve the System.* Substitute $42 + 3\alpha$ for β in $\alpha + \beta = 90$.

$$\begin{aligned} \alpha + \beta &= 90 && \\ \alpha + (42 + 3\alpha) &= 90 && \text{Substitute } 42 + 3\alpha \text{ for } \beta. \\ 4\alpha + 42 &= 90 && \text{Combine like terms.} \\ 4\alpha &= 48 && \text{Subtract 42 from both sides.} \\ \alpha &= 12 && \text{Divide both sides by 4.} \end{aligned}$$

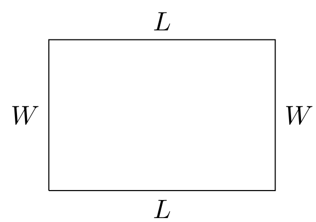
4. *Answer the Question.* The first angle is $\alpha = 12$ degrees. The second angle is:

$$\begin{aligned} \beta &= 42 + 3\alpha \\ \beta &= 42 + 3(12) && \text{Substitute 12 for } \alpha. \\ \beta &= 78 && \text{Simplify.} \end{aligned}$$

5. *Look Back.* Certainly 78° is 42° larger than 3 times 12°. Also, note that $12^\circ + 78^\circ = 90^\circ$, so the angles are complementary. We have the correct solution.

3. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Our variable dictionary will take the form of a diagram, naming the width and length W and L, respectively.



2. *Set up a System of Equations.* The perimeter is found by summing the four sides of the rectangle.

$$\begin{aligned} P &= L + W + L + W \\ P &= 2L + 2W \end{aligned}$$

We're told the perimeter is 116 inches, so we can substitute 116 for P in the last equation.

$$116 = 2L + 2W$$

We can simplify this equation by dividing both sides by 2, giving the following result:

$$L + W = 58$$

Secondly, we're told that the "length is 28 inches more than twice the width." This translates to:

$$L = 28 + 2W$$

3. *Solve the System.* As the last equation is already solved for L, let use the substitution method and substitute $28 + 2W$ for L in the equation $L + W = 58$.

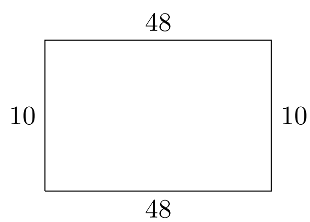
$$\begin{aligned} L + W &= 58 && \text{Perimeter equation.} \\ (28 + 2W) + W &= 58 && \text{Substitute } 28 + 2W \text{ for } L. \\ 3W + 28 &= 58 && \text{Combine like terms.} \\ 3W &= 30 && \text{Subtract 28 from both sides.} \\ W &= 10 && \text{Divide both sides by 3.} \end{aligned}$$

4. *Answer the Question.* The width is $W = 10$ inches. To find the length, substitute 10 for W in the equation $L = 28 + 2W$.

$$\begin{aligned} L &= 28 + 2W && \text{Length equation.} \\ L &= 28 + 2(10) && \text{Substitute 10 for } W. \\ L &= 28 + 20 && \text{Multiply.} \\ L &= 48 && \text{Add.} \end{aligned}$$

Thus, the length is $L = 48$ inches.

5. *Look Back.* Perhaps a picture, labeled with our answers might best demonstrate that we have the correct solution. Remember, we found that the width was 10 inches and the length was 48 inches.



Note that the perimeter is $P = 48 + 10 + 48 + 10 = 116$ inches. Secondly, note that the length (48 inches) is 28 inches more than twice the width. So we have the correct solution.

5. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let N represent the number of nickels and let Q represent the number of quarters.

2. *Set up a System of Equations.* Using a table to summarize information is a good strategy. In the first column, we list the type of coin. The second column gives the number of each type of coin, and the third column contains the value (in cents) of the number of coins in her pocket.

	Number of Coins	Value (in cents)
Nickels	N	$5N$
Quarters	Q	$25Q$
Totals	59	635

Note that N nickels, valued at 5 cents apiece, are worth $5N$ cents. Similarly, Q quarters, valued at 25 cents apiece, are worth $25Q$ cents. Note also how we've change \$6.35 to 635 cents.

The second column of the table gives us our first equation.

$$N + Q = 59$$

The third column of the table gives us our second equation.

$$5N + 25Q = 635$$

3. *Solve the System.* Because both equations are in standard form $Ax+By = C$, we'll use the elimination method to find a solution. Because the question asks us to find the number of quarters in her pocket, we'll focus on eliminating the N-terms and keeping the Q-terms.

$$\begin{array}{rcrcr} -5N & - & 5Q & = & -295 \\ 5N & + & 25Q & = & 635 \\ \hline 20Q & & & = & 340 \end{array}$$

Multiply first equation by -5.
Second equation.
Add the equations.

Dividing both sides of the last equation by 20 gives us $Q = 17$.

4. *Answer the Question.* The previous solution tells us that Maria has 17 quarters in her pocket.

5. *Look Back.* Again, summarizing results in a table might help us see if we have the correct solution. First, because we're told that Maria has 59 coins in all, and we found that she had 17 quarters, this means that she must have 42 nickels.

	Number of Coins	Value (in cents)
Nickels	42	210
Quarters	17	425
Totals	59	635

42 nickels are worth 210 cents, and 17 quarters are worth 425 cents. That's a total of 59 coins and 635 cents, or \$6.35. Thus we have the correct solution.

7. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let x be the number of pounds of cashews used and let y be the number of pounds of raisins used.

2. *Set up a System of Equations.* The following table summarizes the information given in the problem:

	Cost per pound	Amount (pounds)	Cost
cashews	\$6.00	x	$6.00x$
raisins	\$7.00	y	$7.00y$
Totals	\$6.42	50	$6.42(50) = 321.00$

At \$6.00 per pound, x pounds of cashews cost $6.00x$. At \$7.00 per pound, y pounds of raisins cost $7.00y$. Finally, at \$6.42 per pound, 50 pounds of a mixture of cashews and raisins will cost \$6.42(50), or \$321.00.

The third column of the table gives us our first equation. The total number of pounds of mixture is given by the following equation:

$$x + y = 50$$

The fourth column of the table gives us our second equation. The total cost is the sum of the costs for purchasing the cashews and raisins.

$$6.00x + 7.00y = 321.00$$

Therefore, we have the following system of equations:

$$\begin{aligned} x + y &= 50 \\ 6.00x + 7.00y &= 321.00 \end{aligned}$$

3. *Solve the System.* We can solve this system by substitution. Solve the first equation for x.

$$\begin{aligned} x + y &= 50 && \text{First Equation.} \\ x &= 50 - y && \text{Subtract } y \text{ from both sides.} \end{aligned}$$

Next, substitute $50 - y$ for x in the second equation and solve for y.

$$\begin{aligned} 6.00x + 7.00y &= 321.00 && \text{Second Equation.} \\ 6.00(50 - y) + 7.00y &= 321.00 && \text{Substitute } 50 - y \text{ for } x. \\ 300.00 - 6.00y + 7.00y &= 321.00 && \text{Distribute the 6.00.} \\ 300.00 + 1.00y &= 321.00 && \text{Combine like terms.} \\ 1.00y &= 21.00 && \text{Subtract 300.00 from both sides.} \\ y &= 21 && \text{Divide both sides by 1.00.} \end{aligned}$$

Thus, there are 21 pounds of raisins in the mix.

4. *Answer the Question.* The question asks for both amounts, cashews and raisins. Substitute 21 for y in the first equation and solve for x.

$$\begin{aligned} x + y &= 50 && \text{First Equation.} \\ x + 21 &= 50 && \text{Substitute 21 for } y. \\ x &= 29 && \text{Subtract 21 from both sides.} \end{aligned}$$

   Thus, there are 29 pounds of cashews in the mix.

5. *Look Back.* First, note that the amount of cashews and raisins in the mix is 29 and 21 pounds respectively, so that the total mixture weighs 50 pounds as required. Let's calculate the costs: for the cashews, 6.00(29), or \$174.00, for the raisins, 7.00(21), or \$147.00.

	Cost per pound	Amount (pounds)	Cost
cashews	\$6.00	29	\$174.00
raisins	\$7.00	21	\$147.00
Totals	\$6.42	50	\$321.00

   Note that the total cost is \$321.00, as required in the problem statement. Thus, our solution is correct.

9. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let D represent the number of dimes and let Q represent the number of quarters.

2. *Set up a System of Equations.* Using a table to summarize information is a good strategy. In the first column, we list the type of coin. The second column gives the number of each type of coin, and the third column contains the value (in cents) of the number of coins in his pocket.

	Number of Coins	Value (in cents)
Dimes	D	$10D$
Quarters	Q	$25Q$
Totals	38	545

   Note that D times, valued at 10 cents apiece, are worth $10D$ cents. Similarly, Q quarters, valued at 25 cents apiece, are worth $25Q$ cents. Note also how we've change \$5.45 to 545 cents.

The second column of the table gives us our first equation.

$$D + Q = 38$$

The third column of the table gives us our second equation.

$$10D + 25Q = 545$$

3. *Solve the System.* Because both equations are in standard form $Ax+By = C$, we'll use the elimination method to find a solution. Because the question asks us to find the number of dimes in his pocket, we'll focus on eliminating the Q-terms and keeping the D-terms.

$$\begin{array}{rcrcrl} -25D & - & 25Q & = & -950 & \quad \text{Multiply first equation by } -25. \\ 10D & + & 25Q & = & 545 & \quad \text{Second equation.} \\ \hline -15D & & & = & -405 & \quad \text{Add the equations.} \end{array}$$

Dividing both sides of the last equation by -15 gives us $D = 27$.

4. *Answer the Question.* The previous solution tells us that Roberto has 27 dimes in his pocket.

5. *Look Back.* Again, summarizing results in a table might help us see if we have the correct solution. First, because we're told that Roberto has 38 coins in all, and we found that he had 27 dimes, this means that he must have 11 quarters.

	Number of Coins	Value (in cents)
Dimes	27	270
Quarters	11	275
Totals	38	545

27 dimes are worth 270 cents, and 11 quarters are worth 275 cents. That's a total of 38 coins and 545 cents, or \$5.45. Thus we have the correct solution.

11. In geometry, two angles that sum to 180° are called *supplementary angles.* If the second of two supplementary angles is 40 degrees larger than 3 times the first angle, find the degree measure of both angles.

13. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Let C represent the amount invested in the certificate of deposit and M represent the amount invested in the mutual fund.

2. *Set up a System of Equations.* The following table summarizes the information given in the problem:

	Rate	Amount invested	Interest
Certificate of Deposit	3%	C	$0.03C$
Mutual Fund	5%	M	$0.05M$
Totals		20,000	780

   At 3%, the interest earned on a C dollars investment is found by taking 3% of C (i.e., $0.03C$). Similarly, the interest earned on the mutual fund is $0.05M$.

   The third column of the table gives us our first equation. The total investment is $20,000.

$$C + M = 20000$$

   The fourth column of the table gives us our second equation. The total interest earned is the sum of the interest earned in each account.

$$0.03C + 0.05M = 780$$

   Therefore, we have the following system of equations:

$$\begin{aligned} C + M &= 20000 \\ 0.03C + 0.05M &= 780 \end{aligned}$$

3. *Solve the System.* We can solve this system by substitution. Solve the first equation for C.

$$\begin{aligned} C + M &= 20000 && \text{First Equation.} \\ C &= 20000 - M && \text{Subtract } M \text{ from both sides.} \end{aligned}$$

   Next, substitute $20000 - M$ for C in the second equation and solve for M.

$$\begin{aligned} 0.03C + 0.05M &= 780 && \text{Second Equation.} \\ 0.03(20000 - M) + 0.05M &= 780 && \text{Substitute } 20000 - M \text{ for } C. \\ 600 - 0.03M + 0.05M &= 780 && \text{Distribute the 0.03.} \\ 600 + 0.02M &= 780 && \text{Combine like terms.} \\ 0.02M &= 180 && \text{Subtract 600 from both sides.} \\ M &= 9000 && \text{Divide both sides by 0.02.} \end{aligned}$$

Thus, the amount invested in the mutual fund is $M = \$9,000$.

4. *Answer the Question.* The question asks us to find the amount invested in each account. So, substitute 9000 for M in the first equation and solve for C.

$$\begin{aligned} C + M &= 20000 && \text{First Equation.} \\ C + 9000 &= 20000 && \text{Substitute 9000 for } M. \\ C &= 11000 && \text{Subtract 9000 from both sides.} \end{aligned}$$

Thus, the amount invested in the certificate of deposit is $\$11,000$.

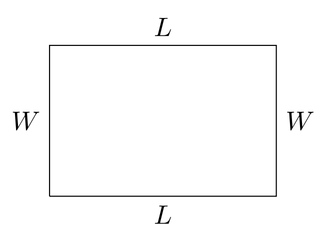
5. *Look Back.* First, note that the investments in the certificate of deposit and the mutual fund, \$11000 and \$9000 respectively, total \$20,000. Let's calculate the interest on each investment: 3% of \$11000 is \$330 and 5% of \$9000 is \$450.

	Rate	Amount invested	Interest
Certificate of Deposit	3%	11,000	330
Mutual Fund	5%	9,000	450
Totals		20,000	780

Note that the total interest is \$780, as required in the problem statement. Thus, our solution is correct.

15. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a Variable Dictionary.* Our variable dictionary will take the form of a diagram, naming the width and length W and L, respectively.



2. *Set up a System of Equations.* The perimeter is found by summing the four sides of the rectangle.

$$\begin{aligned} P &= L + W + L + W \\ P &= 2L + 2W \end{aligned}$$

We're told the perimeter is 376 centimeters, so we can substitute 376 for P in the last equation.

$$376 = 2L + 2W$$

We can simplify this equation by dividing both sides by 2, giving the following result:

$$L + W = 188$$

Secondly, we're told that the "length is 12 centimeters less than three times the width." This translates to:

$$L = 3W - 12$$

3. *Solve the System.* As the last equation is already solved for L, let use the substitution method and substitute $3W - 12$ for L in the equation $L + W = 188$.

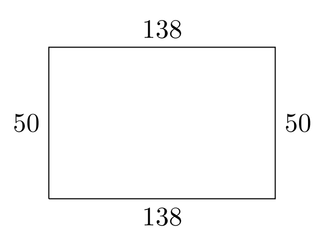
$$\begin{aligned}
L + W &= 188 && \text{Perimeter equation.} \\
(3W - 12) + W &= 188 && \text{Substitute } 3W - 12 \text{ for } L. \\
4W - 12 &= 188 && \text{Combine like terms.} \\
4W &= 200 && \text{Add 12 to both sides.} \\
W &= 50 && \text{Divide both sides by 4.}
\end{aligned}$$

4. *Answer the Question.* The width is $W = 50$ centimeters. To find the length, substitute 50 for W in the equation $L = 3W - 12$.

$$\begin{aligned}
L &= 3W - 12 && \text{Length equation.} \\
L &= 3(50) - 12 && \text{Substitute 50 for } W. \\
L &= 150 - 12 && \text{Multiply.} \\
L &= 138 && \text{Subtract.}
\end{aligned}$$

Thus, the length is $L = 138$ centimeters.

5. *Look Back.* Perhaps a picture, labeled with our answers might best demonstrate that we have the correct solution. Remember, we found that the width was 50 centimeters and the length was 138 centimeters.



Note that the perimeter is $P = 138 + 50 + 138 + 50 = 376$ centimeters. Secondly, note that the length (138 centimeters) is 12 centimeters less than three times the width. So we have the correct solution.

Chapter 5

Polynomials

5.1 Functions

1. Consider again the relation R.

$$R = \{(7,4),\ (2,4),\ (4,2),\ (8,5)\}$$

To form the domain, we take the first element of each ordered pair and put it into a set.

$$\{7,2,4,8\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Domain} = \{2,4,7,8\}$$

To find the range, we take the second element of each ordered pair and put it in a set.

$$\{4,4,2,5\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Range} = \{2,4,5\}$$

3. Consider again the relation T.

$$T = \{(7,2),\ (3,1),\ (9,4),\ (8,1)\}$$

To form the domain, we take the first element of each ordered pair and put it into a set.

$$\{7,3,9,8\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Domain} = \{3, 7, 8, 9\}$$

To find the range, we take the second element of each ordered pair and put it in a set.

$$\{2, 1, 4, 1\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Range} = \{1, 2, 4\}$$

5. Consider again the relation T.

$$T = \{(4, 7),\ (4, 8),\ (5, 0),\ (0, 7)\}$$

To form the domain, we take the first element of each ordered pair and put it into a set.

$$\{4, 4, 5, 0\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Domain} = \{0, 4, 5\}$$

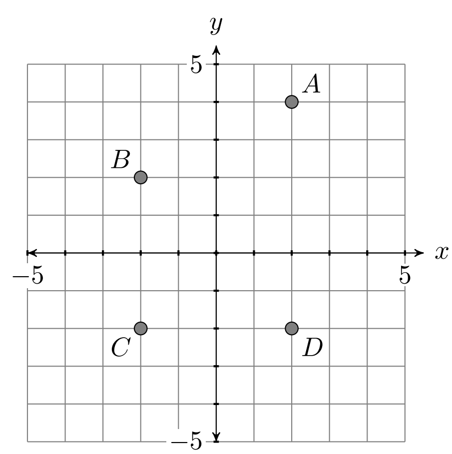
To find the range, we take the second element of each ordered pair and put it in a set.

$$\{7, 8, 0, 7\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Range} = \{0, 7, 8\}$$

7. Consider again the relation given graphically.



Note the coordinates of each point: $A = (2, 4)$, $B = (-2, 2)$, $C = (-2, -2)$, and $D = (2, -2)$. To form the domain, we take the first element of each ordered pair and put it into a set.

$$\{2, -2, -2, 2\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Domain} = \{-2, 2\}$$

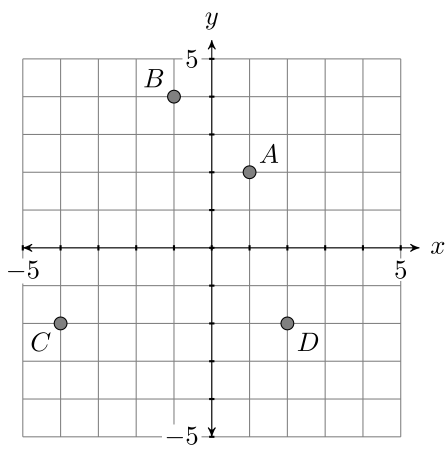
To find the range, we take the second element of each ordered pair and put it in a set.

$$\{4, 2, -2, -2\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Range} = \{-2, 2, 4\}$$

9. Consider again the relation given graphically.



Note the coordinates of each point: $A = (1, 2)$, $B = (-1, 4)$, $C = (-4, -2)$, and $D = (2, -2)$. To form the domain, we take the first element of each ordered pair and put it into a set.

$$\{1, -1, -4, 2\}$$

However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Domain} = \{-4, -1, 1, 2\}$$

To find the range, we take the second element of each ordered pair and put it in a set.

$$\{2, 4, -2, -2\}$$

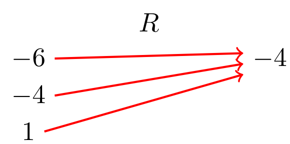
However, in listing the final answer, we should eliminate duplicate elements and then sort the numbers in numerical order, from smallest to largest.

$$\text{Range} = \{-2, 2, 4\}$$

11. Consider again the relation.

$$R = \{(-6, -4), (-4, -4), (1, -4)\}$$

List the elements of the domain on the left, the elements of the range on the right, then use arrows to indicate the connection between the first and second elements of each ordered pair.

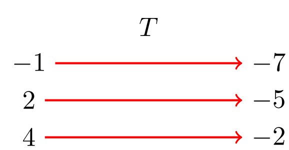


Note that each domain element is paired with exactly one range element. Hence, R is a function.

13. Consider again the relation.

$$T = \{(-1, -7), (2, -5), (4, -2)\}$$

List the elements of the domain on the left, the elements of the range on the right, then use arrows to indicate the connection between the first and second elements of each ordered pair.

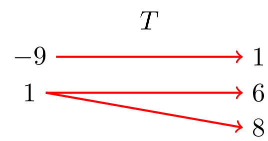


Note that each domain element is paired with exactly one range element. Hence, T is a function.

15. Consider again the relation.

$$T = \{(-9, 1), (1, 6), (1, 8)\}$$

List the elements of the domain on the left, the elements of the range on the right, then use arrows to indicate the connection between the first and second elements of each ordered pair.

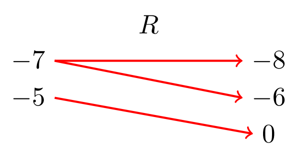


Note that the domain element 1 is paired with two range elements, 1 and 6. Hence, the relation T is **not** a function.

17. Consider again the relation.

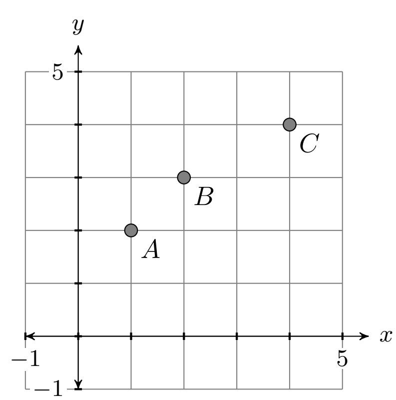
$$R = \{(-7, -8), (-7, -6), (-5, 0)\}$$

List the elements of the domain on the left, the elements of the range on the right, then use arrows to indicate the connection between the first and second elements of each ordered pair.



Note that the domain element -7 is paired with two range elements, -8 and -6. Hence, the relation R is **not** a function.

19. Consider again the relation.

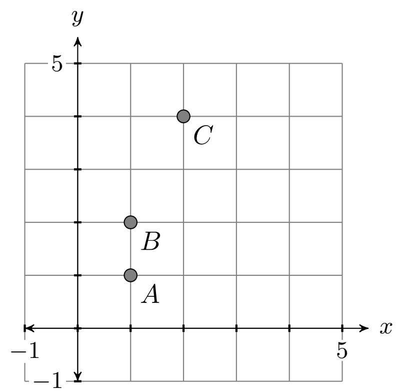


Create a mapping diagram for the points $A = (1, 2)$, $B = (2, 3)$, and $C = (4, 4)$. List the elements of the domain on the left, the elements of the range on the right, then use arrows to indicate the connection between the first and second elements of each ordered pair.

$$\begin{array}{lcl} 1 & \longrightarrow & 2 \\ 2 & \longrightarrow & 3 \\ 4 & \longrightarrow & 4 \end{array}$$

Note that each domain element is paired with exactly one range element. Hence, the relation is a function.

21. Consider again the relation.



Create a mapping diagram for the points $A = (1, 1)$, $B = (1, 2)$, and $C = (2, 4)$. List the elements of the domain on the left, the elements of the range on the right, then use arrows to indicate the connection between the first and second elements of each ordered pair.

$$\begin{array}{lcl} 1 & \longrightarrow & 1 \\ 1 & \longrightarrow & 2 \\ 2 & \longrightarrow & 4 \end{array}$$

Note that the domain element 1 is paired with two range elements, 1 and 2. Hence, the relation is **not** a function.

23. Given $f(x) = |6x - 9|$, to evaluate $f(8)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = \lvert 6x - 9 \rvert$	Original function notation.
$f(\ \ ) = \lvert 6(\ \ ) - 9 \rvert$	Replace each occurrence of x with open parentheses.

Now substitute 8 for x in the open parentheses prepared in the last step.

$f(\mathbf{8}) = \lvert 6(\mathbf{8}) - 9 \rvert$	Substitute 8 for x in the open parentheses positions.
$f(8) = \lvert 48 - 9 \rvert$	Multiply: $6(8) = 48$.
$f(8) = \lvert 39 \rvert$	Simplify.
$f(8) = 39$	Take absolute value.

Hence, $f(8) = 39$; i.e., f sends 8 to 39.

25. Given $f(x) = -2x^2+8$, to evaluate $f(3)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = -2x^2 + 8$	Original function notation.
$f(\ \ ) = -2(\ \ )^2 + 8$	Replace each occurrence of x with open parentheses.

Now substitute 3 for x in the open parentheses prepared in the last step.

$f(\mathbf{3}) = -2(\mathbf{3})^2 + 8$	Substitute 3 for x in the open parentheses positions.
$f(3) = -2(9) + 8$	Exponent first: $(3)^2 = 9$
$f(3) = -18 + 8$	Multiply: $-2(9) = -18$ and $0(3) = 0$
$f(3) = -10$	Simplify.

Hence, $f(3) = -10$; i.e., f sends 3 to -10.

27. Given $f(x) = -3x^2 + 4x + 1$, to evaluate $f(2)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = -3x^2 + 4x + 1$	Original function notation.
$f(\ \ ) = -3(\ \ )^2 + 4(\ \ ) + 1$	Replace each occurrence of x with open parentheses.

Now substitute 2 for x in the open parentheses prepared in the last step.

$f(\mathbf{2}) = -3(\mathbf{2})^2 + 4(\mathbf{2}) + 1$	Substitute 2 for x in the open parentheses positions.
$f(2) = -3(4) + 4(2) + 1$	Exponent first: $(2)^2 = 4$
$f(2) = -12 + 8 + 1$	Multiply: $-3(4) = -12$ and $4(2) = 8$
$f(2) = -3$	Simplify.

Hence, $f(2) = -3$; i.e., f sends 2 to -3.

29. Given $f(x) = |5x+9|$, to evaluate $f(-8)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$$f(x) = |5x + 9| \quad \text{Original function notation.}$$
$$f(\ ) = |5(\ ) + 9| \quad \text{Replace each occurrence of } x \text{ with open parentheses.}$$

Now substitute -8 for x in the open parentheses prepared in the last step.

$$f(\mathbf{-8}) = |5(\mathbf{-8}) + 9| \quad \text{Substitute } -8 \text{ for } x \text{ in the open parentheses positions.}$$
$$f(-8) = |-40 + 9| \quad \text{Multiply: } 5(-8) = -40.$$
$$f(-8) = |-31| \quad \text{Simplify.}$$
$$f(-8) = 31 \quad \text{Take absolute value.}$$

Hence, $f(-8) = 31$; i.e., f sends -8 to 31.

31. Given $f(x) = \sqrt{x-6}$, to evaluate $f(42)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$$f(x) = \sqrt{x-6} \quad \text{Original function notation.}$$
$$f(\ ) = \sqrt{(\ ) - 6} \quad \text{Replace each occurrence of } x \text{ with open parentheses.}$$

Now substitute 42 for x in the open parentheses prepared in the last step.

$$f(\mathbf{42}) = \sqrt{(\mathbf{42}) - 6} \quad \text{Substitute 42 for } x \text{ in the open parentheses positions.}$$
$$f(42) = \sqrt{36} \quad \text{Simplify.}$$
$$f(42) = 6 \quad \text{Take square root: } \sqrt{36} = 6$$

Hence, $f(42) = 6$; i.e., f sends 42 to 6.

33. Given $f(x) = \sqrt{x-7}$, to evaluate $f(88)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$$f(x) = \sqrt{x-7} \quad \text{Original function notation.}$$
$$f(\ ) = \sqrt{(\ ) - 7} \quad \text{Replace each occurrence of } x \text{ with open parentheses.}$$

Now substitute 88 for x in the open parentheses prepared in the last step.

$$f(\mathbf{88}) = \sqrt{(\mathbf{88}) - 7} \quad \text{Substitute 88 for } x \text{ in the open parentheses positions.}$$
$$f(88) = \sqrt{81} \quad \text{Simplify.}$$
$$f(88) = 9 \quad \text{Take square root: } \sqrt{81} = 9$$

Hence, $f(88) = 9$; i.e., f sends 88 to 9.

35. Given $f(x) = -4x + 6$, to evaluate $f(8)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$$
\begin{aligned}
f(x) &= -4x + 6 && \text{Original function notation.} \\
f(\ ) &= -4(\ ) + 6 && \text{Replace each occurrence of } x \text{ with open parentheses.}
\end{aligned}
$$

Now substitute 8 for x in the open parentheses prepared in the last step.

$$
\begin{aligned}
f(\mathbf{8}) &= -4(\mathbf{8}) + 6 && \text{Substitute 8 for } x \text{ in the open parentheses positions.} \\
f(8) &= -32 + 6 && \text{Multiply: } -4(8) = -32 \\
f(8) &= -26 && \text{Add: } -32 + 6 = -26
\end{aligned}
$$

Hence, $f(8) = -26$; i.e., f sends 8 to -26.

37. Given $f(x) = -6x + 7$, to evaluate $f(8)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$$
\begin{aligned}
f(x) &= -6x + 7 && \text{Original function notation.} \\
f(\ ) &= -6(\ ) + 7 && \text{Replace each occurrence of } x \text{ with open parentheses.}
\end{aligned}
$$

Now substitute 8 for x in the open parentheses prepared in the last step.

$$
\begin{aligned}
f(\mathbf{8}) &= -6(\mathbf{8}) + 7 && \text{Substitute 8 for } x \text{ in the open parentheses positions.} \\
f(8) &= -48 + 7 && \text{Multiply: } -6(8) = -48 \\
f(8) &= -41 && \text{Add: } -48 + 7 = -41
\end{aligned}
$$

Hence, $f(8) = -41$; i.e., f sends 8 to -41.

39. Given $f(x) = -2x^2 + 3x + 2$ and $g(x) = 3x^2 + 5x - 5$, to evaluate $f(3)$, first choose the function f, then replace each occurrence of the variable with open parentheses.

$$
\begin{aligned}
f(x) &= -2x^2 + 3x + 2 && \text{Original function notation.} \\
f(\ ) &= -2(\ )^2 + 3(\ ) + 2 && \text{Replace each occurrence of } x \text{ with open parentheses.}
\end{aligned}
$$

Now substitute 3 for x in the open parentheses prepared in the last step.

$f(\mathbf{3}) = -2(\mathbf{3})^2 + 3(\mathbf{3}) + 2$	Substitute 3 for x in the open parentheses positions.
$f(3) = -2(9) + 3(3) + 2$	Exponent first: $(3)^2 = 9$
$f(3) = -18 + 9 + 2$	Multiply: $-2(9) = -18$ and $3(3) = 9$
$f(3) = -7$	Simplify.

Hence, $f(3) = -7$; i.e., f sends 3 to -7. To evaluate $g(3)$, repeat the same procedure, this time using the function g.

$g(x) = 3x^2 + 5x - 5$	Original function notation.
$g(\ \ ) = 3(\ \ )^2 + 5(\ \ ) - 5$	Replace each occurrence of x with open parentheses.

Now substitute 3 for x in the open parentheses prepared in the last step.

$g(\mathbf{3}) = 3(\mathbf{3})^2 + 5(\mathbf{3}) - 5$	Substitute 3 for x in the open parentheses positions.
$g(3) = 3(9) + 5(3) - 5$	Exponent first: $(3)^2 = 9$
$g(3) = 27 + 15 - 5$	Multiply: $3(9) = 27$ and $5(3) = 15$
$g(3) = 37$	Simplify.

Hence, $g(3) = 37$; i.e., g sends 3 to 37.

41. Given $f(x) = 6x - 2$ and $g(x) = -8x + 9$, to evaluate $f(-7)$, first choose the function $f(x) = 6x - 2$, then replace each occurrence of the variable with open parentheses.

$f(x) = 6x - 2$	Original function notation.
$f(\ \ ) = 6(\ \ ) - 2$	Replace each occurrence of x with open parentheses.

Now substitute -7 for x in the open parentheses prepared in the last step.

$f(\mathbf{-7}) = 6(\mathbf{-7}) - 2$	Substitute -7 for x in the open parentheses positions.
$f(-7) = -42 - 2$	Multiply: $6(-7) = -42$
$f(-7) = -44$	Simplify.

Hence, $f(-7) = -44$; i.e., f sends -7 to -44. Now, repeat the procedure, using the function g.

$g(x) = -8x + 9$	Original function notation.
$g(\ \ ) = -8(\ \ ) + 9$	Replace each occurrence of x with open parentheses.

Now substitute -7 for x in the open parentheses prepared in the last step.

$g(\mathbf{-7}) = -8(\mathbf{-7}) + 9$	Substitute -7 for x in the open parentheses positions.
$g(-7) = 56 + 9$	Multiply: $-8(-7) = 56$
$g(-7) = 65$	Simplify.

Hence, $g(-7) = 65$; i.e., g sends -7 to 65.

43. Given $f(x) = 4x-3$ and $g(x) = -3x+8$, to evaluate $f(-3)$, first choose the function f, then replace each occurrence of the variable with open parentheses.

$f(x) = 4x - 3$	Original function notation.
$f(\ \ ) = 4(\ \ ) - 3$	Replace each occurrence of x with open parentheses.

Now substitute -3 for x in the open parentheses prepared in the last step.

$f(\mathbf{-3}) = 4(\mathbf{-3}) - 3$	Substitute -3 for x in the open parentheses positions.
$f(-3) = -12 - 3$	Multiply: $4(-3) = -12$
$f(-3) = -15$	Simplify.

Hence, $f(-3) = -15$; i.e., f sends -3 to -15. To evaluate $g(-3)$, repeat the same procedure, this time using the function g.

$g(x) = -3x + 8$	Original function notation.
$g(\ \ ) = -3(\ \ ) + 8$	Replace each occurrence of x with open parentheses.

Now substitute -3 for x in the open parentheses prepared in the last step.

$g(\mathbf{-3}) = -3(\mathbf{-3}) + 8$	Substitute -3 for x in the open parentheses positions.
$g(-3) = 9 + 8$	Multiply: $-3(-3) = 9$
$g(-3) = 17$	Simplify.

Hence, $g(-3) = 17$; i.e., g sends -3 to 17.

45. Given $f(x) = -2x^2 + 5x - 9$ and $g(x) = -2x^2 + 3x - 4$, to evaluate $g(-2)$, first choose the function g, then replace each occurrence of the variable with open parentheses.

$g(x) = -2x^2 + 3x - 4$	Original function notation.
$g(\ \ ) = -2(\ \ )^2 + 3(\ \ ) - 4$	Replace each occurrence of x with open parentheses.

Now substitute -2 for x in the open parentheses prepared in the last step.

$g(\mathbf{-2}) = -2(\mathbf{-2})^2 + 3(\mathbf{-2}) - 4$	Substitute -2 for x in the open parentheses positions.
$g(-2) = -2(4) + 3(-2) - 4$	Exponent first: $(-2)^2 = 4$
$g(-2) = -8 - 6 - 4$	Multiply: $-2(4) = -8$ and $3(-2) = -6$
$g(-2) = -18$	Simplify.

Hence, $g(-2) = -18$; i.e., g sends -2 to -18. To find $f(-2)$, repeat the procedure, this time using f.

$f(x) = -2x^2 + 5x - 9$	Original function notation.
$f(\ \ ) = -2(\ \ )^2 + 5(\ \ ) - 9$	Replace each occurrence of x with open parentheses.

Now substitute -2 for x in the open parentheses prepared in the last step.

$f(\mathbf{-2}) = -2(\mathbf{-2})^2 + 5(\mathbf{-2}) - 9$	Substitute -2 for x in the open parentheses positions.
$f(-2) = -2(4) + 5(-2) - 9$	Exponent first: $(-2)^2 = 4$
$f(-2) = -8 - 10 - 9$	Multiply: $-2(4) = -8$ and $5(-2) = -10$
$f(-2) = -27$	Simplify.

Hence, $f(-2) = -27$; i.e., f sends -2 to -27.

5.2 Polynomials

1. When a term is a product of a number and one or more variables, the number in front of the variables is called the coefficient of the term. Consequently, the coefficient of the term $3v^5u^6$ is 3.

The degree of a term is the sum of the exponents on each variable of the term. Consequently, the degree of the term $3v^5u^6$ is:

$$\begin{aligned}\text{Degree} &= 5 + 6\\ &= 11\end{aligned}$$

3. When a term is a product of a number and one or more variables, the number in front of the variables is called the coefficient of the term. Consequently, the coefficient of the term $-5v^6$ is -5.

The degree of a term is the sum of the exponents on each variable of the term. Consequently, the degree of the term $-5v^6$ is 6.

5. When a term is a product of a number and one or more variables, the number in front of the variables is called the coefficient of the term. Consequently, the coefficient of the term $2u^7x^4d^5$ is 2.

The degree of a term is the sum of the exponents on each variable of the term. Consequently, the degree of the term $2u^7x^4d^5$ is:

$$\begin{aligned} \text{Degree} &= 7 + 4 + 5 \\ &= 16 \end{aligned}$$

7. The terms in an expression are separated by plus or minus signs. There is only one term in the expression, so $-7b^9c^3$ is a monomial.

9. The terms in an expression are separated by plus or minus signs. There are exactly two terms in the expression, so $4u + 7v$ is a binomial.

11. The terms in an expression are separated by plus or minus signs. There are exactly three terms in the expression, so $3b^4 - 9bc + 9c^2$ is a trinomial.

13. The terms in an expression are separated by plus or minus signs. There are exactly two terms in the expression, so $5s^2 + 9t^7$ is a binomial.

15. The terms in an expression are separated by plus or minus signs. There are exactly three terms in the expression, so $2u^3 - 5uv - 4v^4$ is a trinomial.

17. To arrange $-2x^7 - 9x^{13} - 6x^{12} - 7x^{17}$ in descending powers of x, we must begin with the term with the largest exponent, then the next largest exponent, etc. Thus, arranging in descending powers, we arrive at:

$$-7x^{17} - 9x^{13} - 6x^{12} - 2x^7$$

19. To arrange $8x^6 + 2x^{15} - 3x^{11} - 2x^2$ in descending powers of x, we must begin with the term with the largest exponent, then the next largest exponent, etc. Thus, arranging in descending powers, we arrive at:

$$2x^{15} - 3x^{11} + 8x^6 - 2x^2$$

21. To arrange $7x^{17}+3x^4-2x^{12}+8x^{14}$ in ascending powers of x, we must begin with the term with the smallest exponent, then the next smallest exponent, etc. Thus, arranging in ascending powers, we arrive at:

$$3x^4 - 2x^{12} + 8x^{14} + 7x^{17}$$

23. To arrange $2x^{13}+3x^{18}+8x^7+5x^4$ in ascending powers of x, we must begin with the term with the smallest exponent, then the next smallest exponent, etc. Thus, arranging in ascending powers, we arrive at:

$$5x^4 + 8x^7 + 2x^{13} + 3x^{18}$$

25. In order to arrange our answer in descending powers of x, we want to place the term with the highest power of x first and the term with the lowest power of x last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&-5x + 3 - 6x^3 + 5x^2 - 9x + 3 - 3x^2 + 6x^3 \\
&\quad = (-6x^3 + 6x^3) + (5x^2 - 3x^2) + (-5x - 9x) + (3 + 3) \\
&\quad = 2x^2 - 14x + 6
\end{aligned}$$

27. In order to arrange our answer in descending powers of x, we want to place the term with the highest power of x first and the term with the lowest power of x last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&4x^3 + 6x^2 - 8x + 1 + 8x^3 - 7x^2 + 5x - 8 \\
&\quad = (4x^3 + 8x^3) + (6x^2 - 7x^2) + (-8x + 5x) + (1 - 8) \\
&\quad = 12x^3 - x^2 - 3x - 7
\end{aligned}$$

29. In order to arrange our answer in descending powers of x, we want to place the term with the highest power of x first and the term with the lowest power of x last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&x^2 + 9x - 3 + 7x^2 - 3x - 8 \\
&\quad = (x^2 + 7x^2) + (9x - 3x) + (-3 - 8) \\
&\quad = 8x^2 + 6x - 11
\end{aligned}$$

31. In order to arrange our answer in descending powers of x, we want to place the term with the highest power of x first and the term with the lowest power of x last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&8x + 7 + 2x^2 - 8x - 3x^3 - x^2 \\
&\quad = (-3x^3) + (2x^2 - x^2) + (8x - 8x) + (7) \\
&\quad = -3x^3 + x^2 + 7
\end{aligned}$$

33. We'll arrange our answer in descending powers of x, so we place the term with the highest power of x first and the term with the lowest power of x last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&-8x^2 - 4xz - 2z^2 - 3x^2 - 8xz + 2z^2 \\
&\quad = (-8x^2 - 3x^2) + (-4xz - 8xz) + (-2z^2 + 2z^2) \\
&\quad = -11x^2 - 12xz
\end{aligned}$$

35. We'll arrange our answer in descending powers of u, so we place the term with the highest power of u first and the term with the lowest power of u last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&-6u^3 + 4uv^2 - 2v^3 - u^3 + 6u^2v - 5uv^2 \\
&\quad = (-6u^3 - u^3) + (6u^2v) + (4uv^2 - 5uv^2) + (-2v^3) \\
&\quad = -7u^3 + 6u^2v - uv^2 - 2v^3
\end{aligned}$$

37. We'll arrange our answer in descending powers of b, so we place the term with the highest power of b first and the term with the lowest power of b last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&-4b^2c - 3bc^2 - 5c^3 + 9b^3 - 3b^2c + 5bc^2 \\
&\quad = (9b^3) + (-4b^2c - 3b^2c) + (-3bc^2 + 5bc^2) + (-5c^3) \\
&\quad = 9b^3 - 7b^2c + 2bc^2 - 5c^3
\end{aligned}$$

39. We'll arrange our answer in descending powers of y, so we place the term with the highest power of y first and the term with the lowest power of y last.

We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&-8y^2 + 6yz - 7z^2 - 2y^2 - 3yz - 9z^2 \\
&\qquad = (-8y^2 - 2y^2) + (6yz - 3yz) + (-7z^2 - 9z^2) \\
&\qquad = -10y^2 + 3yz - 16z^2
\end{aligned}$$

41. We'll arrange our answer in descending powers of b, so we place the term with the highest power of b first and the term with the lowest power of b last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&7b^2c + 8bc^2 - 6c^3 - 4b^3 + 9bc^2 - 6c^3 \\
&\qquad = (-4b^3) + (7b^2c) + (8bc^2 + 9bc^2) + (-6c^3 - 6c^3) \\
&\qquad = -4b^3 + 7b^2c + 17bc^2 - 12c^3
\end{aligned}$$

43. We'll arrange our answer in descending powers of a, so we place the term with the highest power of a first and the term with the lowest power of a last. We use the commutative and associative properties to change the order and regroup, then we combine like terms.

$$\begin{aligned}
&9a^2 + ac - 9c^2 - 5a^2 - 2ac + 2c^2 \\
&\qquad = (9a^2 - 5a^2) + (ac - 2ac) + (-9c^2 + 2c^2) \\
&\qquad = 4a^2 - ac - 7c^2
\end{aligned}$$

45. To help determine the degree, take the polynomial

$$3x^{15} + 4 + 8x^3 - 8x^{19}$$

and arrange it in descending powers of x:

$$-8x^{19} + 3x^{15} + 8x^3 + 4$$

Thus, its now easy to see that the term $-8x^{19}$ is the term with the highest degree. Hence, the degree of $-8x^{19} + 3x^{15} + 8x^3 + 4$ is 19.

47. To help determine the degree, take the polynomial

$$7x^{10} - 3x^{18} + 9x^4 - 6$$

and arrange it in descending powers of x:

$$-3x^{18} + 7x^{10} + 9x^4 - 6$$

Thus, its now easy to see that the term $-3x^{18}$ is the term with the highest degree. Hence, the degree of $-3x^{18} + 7x^{10} + 9x^4 - 6$ is 18.

49. To help determine the degree, take the polynomial

$$-2 - x^7 - 5x^5 + x^{10}$$

and arrange it in descending powers of x:

$$x^{10} - x^7 - 5x^5 - 2$$

Thus, its now easy to see that the term x^{10} is the term with the highest degree. Hence, the degree of $x^{10} - x^7 - 5x^5 - 2$ is 10.

51. Given $f(x) = 5x^3 + 4x^2 - 6$, to evaluate $f(-1)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = 5x^3 + 4x^2 - 6$	Original function notation.
$f(\ ) = 5(\ )^3 + 4(\ )^2 - 6$	Replace each occurrence of x with open parentheses.

Now substitute -1 for x in the open parentheses prepared in the last step.

$f(\mathbf{-1}) = 5(\mathbf{-1})^3 + 4(\mathbf{-1})^2 - 6$	Substitute -1 for x in the open parentheses positions.
$f(-1) = 5(-1) + 4(1) - 6$	Exponents first: $(-1)^3 = -1$ and $(-1)^2 = 1$
$f(-1) = -5 + 4 - 6$	Multiply: $5(-1) = -5$, and $4(1) = 4$
$f(-1) = -7$	Simplify.

Hence, $f(-1) = -7$; i.e., f sends -1 to -7.

53. Given $f(x) = 5x^4 - 4x - 6$, to evaluate $f(-2)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = 5x^4 - 4x - 6$	Original function notation.
$f(\ ) = 5(\ )^4 - 4(\ ) - 6$	Replace each occurrence of x with open parentheses.

Now substitute -2 for x in the open parentheses prepared in the last step.

$f(\mathbf{-2}) = 5(\mathbf{-2})^4 - 4(\mathbf{-2}) - 6$	Substitute -2 for x in the open parentheses positions.
$f(-2) = 5(16) - 4(-2) - 6$	Exponents first: $(-2)^4 = 16$
$f(-2) = 80 + 8 - 6$	Multiply: $5(16) = 80$, and $-4(-2) = 8$
$f(-2) = 82$	Simplify.

Hence, $f(-2) = 82$; i.e., f sends -2 to 82.

55. Given $f(x) = 3x^4 + 5x^3 - 9$, to evaluate $f(-2)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = 3x^4 + 5x^3 - 9$	Original function notation.
$f(\ ) = 3(\ )^4 + 5(\ )^3 - 9$	Replace each occurrence of x with open parentheses.

Now substitute -2 for x in the open parentheses prepared in the last step.

$f(\mathbf{-2}) = 3(\mathbf{-2})^4 + 5(\mathbf{-2})^3 - 9$	Substitute -2 for x in the open parentheses positions.
$f(-2) = 3(16) + 5(-8) - 9$	Exponents first: $(-2)^4 = 16$ and $(-2)^3 = -8$
$f(-2) = 48 - 40 - 9$	Multiply: $3(16) = 48$, and $5(-8) = -40$
$f(-2) = -1$	Simplify.

Hence, $f(-2) = -1$; i.e., f sends -2 to -1.

57. Given $f(x) = 3x^4 - 5x^2 + 8$, to evaluate $f(-1)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

$f(x) = 3x^4 - 5x^2 + 8$	Original function notation.
$f(\ ) = 3(\ )^4 - 5(\ )^2 + 8$	Replace each occurrence of x with open parentheses.

Now substitute -1 for x in the open parentheses prepared in the last step.

$f(\mathbf{-1}) = 3(\mathbf{-1})^4 - 5(\mathbf{-1})^2 + 8$	Substitute -1 for x in the open parentheses positions.
$f(-1) = 3(1) - 5(1) + 8$	Exponents first: $(-1)^4 = 1$ and $(-1)^2 = 1$
$f(-1) = 3 - 5 + 8$	Multiply: $3(1) = 3$, and $-5(1) = -5$
$f(-1) = 6$	Simplify.

Hence, $f(-1) = 6$; i.e., f sends -1 to 6.

59. Given $f(x) = -2x^3 + 4x - 9$, to evaluate $f(2)$, first restate the function notation, then replace each occurrence of the variable with open parentheses.

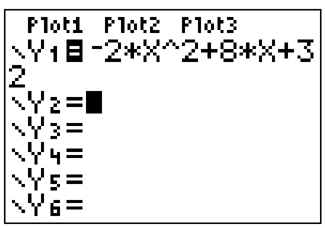
$f(x) = -2x^3 + 4x - 9$	Original function notation.
$f(\ ) = -2(\ )^3 + 4(\ ) - 9$	Replace each occurrence of x with open parentheses.

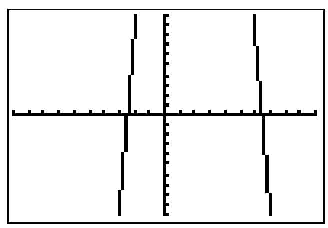
Now substitute 2 for x in the open parentheses prepared in the last step.

$f(\mathbf{2}) = -2(\mathbf{2})^3 + 4(\mathbf{2}) - 9$	Substitute 2 for x in the open parentheses positions.
$f(2) = -2(8) + 4(2) - 9$	Exponents first: $(2)^3 = 8$
$f(2) = -16 + 8 - 9$	Multiply: $-2(8) = -16$, and $4(2) = 8$
$f(2) = -17$	Simplify.

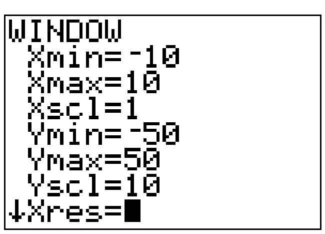
Hence, $f(2) = -17$; i.e., f sends 2 to -17.

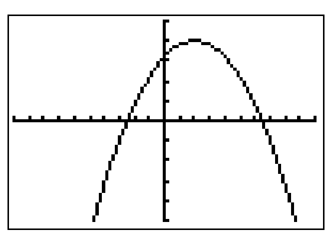
61. Enter the equation $p(x) = -2x^2 + 8x + 32$ into **Y1** in the Y= menu, then select **6:ZStandard** to produce the following image.



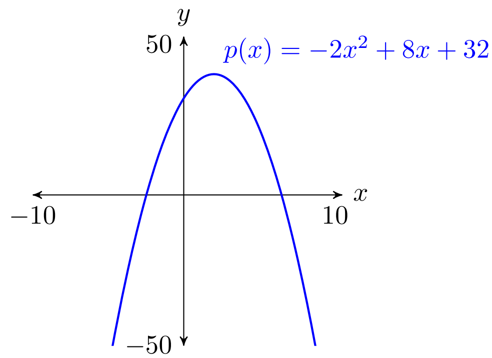


Because the coefficient of the leading term is -2, the parabola opens downward. Hence, the vertex must lie off the top of the screen. After some experimentation, we settled on the following WINDOW parameters, then pushed the GRAPH button to produce the accompanying graph.

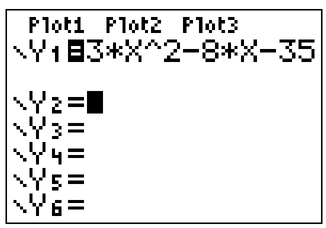


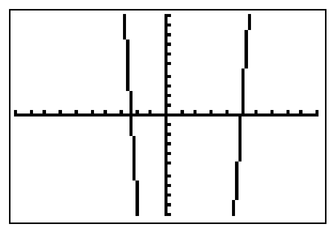


Follow the *Calculator Submission Guidelines* in reporting the answer on our homework.

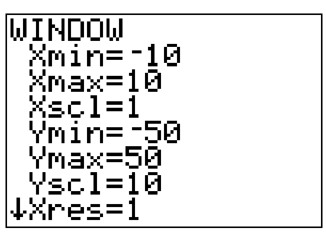


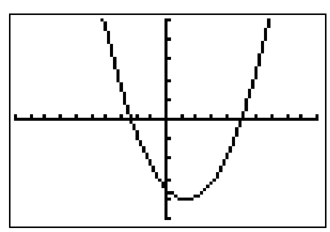
63. Enter the equation $p(x) = 3x^2 - 8x - 35$ into **Y1** in the Y= menu, then select **6:ZStandard** to produce the following image.



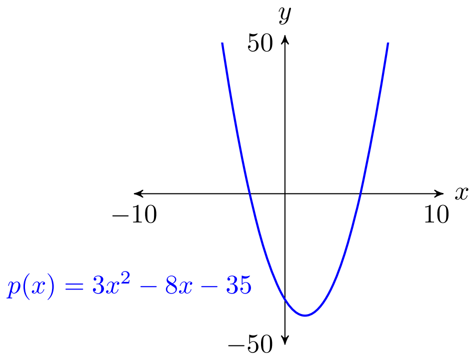


Because the coefficient of the leading term is 3, the parabola opens upward. Hence, the vertex must lie off the bottom of the screen. After some experimentation, we settled on the following WINDOW parameters, then pushed the GRAPH button to produce the accompanying graph.

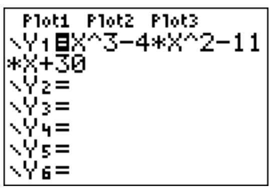


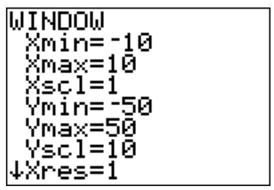


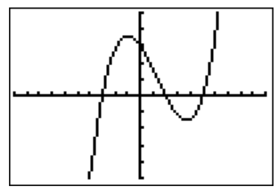
Follow the *Calculator Submission Guidelines* in reporting the answer on our homework.



65. Enter the function $p(x) = x^3 - 4x^2 - 11x + 30$ in the Y= menu (see first image below), then set the given WINDOW parameters (see second image below). Push the GRAPH button to produce the graph (see third image below).

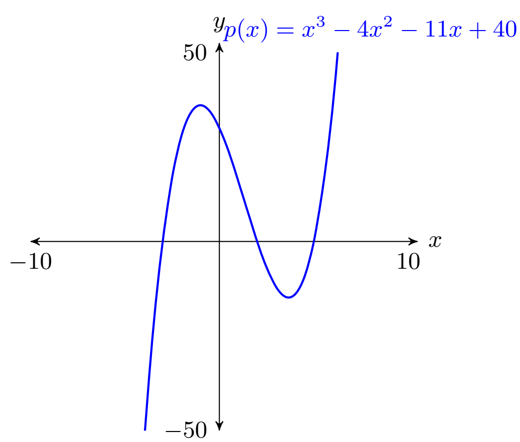




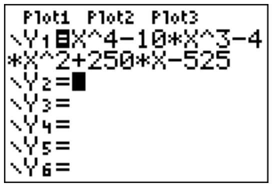


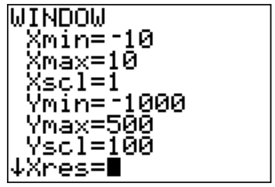
Use the *Calculator Submission Guidelines* when reporting your solution.

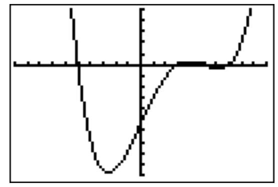
1. Draw axes with a ruler.
2. Label the horizontal axis x and the vertical axis y.
3. Indicate the WINDOW parameters **Xmin**, **Xmax**, **Ymin**, and **Ymax**, at the end of each axis.
4. Freehand the curve and label it with its equation.



67. Enter the function $p(x) = x^4 - 10x^3 + 250x - 525$ in the Y= menu (see first image below), then set the given WINDOW parameters (see second image below). Push the GRAPH button to produce the graph (see third image below).

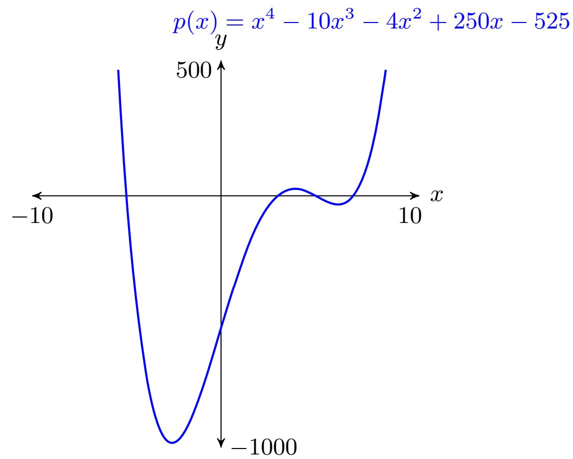






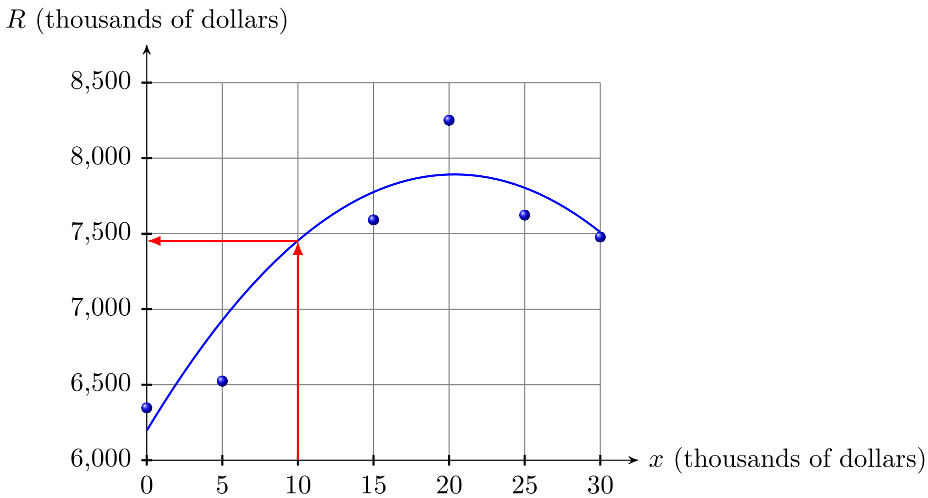
Use the *Calculator Submission Guidelines* when reporting your solution.

1. Draw axes with a ruler.
2. Label the horizontal axis x and the vertical axis y.
3. Indicate the WINDOW parameters **Xmin**, **Xmax**, **Ymin**, and **Ymax**, at the end of each axis.
4. Freehand the curve and label it with its equation.



5.3 Applications of Polynomials

1. To find the firm's revenue when it spends \$10,000 on advertising, locate 10 on the horizontal axis (the horizontal axis is measured in thousands of dollars), draw a vertical arrow to the curve, then a horizontal arrow to the vertical axis. It appears that this last arrow points at a number between 7,400 and 7,500 on the vertical axis. We'll guess 7,450. Because the vertical axis is measured in thousands of dollars, the firm's revenue is approximately \$7,450,000.



To use the polynomial to find the firm's revenue, substitute 10 for x.

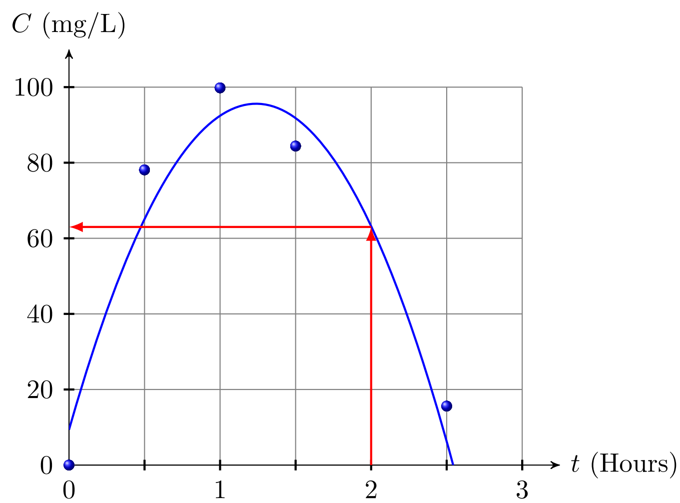
$$R(x) = -4.1x^2 + 166.8x + 6196$$
$$R(10) = -4.1(10)^2 + 166.8(10) + 6196$$

Using a calculator, we estimate:

$$R(1) \approx 7454$$

Hence, when the firm invests \$10,000 in advertising, their revenue is approximately \$7,454,000.

3. To find the concentration of medication in the patient's blood after 2 hours, locate the number 2 on the horizontal axis, draw a vertical arrow to the graph, then a horizontal arrow to the vertical axis. It appears that this last arrow points at a number slightly above 60 mg/L. We'll guess 63 mg/L.



To use the polynomial to find the medication concentration after 2 hours, substitute 2 for t in the given polynomial.

$$\begin{aligned} C(t) &= -56.214t^2 + 139.31t + 9.35 \\ C(2) &= -56.214(2)^2 + 139.31(2) + 9.35 \end{aligned}$$

Using a calculator, we estimate:

$$N(3) \approx 63$$

Hence, the concentration of medication in the patient's blood after 2 hours is approximately 63 mg/L.

5. The projectile's height above ground is given by the formula

$$y = y_0 + v_0 t - \frac{1}{2} g t^2,$$

where the initial height is 75 meters, the initial velocity is $v_0 = 457$ meters per second, and the acceleration due to gravity is $g = 9.8$ meters per second per second. We're asked to find when the object first reaches a height of $y = 6592$ meters. Substituting these numbers, the formula becomes

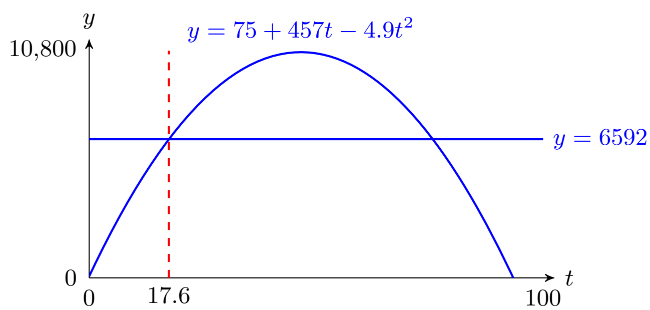
$$6592 = 75 + 457t - \frac{1}{2}(9.8)t^2,$$

or equivalently,

$$6592 = 75 + 457t - 4.9t^2.$$

Load each side of this equation into the Y= menu of your graphing calculator, then set the WINDOW parameters as follows: Xmin=0, Xmax=100, Ymin=0,

and Ymax=10800. Push the GRAPH button to produce the graph, then use the **5:intersect** tool from the CALC menu to find the first time the height of the object reaches 6592 meters. Report the results on your homework as shown in the following graph.



Hence, the projectile first reaches a height of 6592 meters at approximately 17.6 seconds.

7. The projectile's height above ground is given by the formula

$$y = y_0 + v_0 t - \frac{1}{2} g t^2,$$

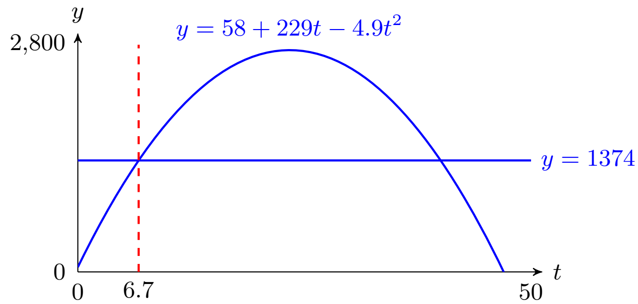
where the initial height is 58 meters, the initial velocity is $v_0 = 229$ meters per second, and the acceleration due to gravity is $g = 9.8$ meters per second per second. We're asked to find when the object first reaches a height of $y = 1374$ meters. Substituting these numbers, the formula becomes

$$1374 = 58 + 229t - \frac{1}{2}(9.8)t^2,$$

or equivalently,

$$1374 = 58 + 229t - 4.9t^2.$$

Load each side of this equation into the Y= menu of your graphing calculator, then set the WINDOW parameters as follows: Xmin=0, Xmax=50, Ymin=0, and Ymax=2800. Push the GRAPH button to produce the graph, then use the **5:intersect** tool from the CALC menu to find the first time the height of the object reaches 1374 meters. Report the results on your homework as shown in the following graph.



Hence, the projectile first reaches a height of 1374 meters at approximately 6.7 seconds.

9. To find the zero of $f(x) = 3.25x - 4.875$ algebraically, set $f(x) = 0$ and solve for x.

$$f(x) = 0 \quad \text{We want the value of } x \text{ that makes the function equal to zero.}$$

$$3.25x - 4.875 = 0 \quad \text{Replace } f(x) \text{ with } 3.25x - 4.875.$$

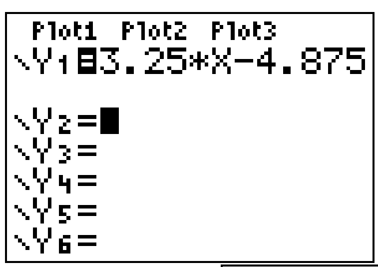
Now, solve for x.

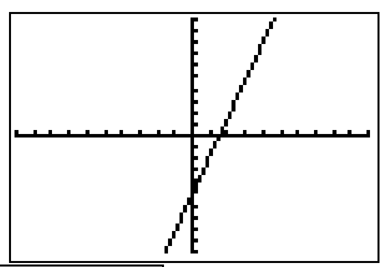
$$3.25x = 4.875 \quad \text{Add 4.875 to both sides.}$$

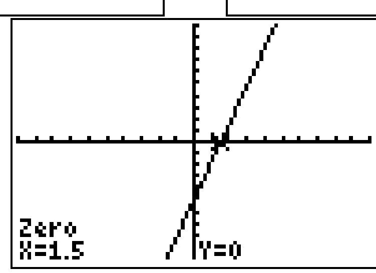
$$\frac{3.25x}{3.25} = \frac{4.875}{3.25} \quad \text{Divide both sides by 3.25.}$$

$$x = 1.5 \quad \text{Simplify.}$$

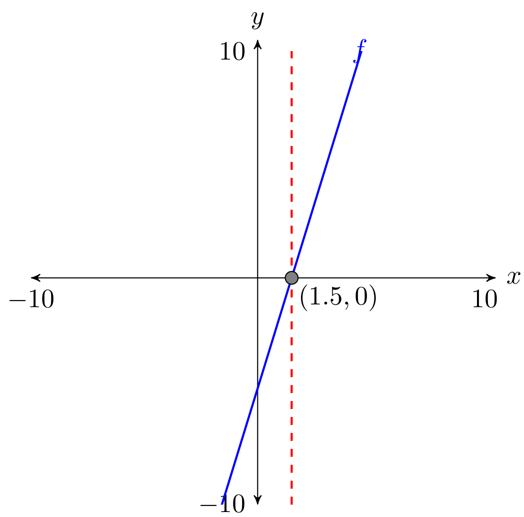
Hence, 1.5 is a zero of $f(x) = 3.25x - 4.875$. Next, load the function into the Y= menu, select **6:ZStandard** from the ZOOM menu, then use the utility **2:zero** from the CALC menu to find the zero of the function.







Next, use the *Calculator Submission Guidelines* when crafting the following report on your homework paper. Draw a dashed vertical line through the x-intercept and label it with its coordinates.



Note how the graphical solution agrees with the algebraic solution.

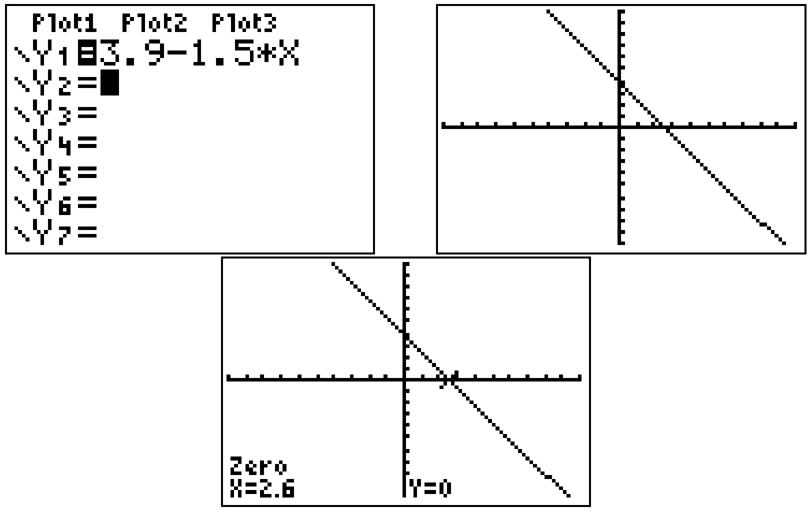
11. To find the zero of $f(x) = 3.9 - 1.5x$ algebraically, set $f(x) = 0$ and solve for x.

$f(x) = 0$	We want the value of x that makes the function equal to zero.
$3.9 - 1.5x = 0$	Replace $f(x)$ with $3.9 - 1.5x$.

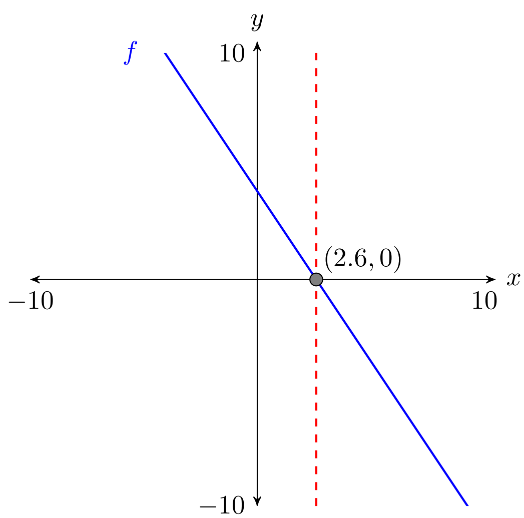
Now, solve for x.

$-1.5x = -3.9$	Subtract 3.9 from both sides.
$\dfrac{-1.5x}{-1.5} = \dfrac{-3.9}{-1.5}$	Divide both sides by -1.5.
$x = 2.6$	Simplify.

Hence, 2.6 is a zero of $f(x) = 3.9 - 1.5x$. Next, load the function into the Y= menu, select **6:ZStandard** from the ZOOM menu, then use the utility **2:zero** from the CALC menu to find the zero of the function.



Next, use the *Calculator Submission Guidelines* when crafting the following report on your homework paper. Draw a dashed vertical line through the x-intercept and label it with its coordinates.



Note how the graphical solution agrees with the algebraic solution.

13. The projectile's height above ground is given by the formula

$$y = y_0 + v_0 t - \frac{1}{2} g t^2,$$

where the initial height is 52 meters, the initial velocity is $v_0 = 203$ meters per second, and the acceleration due to gravity is $g = 9.8$ meters per second per

second. Substituting these numbers, the formula becomes

$$y = 52 + 203t - \frac{1}{2}(9.8)t^2,$$

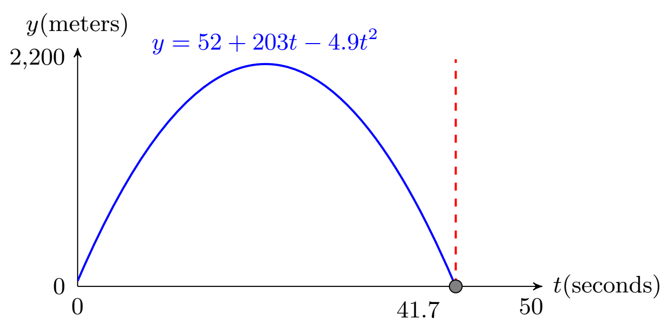
or equivalently,

$$y = 52 + 203t - 4.9t^2.$$

We're asked to find the time it takes the projectile to return to ground level. When this happens, its height y above ground level will equal zero. Substitute 0 for y in the last equation.

$$0 = 52 + 203t - 4.9t^2.$$

Load the right-hand side of this equation into the Y= menu of your graphing calculator, then set the WINDOW parameters as follows: Xmin=0, Xmax=50, Ymin=0, and Ymax=2200. Push the GRAPH button to produce the graph, then use the **2:zero** tool from the CALC menu to find where the graph of f crosses the x-axis. Report the results on your homework as shown in the following graph.



Hence, the projectile reaches ground level at approximately 41.7 seconds.

15. The projectile's height above ground is given by the formula

$$y = y_0 + v_0 t - \frac{1}{2}gt^2,$$

where the initial height is 52 meters, the initial velocity is $v_0 = 276$ meters per second, and the acceleration due to gravity is $g = 9.8$ meters per second per second. Substituting these numbers, the formula becomes

$$y = 52 + 276t - \frac{1}{2}(9.8)t^2,$$

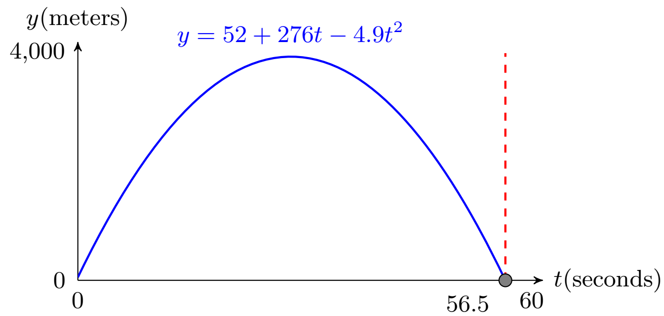
or equivalently,

$$y = 52 + 276t - 4.9t^2.$$

We're asked to find the time it takes the projectile to return to ground level. When this happens, its height y above ground level will equal zero. Substitute 0 for y in the last equation.

$$0 = 52 + 276t - 4.9t^2.$$

Load the right-hand side of this equation into the Y= menu of your graphing calculator, then set the WINDOW parameters as follows: Xmin=0, Xmax=60, Ymin=0, and Ymax=4000. Push the GRAPH button to produce the graph, then use the **2:zero** tool from the CALC menu to find where the graph of f crosses the x-axis. Report the results on your homework as shown in the following graph.



Hence, the projectile reaches ground level at approximately 56.5 seconds.

5.4 Adding and Subtracting Polynomials

1. Let's arrange our answer in descending powers of r by listing the highest powers of r first, then the next highest, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned}(-8r^2t + 7rt^2 + 3t^3) &+ (9r^3 + 2rt^2 + 4t^3)\\ &= (9r^3) + (-8r^2t) + (7rt^2 + 2rt^2) + (3t^3 + 4t^3)\\ &= 9r^3 - 8r^2t + 9rt^2 + 7t^3\end{aligned}$$

3. Let's arrange our answer in descending powers of x by listing the highest powers of x first, then the next highest, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned}(7x^2 - 6x - 9) + (8x^2 + 10x + 9)&\\ &= (7x^2 + 8x^2) + (-6x + 10x) + (-9 + 9)\\ &= 15x^2 + 4x\end{aligned}$$

5. Let's arrange our answer in descending powers of r by listing the highest powers of r first, then the next highest, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned}
&(-2r^2 + 7rs + 4s^2) + (-9r^2 + 7rs - 2s^2) \\
&\quad = (-2r^2 - 9r^2) + (7rs + 7rs) + (4s^2 - 2s^2) \\
&\quad = -11r^2 + 14rs + 2s^2
\end{aligned}$$

7. Let's arrange our answer in descending powers of y by listing the highest powers of y first, then the next highest, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned}
&(-8y^3 - 3y^2z - 6z^3) + (-3y^3 + 7y^2z - 9yz^2) \\
&\quad = (-8y^3 - 3y^3) + (-3y^2z + 7y^2z) + (-9yz^2) + (-6z^3) \\
&\quad = -11y^3 + 4y^2z - 9yz^2 - 6z^3
\end{aligned}$$

9. Negating a polynomial is accomplished by reversing the sign of each term of the polynomial. Thus,

$$-(5x^2 - 4) = -5x^2 + 4$$

11. Negating a polynomial is accomplished by reversing the sign of each term of the polynomial. Thus,

$$-(9r^3 - 4r^2t - 3rt^2 + 4t^3) = -9r^3 + 4r^2t + 3rt^2 - 4t^3$$

13. Negating a polynomial is accomplished by reversing the sign of each term of the polynomial. Thus,

$$-(-5x^2 + 9xy + 6y^2) = 5x^2 - 9xy - 6y^2$$

15. To arrange the answer in descending powers of u, place the highest power of u first, then the next highest, and so on. First, distribute the minus sign, changing the sign of each term of the second polynomial.

$$\begin{aligned}
&(-u^3 - 4u^2w + 7w^3) - (u^2w + uw^2 + 3w^3) \\
&\quad = -u^3 - 4u^2w + 7w^3 - u^2w - uw^2 - 3w^3
\end{aligned}$$

Now use the commutative and associative properties to change the order and regroup. Combine like terms.

$$\begin{aligned}
&= (-u^3) + (-4u^2w - u^2w) + (-uw^2) + (7w^3 - 3w^3) \\
&= -u^3 - 5u^2w - uw^2 + 4w^3
\end{aligned}$$

17. To arrange the answer in descending powers of y, place the highest power of y first, then the next highest, and so on. First, distribute the minus sign, changing the sign of each term of the second polynomial.

$$\begin{aligned}&(2y^3 - 2y^2z + 3z^3) - (-8y^3 + 5yz^2 - 3z^3)\\&\quad = 2y^3 - 2y^2z + 3z^3 + 8y^3 - 5yz^2 + 3z^3\end{aligned}$$

Now use the commutative and associative properties to change the order and regroup. Combine like terms.

$$\begin{aligned}&= (2y^3 + 8y^3) + (-2y^2z) + (-5yz^2) + (3z^3 + 3z^3)\\&= 10y^3 - 2y^2z - 5yz^2 + 6z^3\end{aligned}$$

19. To arrange the answer in descending powers of r, place the highest power of r first, then the next highest, and so on. First, distribute the minus sign, changing the sign of each term of the second polynomial.

$$\begin{aligned}&(-7r^2 - 9rs - 2s^2) - (-8r^2 - 7rs + 9s^2)\\&\qquad = -7r^2 - 9rs - 2s^2 + 8r^2 + 7rs - 9s^2\end{aligned}$$

Now use the commutative and associative properties to change the order and regroup. Combine like terms.

$$\begin{aligned}&= (-7r^2 + 8r^2) + (-9rs + 7rs) + (-2s^2 - 9s^2)\\&= r^2 - 2rs - 11s^2\end{aligned}$$

21. To arrange the answer in descending powers of x, place the highest power of x first, then the next highest, and so on. First, distribute the minus sign, changing the sign of each term of the second polynomial.

$$\begin{aligned}&(10x^2 + 2x - 6) - (-8x^2 + 14x + 17)\\&\qquad = 10x^2 + 2x - 6 + 8x^2 - 14x - 17\end{aligned}$$

Now use the commutative and associative properties to change the order and regroup. Combine like terms.

$$\begin{aligned}&= (10x^2 + 8x^2) + (2x - 14x) + (-6 - 17)\\&= 18x^2 - 12x - 23\end{aligned}$$

23. To arrange our answer in descending powers of x, we need to first list the highest power of x, then the next highest power, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned} f(x)+g(x) &= (-2x^2+9x+7)+(8x^3-7x^2+5) \\ &= (8x^3)+(-2x^2-7x^2)+(9x)+(7+5) \\ &= 8x^3-9x^2+9x+12 \end{aligned}$$

25. To arrange our answer in descending powers of x, we need to first list the highest power of x, then the next highest power, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned} f(x)+g(x) &= (5x^3-5x^2+8x)+(7x^2-2x-9) \\ &= (5x^3)+(-5x^2+7x^2)+(8x-2x)+(-9) \\ &= 5x^3+2x^2+6x-9 \end{aligned}$$

27. To arrange our answer in descending powers of x, we need to first list the highest power of x, then the next highest power, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned} f(x)+g(x) &= (-3x^2-8x-9)+(5x^2-4x+4) \\ &= (-3x^2+5x^2)+(-8x-4x)+(-9+4) \\ &= 2x^2-12x-5 \end{aligned}$$

29. To arrange our answer in descending powers of x, we need to first list the highest power of x, then the next highest power, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned} f(x)-g(x) &= (-6x^3-7x+7)-(-3x^3-3x^2-8x) \\ &= -6x^3-7x+7+3x^3+3x^2+8x \\ &= (-6x^3+3x^3)+(3x^2)+(-7x+8x)+(7) \\ &= -3x^3+3x^2+x+7 \end{aligned}$$

31. To arrange the answer in descending powers of x, place the highest power of x first, then the next highest, and so on. First, distribute the minus sign, changing the sign of each term of the second polynomial.

$$\begin{aligned} f(x) - g(x) &= (12x^2 - 5x + 4) - (8x^2 - 16x - 7) \\ &= 12x^2 - 5x + 4 - 8x^2 + 16x + 7 \end{aligned}$$

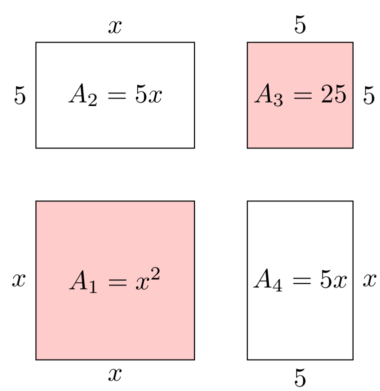
Now use the commutative and associative properties to change the order and regroup. Combine like terms.

$$\begin{aligned} &= (12x^2 - 8x^2) + (-5x + 16x) + (4 + 7) \\ &= 4x^2 + 11x + 11 \end{aligned}$$

33. To arrange our answer in descending powers of x, we need to first list the highest power of x, then the next highest power, and so on. Use the commutative and associative properties to change the order and regroup. Then combine like terms.

$$\begin{aligned} f(x) - g(x) &= (-3x^3 - 4x + 2) - (-4x^3 - 7x^2 + 6) \\ &= -3x^3 - 4x + 2 + 4x^3 + 7x^2 - 6 \\ &= (-3x^3 + 4x^3) + (7x^2) + (-4x) + (2 - 6) \\ &= x^3 + 7x^2 - 4x - 4 \end{aligned}$$

35. The two shaded squares in have areas $A_1 = x^2$ and $A_3 = 25$, respectively. The two unshaded rectangles have areas $A_2 = 5x$ and $A_4 = 5x$.



Summing these four areas gives us the area of the entire figure.

$$\begin{aligned} A &= A_1 + A_2 + A_3 + A_4 \\ &= x^2 + 5x + 25 + 5x \\ &= x^2 + 10x + 25 \end{aligned}$$

37. The profit made from selling x wicker baskets is found by subtracting the costs incurred from the revenue received. In symbols:

$$P(x) = R(x) - C(x)$$

Next, replace $R(x)$ and $C(x)$ with their definitions. Because we are supposed to subtract *all* of the cost from the revenue, be sure to surround the cost polynomial with parentheses.

$$P(x) = 33.45x - (232 + 7x - 0.0085x^2)$$

Distribute the minus sign and combine like terms.

$$\begin{aligned} &= 33.45x - 232 - 7x + 0.0085x^2 \\ &= -232 + 26.45x + 0.0085x^2 \end{aligned}$$

Thus, the profit function is $P(x) = -232 + 26.45x + 0.0085x^2$. Next, to determine the profit if 233 wicker baskets are sold, subsitute 233 for x in the profit function $P(x)$.

$$\begin{aligned} P(x) &= -232 + 26.45x + 0.0085x^2 \\ P(233) &= -232 + 26.45(233) + 0.0085(233)^2 \end{aligned}$$

You can now use your graphing calculator to determine the profit.

$$P(233) = 6392.3065$$

Rounding to the nearest cent, the profit is \$6,392.31.

5.5 Laws of Exponents

1. The exponent tells us how many times to write the base as a factor.

$$\begin{aligned} (-4)^3 &= (-4)(-4)(-4) && \text{Write } -4 \text{ as a factor 3 times.} \\ &= -64 && \text{Multiply.} \end{aligned}$$

3. If you raise any number (other than zero) to the zero power, the answer is 1.

$$\left(-\frac{5}{7}\right)^0 = 1$$

5. The exponent tells us how many times to write the base as a factor.

$$\begin{aligned}
\left(-\frac{4}{3}\right)^2 &= \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right) && \text{Write } -4/3 \text{ as a factor 2 times.}\\
&= \frac{16}{9} && \text{Multiply.}
\end{aligned}$$

7. If you raise any number (other than zero) to the zero power, the answer is 1.

$$(-19)^0 = 1$$

9. When multiplying like bases, use the law $a^m a^n = a^{m+n}$. That is, repeat the base and add the exponents.

$$\begin{aligned}
(7v-6w)^{18} \cdot (7v-6w)^{17} &= (7v-6w)^{18+17} && \text{Repeat the base, add the exponents.}\\
&= (7v-6w)^{35} && \text{Simplify: } 18+17=35
\end{aligned}$$

11. When multiplying like bases, use the law $a^m a^n = a^{m+n}$. That is, repeat the base and add the exponents.

$$\begin{aligned}
3^4 \cdot 3^0 &= 3^{4+0} && \text{Repeat the base, add the exponents.}\\
&= 3^4 && \text{Simplify: } 4+0=4
\end{aligned}$$

If you wish, you can use your calculator to compute $3^4 = 81$. However, reporting the answer in exponential form 3^4 is far easier.

13. When multiplying like bases, use the law $a^m a^n = a^{m+n}$. That is, repeat the base and add the exponents.

$$\begin{aligned}
4^n \cdot 4^{8n+3} &= 4^{(n)+(8n+3)} && \text{Repeat the base, add the exponents.}\\
&= 4^{(n+8n)+(3)} && \text{Group like terms.}\\
&= 4^{9n+3} && \text{Simplify.}
\end{aligned}$$

15. When multiplying like bases, use the law $a^m a^n = a^{m+n}$. That is, repeat the base and add the exponents.

$$\begin{aligned}
x^8 \cdot x^3 &= x^{8+3} && \text{Repeat the base, add the exponents.}\\
&= x^{11} && \text{Simplify: } 8+3=11
\end{aligned}$$

17. When multiplying like bases, use the law $a^m a^n = a^{m+n}$. That is, repeat the base and add the exponents.

$$\begin{aligned} 2^5 \cdot 2^3 &= 2^{5+3} && \text{Repeat the base, add the exponents.} \\ &= 2^8 && \text{Simplify: } 5+3=8 \end{aligned}$$

If you wish, you can use your calculator to compute $2^8 = 256$. However, reporting the answer in exponential form 2^8 is far easier.

19. When dividing like bases, use the law $a^m/a^n = a^{m-n}$. That is, repeat the base and subtract the exponents.

$$\begin{aligned} \frac{4^{16}}{4^{16}} &= 4^{16-16} && \text{Repeat the base, subtract the exponents.} \\ &= 4^0 && \text{Simplify: } 16-16=0 \\ &= 1 && \text{Any number (except zero) to the zero power equals 1.} \end{aligned}$$

Hence, $4^{16}/4^{16} = 1$.

21. When dividing like bases, use the law $a^m/a^n = a^{m-n}$. That is, repeat the base and subtract the exponents.

$$\begin{aligned} \frac{w^{11}}{w^7} &= w^{11-7} && \text{Repeat the base, subtract the exponents.} \\ &= w^4 && \text{Simplify: } 11-7=4 \end{aligned}$$

23. When dividing like bases, use the law $a^m/a^n = a^{m-n}$. That is, repeat the base and subtract the exponents.

$$\begin{aligned} \frac{(9a-8c)^{15}}{(9a-8c)^8} &= (9a-8c)^{15-8} && \text{Repeat the base, subtract the exponents.} \\ &= (9a-8c)^7 && \text{Simplify: } 15-8=7 \end{aligned}$$

25. When dividing like bases, use the law $a^m/a^n = a^{m-n}$. That is, repeat the base and subtract the exponents.

$$\begin{aligned} \frac{2^{9n+5}}{2^{3n-4}} &= 2^{(9n+5)-(3n-4)} && \text{Repeat the base, subtract the exponents.} \\ &= 2^{9n+5-3n+4} && \text{Distribute minus sign.} \\ &= 2^{(9n-3n)+(5+4)} && \text{Group like terms.} \\ &= 2^{6n+9} && \text{Simplify.} \end{aligned}$$

27. When dividing like bases, use the law $a^m/a^n = a^{m-n}$. That is, repeat the base and subtract the exponents.

$$\begin{aligned} \frac{4^{17}}{4^9} &= 4^{17-9} && \text{Repeat the base, subtract the exponents.} \\ &= 4^8 && \text{Simplify: } 17-9=8 \end{aligned}$$

If you wish, you can use your calculator to compute $4^8 = 65536$. However, reporting the answer in exponential form 4^8 is far easier.

29. When raising a power to a power, use the law $(a^m)^n = a^{mn}$. That is, repeat the base and multiply the exponents.

$$\begin{aligned} \left(4^{8m-6}\right)^7 &= 4^{7(8m-6)} && \text{Repeat the base, multiply the exponents.} \\ &= 4^{56m-42} && \text{Distribute 7.} \end{aligned}$$

31. When raising a power to a power, use the law $(a^m)^n = a^{mn}$. That is, repeat the base and multiply the exponents.

$$\begin{aligned} \left[(9x+5y)^3\right]^7 &= (9x+5y)^{(3)(7)} && \text{Repeat the base, multiply the exponents.} \\ &= (9x+5y)^{21} && \text{Simplify: } (3)(7)=21 \end{aligned}$$

33. When raising a power to a power, use the law $(a^m)^n = a^{mn}$. That is, repeat the base and multiply the exponents.

$$\begin{aligned} \left(4^3\right)^2 &= 4^{(3)(2)} && \text{Repeat the base, multiply the exponents.} \\ &= 4^6 && \text{Simplify: } (3)(2)=6 \end{aligned}$$

If you wish, you can use your calculator to compute $4^6 = 4096$. However, reporting the answer in exponential form 4^6 is far easier.

35. When raising a power to a power, use the law $(a^m)^n = a^{mn}$. That is, repeat the base and multiply the exponents.

$$\begin{aligned} \left(c^4\right)^7 &= c^{(4)(7)} && \text{Repeat the base, multiply the exponents.} \\ &= c^{28} && \text{Simplify: } (4)(7)=28 \end{aligned}$$

37. When raising a power to a power, use the law $(a^m)^n = a^{mn}$. That is, repeat the base and multiply the exponents.

$$\begin{aligned}
\left(6^2\right)^0 &= 6^{(2)(0)} && \text{Repeat the base, multiply the exponents.} \\
&= 6^0 && \text{Simplify: } (2)(0) = 0 \\
&= 1 && \text{Any number (except zero) raised to the zero power is 1.}
\end{aligned}$$

Hence, $\left(6^2\right)^0 = 1$.

39. When raising a product to a power, use the law $(ab)^n = a^n b^n$. That is, raise each factor to the n.

$$(uw)^5 = u^5 w^5 \qquad \text{Raise each factor to the 5.}$$

41. When raising a product to a power, use the law $(ab)^n = a^n b^n$. That is, raise each factor to the n.

$$\begin{aligned}
(-2y)^3 &= (-2)^3 (y)^3 && \text{Raise each factor to the 3.} \\
&= -8y^3 && \text{Simplify: } (-2)^3 = -8
\end{aligned}$$

43. When raising a product to a power, use the law $(ab)^n = a^n b^n$. That is, raise each factor to the n.

$$\begin{aligned}
(3w^9)^4 &= (3)^4 (w^9)^4 && \text{Raise each factor to the 4.} \\
&= 81w^{36} && \text{Simplify: } (3)^4 = 81 \text{ and } (w^9)^4 = w^{36}
\end{aligned}$$

45. When raising a product to a power, use the law $(ab)^n = a^n b^n$. That is, raise each factor to the n.

$$\begin{aligned}
(-3x^8y^2)^4 &= (-3)^4 (x^8)^4 (y^2)^4 && \text{Raise each factor to the 4.} \\
&= 81x^{32}y^8 && \text{Simplify: } (-3)^4 = 81,\ (x^8)^4 = x^{32}, \\
& && \text{and } (y^2)^4 = y^8
\end{aligned}$$

47. When raising a product to a power, use the law $(ab)^n = a^n b^n$. That is, raise each factor to the n.

$$\begin{aligned}
(7s^{6n})^3 &= (7)^3 (s^{6n})^3 && \text{Raise each factor to the 3.} \\
&= 343s^{18n} && \text{Simplify: } (7)^3 = 343,\ (s^{6n})^3 = s^{18n}
\end{aligned}$$

49. When raising a quotient to a power, use the law $(a/b)^n = a^n/b^n$. That is, raise both numerator and denominator to the n.

$$\begin{aligned} \left(\frac{v}{2}\right)^3 &= \frac{v^3}{2^3} && \text{Raise numerator and denominator to the third power.} \\ &= \frac{v^3}{8} && \text{Simplify: } (2)^3 = 8 \end{aligned}$$

51. When raising a quotient to a power, use the law $(a/b)^n = a^n/b^n$. That is, raise both numerator and denominator to the n. When you raise a negative fraction to an even power, the answer is positive.

$$\begin{aligned} \left(-\frac{2}{u}\right)^2 &= \frac{2^2}{u^2} && \text{Raise numerator and denominator to the second power.} \\ &= \frac{4}{u^2} && \text{Simplify: } (2)^2 = 4 \end{aligned}$$

53. When raising a quotient to a power, use the law $(a/b)^n = a^n/b^n$. That is, raise both numerator and denominator to the n. When you raise a negative fraction to an even power, the answer is positive.

$$\begin{aligned} \left(-\frac{r^8}{5}\right)^4 &= \frac{\left(r^8\right)^4}{5^4} && \text{Raise numerator and denominator to the fourth power.} \\ &= \frac{r^{32}}{625} && \text{Simplify: } \left(r^8\right)^4 = r^{32} \text{ and } (5)^4 = 625 \end{aligned}$$

55. When raising a quotient to a power, use the law $(a/b)^n = a^n/b^n$. That is, raise both numerator and denominator to the n.

$$\begin{aligned} \left(\frac{5}{c^9}\right)^4 &= \frac{5^4}{\left(c^9\right)^4} && \text{Raise numerator and denominator to the fourth power.} \\ &= \frac{625}{c^{36}} && \text{Simplify: } (5)^4 = 625 \text{ and } \left(c^9\right)^4 = c^{36} \end{aligned}$$

57. When raising a quotient to a power, use the law $(a/b)^n = a^n/b^n$. That is, raise both numerator and denominator to the n.

$$\begin{aligned} \left(\frac{5}{u^{12}}\right)^2 &= \frac{5^2}{\left(u^{12}\right)^2} && \text{Raise numerator and denominator to the second power.} \\ &= \frac{25}{u^{24}} && \text{Simplify: } (5)^2 = 25 \text{ and } \left(u^{12}\right)^2 = u^{24} \end{aligned}$$

5.6 Multiplying Polynomials

1. Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} -3(7r) &= [(-3)(7)]r && \text{Reorder. Regroup.} \\ &= -21r && \text{Multiply: } (-3)(7) = -21 \end{aligned}$$

3. Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} (-9b^3)(-8b^6) &= [(-9)(-8)](b^3b^6) && \text{Reorder. Regroup.} \\ &= 72b^9 && \text{Multiply: } (-9)(-8) = 72,\ b^3b^6 = b^9. \end{aligned}$$

5. Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} &(-7r^2t^4)(7r^5t^2) \\ &\quad = [(-7)(7)](r^2r^5)(t^4t^2) && \text{Reorder. Regroup.} \\ &\quad = -49r^7t^6 && \text{Multiply: } (-7)(7) = -49,\ r^2r^5 = r^7, \\ &&& \text{and } t^4t^2 = t^6 \end{aligned}$$

7. Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} &(-5b^2c^9)(-8b^4c^4) \\ &\quad = [(-5)(-8)](b^2b^4)(c^9c^4) && \text{Reorder. Regroup.} \\ &\quad = 40b^6c^{13} && \text{Multiply: } (-5)(-8) = 40,\ b^2b^4 = b^6, \\ &&& \text{and } c^9c^4 = c^{13} \end{aligned}$$

9. Use the commutative and associative properties to change the order and regroup.

$$\begin{aligned} (-8v^3)(4v^4) &= [(-8)(4)](v^3v^4) && \text{Reorder. Regroup.} \\ &= -32v^7 && \text{Multiply: } (-8)(4) = -32,\ v^3v^4 = v^7. \end{aligned}$$

11. We need to first distribute the 9 times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} 9(-2b^2 + 2b + 9) &= 9(-2b^2) + 9(2b) + 9(9) \\ &= -18b^2 + 18b + 81 \end{aligned}$$

Alternate solution. Note that it is more efficient to distribute 9 times each term of $-2b^2 + 2b + 9$, performing the calculations mentally as you go.

$$9(-2b^2 + 2b + 9) = -18b^2 + 18b + 81$$

13. We need to first distribute the -4 times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} -4(10t^2 - 7t - 6) &= -4(10t^2) - (-4)(7t) - (-4)(6) \\ &= -40t^2 - (-28t) - (-24) \\ &= -40t^2 + 28t + 24 \end{aligned}$$

Alternate solution. Note that it is more efficient to distribute -4 times each term of $10t^2 - 7t - 6$, performing the calculations mentally as you go.

$$-4(10t^2 - 7t - 6) = -40t^2 + 28t + 24$$

15. We need to first distribute the $-8u^2$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} &- 8u^2(-7u^3 - 8u^2 - 2u + 10) \\ &\quad = -8u^2(-7u^3) - (-8u^2)(8u^2) - (-8u^2)(2u) + (-8u^2)(10) \\ &\quad = 56u^5 - (-64u^4) - (-16u^3) + (-80u^2) \\ &\quad = 56u^5 + 64u^4 + 16u^3 - 80u^2 \end{aligned}$$

Alternate solution. Note that it is more efficient to distribute $-8u^2$ times each term of $-7u^3 - 8u^2 - 2u + 10$, performing the calculations mentally as you go.

$$-8u^2(-7u^3 - 8u^2 - 2u + 10) = 56u^5 + 64u^4 + 16u^3 - 80u^2$$

17. We need to first distribute the $10s^2$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} 10s^2(-10s^3 + 2s^2 + 2s + 8) &= 10s^2(-10s^3) + 10s^2(2s^2) + 10s^2(2s) + 10s^2(8) \\ &= -100s^5 + 20s^4 + 20s^3 + 80s^2 \end{aligned}$$

Alternate solution. Note that it is more efficient to distribute $10s^2$ times each term of $-10s^3 + 2s^2 + 2s + 8$, performing the calculations mentally as you go.

$$10s^2(-10s^3 + 2s^2 + 2s + 8) = -100s^5 + 20s^4 + 20s^3 + 80s^2$$

19. We need to first distribute the $2st$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} 2st(-4s^2 + 8st - 10t^2) &= 2st(-4s^2) + 2st(8st) - 2st(10t^2) \\ &= -8s^3t + 16s^2t^2 - 20st^3 \end{aligned}$$

Alternate solution. Note that it is more efficient to distribute 2 times each term of $-4s^2 + 8st - 10t^2$, performing the calculations mentally as you go.

$$2st(-4s^2 + 8st - 10t^2) = -8s^3t + 16s^2t^2 - 20st^3$$

21. We need to first distribute the $-2uw$ times each term of the polynomial. Then we multiply the resulting monomials mentally.

$$\begin{aligned} -2uw(10u^2 - 7uw - 2w^2) &= -2uw(10u^2) - (-2uw)(7uw) - (-2uw)(2w^2) \\ &= -20u^3w - (-14u^2w^2) - (-4uw^3) \\ &= -20u^3w + 14u^2w^2 + 4uw^3 \end{aligned}$$

Alternate solution. Note that it is more efficient to distribute -2 times each term of $10u^2 - 7uw - 2w^2$, performing the calculations mentally as you go.

$$-2uw(10u^2 - 7uw - 2w^2) = -20u^3w + 14u^2w^2 + 4uw^3$$

23. Let's imagine that $(-9x - 4)(-3x + 2)$ has the form $(b + c)a$ and multiply $-3x + 2$ times both terms of $-9x - 4$.

$$(-9x - 4)(-3x + 2) = -9x(-3x + 2) - 4(-3x + 2)$$

Now we distribute monomials times polynomials, then combine like terms.

$$\begin{aligned} &= 27x^2 - 18x + 12x - 8 \\ &= 27x^2 - 6x - 8 \end{aligned}$$

Thus, $(-9x - 4)(-3x + 2) = 27x^2 - 6x - 8$

25. Let's imagine that $(3x + 8)(3x - 2)$ has the form $(b + c)a$ and multiply $3x - 2$ times both terms of $3x + 8$.

$$(3x + 8)(3x - 2) = 3x(3x - 2) + 8(3x - 2)$$

Now we distribute monomials times polynomials, then combine like terms.

$$\begin{aligned} &= 9x^2 - 6x + 24x - 16 \\ &= 9x^2 + 18x - 16 \end{aligned}$$

Thus, $(3x + 8)(3x - 2) = 9x^2 + 18x - 16$

27. Let's imagine that $(2x-1)(-6x^2+4x+5)$ has the form $(b+c)a$ and multiply $-6x^2+4x+5$ times both terms of $2x-1$.

$$(2x-1)(-6x^2+4x+5) = 2x(-6x^2+4x+5) - 1(-6x^2+4x+5)$$

Now we distribute monomials times polynomials, then combine like terms.

$$\begin{aligned} &= -12x^3+8x^2+10x+6x^2-4x-5 \\ &= -12x^3+14x^2+6x-5 \end{aligned}$$

Thus, $(2x-1)(-6x^2+4x+5) = -12x^3+14x^2+6x-5$

29. Let's imagine that $(x-6)(-2x^2-4x-4)$ has the form $(b+c)a$ and multiply $-2x^2-4x-4$ times both terms of $x-6$.

$$(x-6)(-2x^2-4x-4) = x(-2x^2-4x-4) - 6(-2x^2-4x-4)$$

Now we distribute monomials times polynomials, then combine like terms.

$$\begin{aligned} &= -2x^3-4x^2-4x+12x^2+24x+24 \\ &= -2x^3+8x^2+20x+24 \end{aligned}$$

Thus, $(x-6)(-2x^2-4x-4) = -2x^3+8x^2+20x+24$

31. First, multiply $8u$ times each term of $8u-9w$, then multiply $-9w$ times each term of $8u-9w$. Finally, combine like terms.

$$\begin{aligned} (8u-9w)(8u-9w) &= 8u(8u-9w) - 9w(8u-9w) \\ &= 64u^2-72uw-72uw+81w^2 \\ &= 64u^2-144uw+81w^2 \end{aligned}$$

Thus, $(8u-9w)(8u-9w) = 64u^2-144uw+81w^2$.

33. First, multiply $9r$ times each term of $3r-9t$, then multiply $-7t$ times each term of $3r-9t$. Finally, combine like terms.

$$\begin{aligned} (9r-7t)(3r-9t) &= 9r(3r-9t) - 7t(3r-9t) \\ &= 27r^2-81rt-21rt+63t^2 \\ &= 27r^2-102rt+63t^2 \end{aligned}$$

Thus, $(9r-7t)(3r-9t) = 27r^2-102rt+63t^2$.

35. First, multiply $4r$ times each term of $-10r^2 + 10rs - 7s^2$, then multiply $-10s$ times each term of $-10r^2 + 10rs - 7s^2$. Finally, combine like terms.

$$\begin{aligned}(4r - 10s)(-10r^2 + 10rs - 7s^2) &= 4r(-10r^2 + 10rs - 7s^2) - 10s(-10r^2 + 10rs - 7s^2) \\ &= -40r^3 + 40r^2s - 28rs^2 + 100r^2s - 100rs^2 + 70s^3 \\ &= -40r^3 + 140r^2s - 128rs^2 + 70s^3\end{aligned}$$

Thus, $(4r - 10s)(-10r^2 + 10rs - 7s^2) = -40r^3 + 140r^2s - 128rs^2 + 70s^3$.

37. First, multiply $9x$ times each term of $4x^2 - 4xz - 10z^2$, then multiply $-2z$ times each term of $4x^2 - 4xz - 10z^2$. Finally, combine like terms.

$$\begin{aligned}(9x - 2z)(4x^2 - 4xz - 10z^2) &= 9x(4x^2 - 4xz - 10z^2) - 2z(4x^2 - 4xz - 10z^2) \\ &= 36x^3 - 36x^2z - 90xz^2 - 8x^2z + 8xz^2 + 20z^3 \\ &= 36x^3 - 44x^2z - 82xz^2 + 20z^3\end{aligned}$$

Thus, $(9x - 2z)(4x^2 - 4xz - 10z^2) = 36x^3 - 44x^2z - 82xz^2 + 20z^3$.

39. First, write $9r + 3t$ as a factor two times.

$$(9r + 3t)^2 = (9r + 3t)(9r + 3t)$$

Next, multiply $9r$ times each term of $9r + 3t$, then multiply $3t$ times each term of $9r + 3t$. Finally, combine like terms.

$$\begin{aligned}&= 9r(9r + 3t) + 3t(9r + 3t) \\ &= 81r^2 + 27rt + 27rt + 9t^2 \\ &= 81r^2 + 54rt + 9t^2\end{aligned}$$

Thus, $(9r + 3t)^2 = 81r^2 + 54rt + 9t^2$.

41. First, multiply $4y$ times each term of $4y - 5z$, then multiply $5z$ times each term of $4y - 5z$. Finally, combine like terms.

$$\begin{aligned}(4y + 5z)(4y - 5z) &= 4y(4y - 5z) + 5z(4y - 5z) \\ &= 16y^2 - 20yz + 20yz - 25z^2 \\ &= 16y^2 - 25z^2\end{aligned}$$

Thus, $(4y + 5z)(4y - 5z) = 16y^2 - 25z^2$.

43. First, multiply $7u$ times each term of $7u-8v$, then multiply $8v$ times each term of $7u-8v$. Finally, combine like terms.

$$\begin{aligned}(7u+8v)(7u-8v) &= 7u(7u-8v)+8v(7u-8v)\\ &= 49u^2-56uv+56uv-64v^2\\ &= 49u^2-64v^2\end{aligned}$$

Thus, $(7u+8v)(7u-8v)=49u^2-64v^2$.

45. First, write $7b+8c$ as a factor two times.

$$(7b+8c)^2=(7b+8c)(7b+8c)$$

Next, multiply $7b$ times each term of $7b+8c$, then multiply $8c$ times each term of $7b+8c$. Finally, combine like terms.

$$\begin{aligned}&= 7b(7b+8c)+8c(7b+8c)\\ &= 49b^2+56bc+56bc+64c^2\\ &= 49b^2+112bc+64c^2\end{aligned}$$

Thus, $(7b+8c)^2=49b^2+112bc+64c^2$.

47. Multiply $2t^2$ times each term of $2t^2+9t+4$, multiply $9t$ times each term of $2t^2+9t+4$, and multiply 4 times each term of $2t^2+9t+4$. Finally, combine like terms.

$$\begin{aligned}&(2t^2+9t+4)(2t^2+9t+4)\\ &\quad= 2t^2(2t^2+9t+4)+9t(2t^2+9t+4)+4(2t^2+9t+4)\\ &\quad= 4t^4+18t^3+8t^2+18t^3+81t^2+36t+8t^2+36t+16\\ &\quad= 4t^4+36t^3+97t^2+72t+16\end{aligned}$$

Thus, $(2t^2+9t+4)(2t^2+9t+4)=4t^4+36t^3+97t^2+72t+16$.

49. Multiply $4w^2$ times each term of $3w^2-6w+8$, multiply $3w$ times each term of $3w^2-6w+8$, and multiply 5 times each term of $3w^2-6w+8$. Finally, combine like terms.

$$\begin{aligned}&(4w^2+3w+5)(3w^2-6w+8)\\ &\quad= 4w^2(3w^2-6w+8)+3w(3w^2-6w+8)+5(3w^2-6w+8)\\ &\quad= 12w^4-24w^3+32w^2+9w^3-18w^2+24w+15w^2-30w+40\\ &\quad= 12w^4-15w^3+29w^2-6w+40\end{aligned}$$

Thus, $(4w^2+3w+5)(3w^2-6w+8)=12w^4-15w^3+29w^2-6w+40$.

51. To find the revenue, we must multiply the number of widgets sold by the price for each widget. That is, we must multiply the demand times the unit price.

$$\text{Revenue} = \text{Demand} \times \text{Unit Price}$$

Let R represent the revenue. Because x represents the demand and p represents the unit price, the last equation can be rewritten as follows.

$$R = xp$$

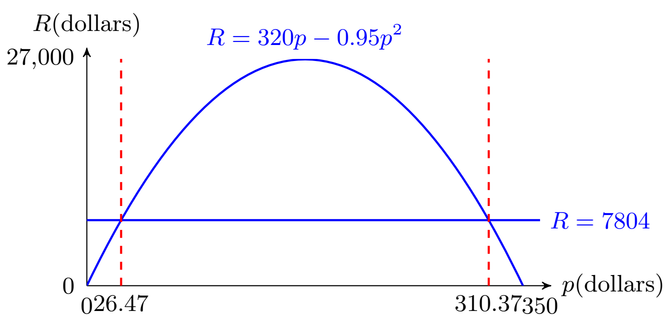
However, the demand is given by the equation $x = 320 - 0.95p$. Substitute $320 - 0.95p$ for x in the last equation to get

$$R = (320 - 0.95p)p,$$

or equivalently,

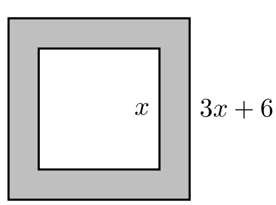
$$R = 320p - 0.95p^2.$$

We're asked to find when the revenue equals $R = 7,804$, so enter $Y1 = 320 * X - 0.95 * X \wedge 2$ and $Y2 = 7804$ into the Y= menu in your calculator, then set the WINDOW parameters as follows: Xmin=0, Xmax=350, Ymin=0, Ymax=27000. Push the GRAPH button to produce the graph, then use the **5:intersect** utility to determine the coordinates of the points of intersection. Report your answer on your homework as follows.



Hence, the revenue will equal $R = \$7,804$ if the unit price is set either at \$26.47 or \$310.37.

53. Because the edge of the outer square is 6 inches longer that 3 times the edge of the inner square, the edge of the outer square is $3x + 6$.



To find the area of the shaded region, subtract the area of the smaller square from the area of the larger square. Recall that the area of a square is found by squaring the length of one of its sides.

$$\text{Area of shaded region} = \text{Area of larger square} - \text{Area of smaller square}$$
$$A(x) = (3x+6)^2 - x^2$$

Use the distributive property to multiply.

$$A(x) = (3x+6)(3x+6) - x^2$$
$$A(x) = 9x^2 + 18x + 18x + 36 - x^2$$

Combine like terms.

$$A(x) = 8x^2 + 36x + 36$$

Hence, the area of the shaded region is given by the polynomial $A(x) = 8x^2 + 36x + 36$. We can now evaluate the polynomial at $x = 5$ inches by substituting 5 for x.

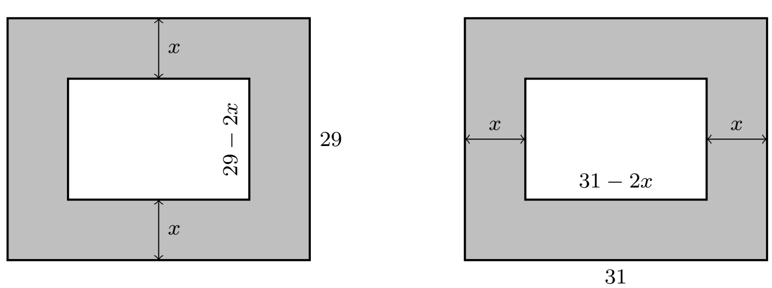
$$A(x) = 8x^2 + 36x + 36$$
$$A(5) = 8(5)^2 + 36(5) + 36$$

Use your calculator to help simplify.

$$A(5) = 416$$

Hence, the area of the shaded region is 416 square inches.

55. If the width of the entire rectangular garden is 29 feet, and the width of the border lawn is x feet, then the width of the interior rectangular garden is $29 - 2x$ feet. Simlarly, if the length of the entire rectangular garden is 31 feet, and the width of the border lawn is x feet, then the length of the interior rectangular garden is $31 - 2x$ feet.



Therefore, the dimensions of the inner rectangular garden are $31 - 2x$ by $29 - 2x$ feet. Thus, the area of the inner rectangular garden is:

$$A(x) = (31 - 2x)(29 - 2x)$$

We are required to express our answer in the standard form $A(x) = ax^2+bx+c$, so we use the distributive property to multiply.

$$\begin{aligned} A(x) &= 899 - 62x - 58x + 4x^2 \\ A(x) &= 899 - 120x + 4x^2 \end{aligned}$$

Now, if the uniform width of the lawn border is 9.3 feet, substitute 9.3 for x in the polynomial that gives the area of the inner rectangular garden.

$$\begin{aligned} A(x) &= 899 - 120x + 4x^2 \\ A(9.3) &= 899 - 120(9.3) + 4(9.3)^2 \end{aligned}$$

Use a calculator to compute the answer.

$$A(9.3) = 128.96$$

Hence, the area of the inner rectangular garden is 128.96 square feet.

5.7 Special Products

1. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(5x)(3x) = 15x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $20x + 6x = 26x$

iii) Multiply the terms in the "Last" positions: $(2)(4) = 8$

Write the answer with no intermediate steps:

$$(5x + 2)(3x + 4) = 15x^2 + 26x + 8$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(5x + 2)(3x + 4) = 15x^2 + 26x + 8$$

3. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(6x)(5x) = 30x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $24x - 15x = 9x$

iii) Multiply the terms in the "Last" positions: $(-3)(4) = -12$

Write the answer with no intermediate steps:

$$(6x - 3)(5x + 4) = 30x^2 + 9x - 12$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(6x - 3)(5x + 4) = 30x^2 + 9x - 12$$

5. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(5x)(3x) = 15x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-20x - 18x = -38x$

iii) Multiply the terms in the "Last" positions: $(-6)(-4) = 24$

Write the answer with no intermediate steps:

$$(5x - 6)(3x - 4) = 15x^2 - 38x + 24$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(5x - 6)(3x - 4) = 15x^2 - 38x + 24$$

7. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(6x)(3x) = 18x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-30x - 6x = -36x$

iii) Multiply the terms in the "Last" positions: $(-2)(-5) = 10$

Write the answer with no intermediate steps:

$$(6x - 2)(3x - 5) = 18x^2 - 36x + 10$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(6x - 2)(3x - 5) = 18x^2 - 36x + 10$$

9. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(6x)(3x) = 18x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $30x + 12x = 42x$

iii) Multiply the terms in the "Last" positions: $(4)(5) = 20$

Write the answer with no intermediate steps:

$$(6x + 4)(3x + 5) = 18x^2 + 42x + 20$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(6x + 4)(3x + 5) = 18x^2 + 42x + 20$$

11. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(4x)(6x) = 24x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $12x - 30x = -18x$

iii) Multiply the terms in the "Last" positions: $(-5)(3) = -15$

Write the answer with no intermediate steps:

$$(4x - 5)(6x + 3) = 24x^2 - 18x - 15$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(4x - 5)(6x + 3) = 24x^2 - 18x - 15$$

13. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(10x)^2 = 100x^2$

ii) Square the term in the "Last" position: $(12)^2 = 144$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(10x-12)(10x+12) &= (10x)^2-(12)^2\\ &= 100x^2-144\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(10x-12)(10x+12) = 100x^2-144$$

15. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(6x)^2 = 36x^2$

ii) Square the term in the "Last" position: $(9)^2 = 81$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(6x+9)(6x-9) &= (6x)^2-(9)^2\\ &= 36x^2-81\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(6x+9)(6x-9) = 36x^2-81$$

17. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(3x)^2 = 9x^2$

ii) Square the term in the "Last" position: $(10)^2 = 100$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(3x+10)(3x-10) &= (3x)^2-(10)^2\\ &= 9x^2-100\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(3x+10)(3x-10) = 9x^2-100$$

19. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(10x)^2 = 100x^2$

ii) Square the term in the "Last" position: $(9)^2 = 81$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(10x-9)(10x+9) &= (10x)^2 - (9)^2 \\ &= 100x^2 - 81\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(10x-9)(10x+9) = 100x^2 - 81$$

21. Follow these steps:

i) Square the first term: $(2x)^2 = 4x^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(2x)(3) = 12x$

iii) Square the "Last" term: $(3)^2 = 9$

Thus:

$$\begin{aligned}(2x+3)^2 &= (2x)^2 + 2(2x)(3) + (3)^2 \\ &= 4x^2 + 12x + 9\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(2x+3)^2 = 4x^2 + 12x + 9$$

23. You can start by writing the difference as a sum:

$$(9x-8)^2 = (9x + (-8))^2$$

Follow these steps:

i) Square the first term: $(9x)^2 = 81x^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(9x)(-8) = -144x$

iii) Square the "Last" term: $(-8)^2 = 64$

Thus:

$$\begin{aligned}(9x-8)^2 &= (9x+(-8))^2 \\ &= (9x)^2 + 2(9x)(-8) + (-8)^2 \\ &= 81x^2 - 144x + 64\end{aligned}$$

Alternately, you can note that $(a-b)^2 = a^2 - 2ab + b^2$, so when the terms of the binomial are separated by a minus sign, the middle term of the result will also be minus. This allows us to move more quickly, writing:

$$\begin{aligned}(9x-8)^2 &= (9x)^2 - 2(9x)(8) + (8)^2 \\ &= 81x^2 - 144x + 64\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $(9x-8)^2 = 81x^2 - 144x + 64$.

25. Follow these steps:

i) Square the first term: $(7x)^2 = 49x^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(7x)(2) = 28x$

iii) Square the "Last" term: $(2)^2 = 4$

Thus:

$$\begin{aligned}(7x+2)^2 &= (7x)^2 + 2(7x)(2) + (2)^2 \\ &= 49x^2 + 28x + 4\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(7x+2)^2 = 49x^2 + 28x + 4$$

27. You can start by writing the difference as a sum:

$$(6x-5)^2 = (6x+(-5))^2$$

Follow these steps:

i) Square the first term: $(6x)^2 = 36x^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(6x)(-5) = -60x$

iii) Square the "Last" term: $(-5)^2 = 25$

Thus:

$$\begin{aligned}(6x-5)^2 &= (6x+(-5))^2\\ &= (6x)^2 + 2(6x)(-5) + (-5)^2\\ &= 36x^2 - 60x + 25\end{aligned}$$

Alternately, you can note that $(a-b)^2 = a^2 - 2ab + b^2$, so when the terms of the binomial are separated by a minus sign, the middle term of the result will also be minus. This allows us to move more quickly, writing:

$$\begin{aligned}(6x-5)^2 &= (6x)^2 - 2(6x)(5) + (5)^2\\ &= 36x^2 - 60x + 25\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in $(6x-5)^2 = 36x^2 - 60x + 25$.

29. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(11x)^2 = 121x^2$

ii) Square the term in the "Last" position: $(2)^2 = 4$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(11x-2)(11x+2) &= (11x)^2 - (2)^2\\ &= 121x^2 - 4\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(11x-2)(11x+2) = 121x^2 - 4$$

31. Follow these steps:

i) Square the first term: $(7r)^2 = 49r^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(7r)(5t) = 70rt$

iii) Square the "Last" term: $(5t)^2 = 25t^2$

Thus:

$$\begin{aligned}(7r - 5t)^2 &= (7r)^2 - 2(7r)(5t) + (5t)^2\\ &= 49r^2 - 70rt + 25t^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(7r - 5t)^2 = 49r^2 - 70rt + 25t^2$$

33. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(5b)(3b) = 15b^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-10bc + 18bc = 8bc$

iii) Multiply the terms in the "Last" positions: $(6c)(-2c) = -12c^2$

Write the answer with no intermediate steps:

$$(5b + 6c)(3b - 2c) = 15b^2 + 8bc - 12c^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(5b + 6c)(3b - 2c) = 15b^2 + 8bc - 12c^2$$

35. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(3u)^2 = 9u^2$

ii) Square the term in the "Last" position: $(5v)^2 = 25v^2$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(3u+5v)(3u-5v) &= (3u)^2-(5v)^2\\ &= 9u^2-25v^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(3u+5v)(3u-5v) = 9u^2-25v^2$$

37. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(9b^3)^2 = 81b^6$

ii) Square the term in the "Last" position: $(10c^5)^2 = 100c^{10}$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(9b^3+10c^5)(9b^3-10c^5) &= (9b^3)^2-(10c^5)^2\\ &= 81b^6-100c^{10}\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(9b^3+10c^5)(9b^3-10c^5) = 81b^6-100c^{10}$$

39. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(9s)^2 = 81s^2$

ii) Square the term in the "Last" position: $(4t)^2 = 16t^2$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(9s-4t)(9s+4t) &= (9s)^2-(4t)^2\\ &= 81s^2-16t^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(9s-4t)(9s+4t) = 81s^2-16t^2$$

41. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(7x)^2 = 49x^2$

ii) Square the term in the "Last" position: $(9y)^2 = 81y^2$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(7x - 9y)(7x + 9y) &= (7x)^2 - (9y)^2 \\ &= 49x^2 - 81y^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(7x - 9y)(7x + 9y) = 49x^2 - 81y^2$$

43. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(6a)(2a) = 12a^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $18ab - 12ab = 6ab$

iii) Multiply the terms in the "Last" positions: $(-6b)(3b) = -18b^2$

Write the answer with no intermediate steps:

$$(6a - 6b)(2a + 3b) = 12a^2 + 6ab - 18b^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(6a - 6b)(2a + 3b) = 12a^2 + 6ab - 18b^2$$

45. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(10x)^2 = 100x^2$

ii) Square the term in the "Last" position: $(10)^2 = 100$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(10x-10)(10x+10) &= (10x)^2-(10)^2\\ &= 100x^2-100\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(10x-10)(10x+10) = 100x^2-100$$

47. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(4a)(6a) = 24a^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-12ab + 12ab = 0$

iii) Multiply the terms in the "Last" positions: $(2b)(-3b) = -6b^2$

Write the answer with no intermediate steps:

$$(4a+2b)(6a-3b) = 24a^2-6b^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(4a+2b)(6a-3b) = 24a^2-6b^2$$

49. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(5b)(3b) = 15b^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $10bc - 12bc = -2bc$

iii) Multiply the terms in the "Last" positions: $(-4c)(2c) = -8c^2$

Write the answer with no intermediate steps:

$$(5b-4c)(3b+2c) = 15b^2-2bc-8c^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(5b-4c)(3b+2c) = 15b^2-2bc-8c^2$$

51. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(4b)(6b) = 24b^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-8bc + -36bc = -44bc$

iii) Multiply the terms in the "Last" positions: $(-6c)(-2c) = 12c^2$

Write the answer with no intermediate steps:

$$(4b - 6c)(6b - 2c) = 24b^2 - 44bc + 12c^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(4b - 6c)(6b - 2c) = 24b^2 - 44bc + 12c^2$$

53. Follow these steps:

i) Square the first term: $(11r^5)^2 = 121r^{10}$

ii) Multiply the "First" and "Last" terms and double the result: $2(11r^5)(9t^2) = 198r^5t^2$

iii) Square the "Last" term: $(9t^2)^2 = 81t^4$

Thus:

$$\begin{aligned}(11r^5 + 9t^2)^2 &= (11r^5)^2 - 2(11r^5)(9t^2) + (9t^2)^2 \\ &= 121r^{10} + 198r^5t^2 + 81t^4\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(11r^5 + 9t^2)^2 = 121r^{10} + 198r^5t^2 + 81t^4$$

55. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(4u)(2u) = 8u^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $-24uv + -8uv = -32uv$

iii) Multiply the terms in the "Last" positions: $(-4v)(-6v) = 24v^2$

Write the answer with no intermediate steps:

$$(4u - 4v)(2u - 6v) = 8u^2 - 32uv + 24v^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(4u - 4v)(2u - 6v) = 8u^2 - 32uv + 24v^2$$

57. Follow these steps:

i) Square the first term: $(8r^4)^2 = 64r^8$

ii) Multiply the "First" and "Last" terms and double the result: $2(8r^4)(7t^5) = 112r^4t^5$

iii) Square the "Last" term: $(7t^5)^2 = 49t^{10}$

Thus:

$$\begin{aligned}(8r^4 + 7t^5)^2 &= (8r^4)^2 - 2(8r^4)(7t^5) + (7t^5)^2 \\ &= 64r^8 + 112r^4t^5 + 49t^{10}\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(8r^4 + 7t^5)^2 = 64r^8 + 112r^4t^5 + 49t^{10}$$

59. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(4r)^2 = 16r^2$

ii) Square the term in the "Last" position: $(3t)^2 = 9t^2$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(4r + 3t)(4r - 3t) &= (4r)^2 - (3t)^2 \\ &= 16r^2 - 9t^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(4r + 3t)(4r - 3t) = 16r^2 - 9t^2$$

61. Follow these steps:

i) Square the first term: $(5r)^2 = 25r^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(5r)(6t) = 60rt$

iii) Square the "Last" term: $(6t)^2 = 36t^2$

Thus:

$$\begin{aligned}(5r+6t)^2 &= (5r)^2 + 2(5r)(6t) + (6t)^2\\ &= 25r^2 + 60rt + 36t^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(5r+6t)^2 = 25r^2 + 60rt + 36t^2$$

63. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(3x)(2x) = 6x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $15x - 8x = 7x$

iii) Multiply the terms in the "Last" positions: $(-4)(5) = -20$

Write the answer with no intermediate steps:

$$(3x-4)(2x+5) = 6x^2 + 7x - 20$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(3x-4)(2x+5) = 6x^2 + 7x - 20$$

65. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(6b)(2b) = 12b^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $18bc + 8bc = 26bc$

iii) Multiply the terms in the "Last" positions: $(4c)(3c) = 12c^2$

Write the answer with no intermediate steps:

$$(6b+4c)(2b+3c) = 12b^2 + 26bc + 12c^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(6b+4c)(2b+3c) = 12b^2 + 26bc + 12c^2$$

67. Note how the terms in the "First" position are identical, as are the terms in the "Last" position, with one set separated by a plus sign and the other with a minus sign. Hence, this is the difference of squares pattern and we proceed as follows:

i) Square the term in the "First" position: $(11u^2)^2 = 121u^4$

ii) Square the term in the "Last" position: $(8w^3)^2 = 64w^6$

iii) Separate the squares with a minus sign.

That is:

$$\begin{aligned}(11u^2 + 8w^3)(11u^2 - 8w^3) &= (11u^2)^2 - (8w^3)^2 \\ &= 121u^4 - 64w^6\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(11u^2 + 8w^3)(11u^2 - 8w^3) = 121u^4 - 64w^6$$

69. Follow these steps:

i) Square the first term: $(4y)^2 = 16y^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(4y)(3z) = 24yz$

iii) Square the "Last" term: $(3z)^2 = 9z^2$

Thus:

$$\begin{aligned}(4y + 3z)^2 &= (4y)^2 + 2(4y)(3z) + (3z)^2 \\ &= 16y^2 + 24yz + 9z^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(4y + 3z)^2 = 16y^2 + 24yz + 9z^2$$

71. Follow these steps:

i) Square the first term: $(7u)^2 = 49u^2$

ii) Multiply the "First" and "Last" terms and double the result: $2(7u)(2v) = 28uv$

iii) Square the "Last" term: $(2v)^2 = 4v^2$

Thus:

$$\begin{aligned}(7u-2v)^2 &= (7u)^2 - 2(7u)(2v) + (2v)^2\\ &= 49u^2 - 28uv + 4v^2\end{aligned}$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(7u-2v)^2 = 49u^2 - 28uv + 4v^2$$

73. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(3v)(5v) = 15v^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $18vw + 10vw = 28vw$

iii) Multiply the terms in the "Last" positions: $(2w)(6w) = 12w^2$

Write the answer with no intermediate steps:

$$(3v+2w)(5v+6w) = 15v^2 + 28vw + 12w^2$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(3v+2w)(5v+6w) = 15v^2 + 28vw + 12w^2$$

75. Each of the following steps is performed mentally.

i) Multiply the terms in the "First" positions: $(5x)(6x) = 30x^2$

ii) Multiply the terms in the "Outer" and "Inner" positions and add the results mentally: $10x - 18x = -8x$

iii) Multiply the terms in the "Last" positions: $(-3)(2) = -6$

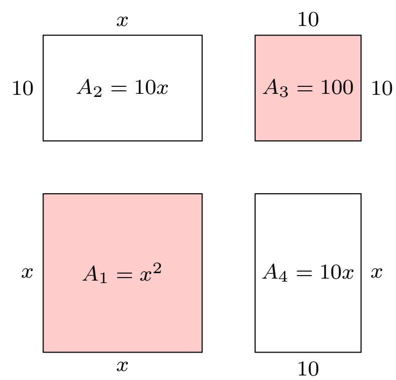
Write the answer with no intermediate steps:

$$(5x-3)(6x+2) = 30x^2 - 8x - 6$$

Note: You should practice this pattern until you can go straight from the problem statement to the answer without writing down any intermediate work, as in:

$$(5x-3)(6x+2) = 30x^2 - 8x - 6$$

77. The two shaded squares in have areas $A_1 = x^2$ and $A_3 = 100$, respectively. The two unshaded rectangles have areas $A_2 = 10x$ and $A_4 = 10x$.



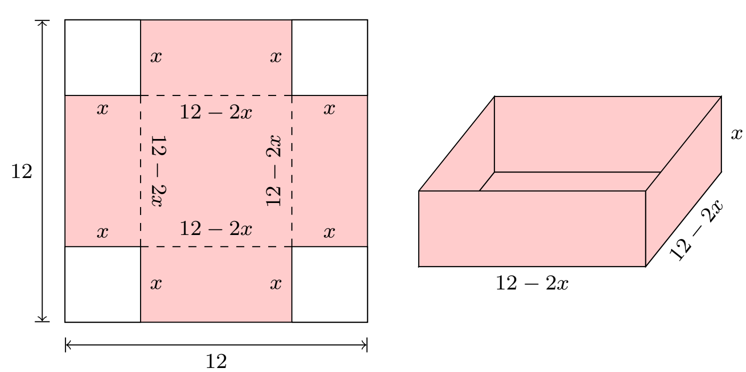
Summing these four areas gives us the area of the entire figure.

$$\begin{aligned} A &= A_1 + A_2 + A_3 + A_4 \\ &= x^2 + 10x + 100 + 10x \\ &= x^2 + 20x + 100 \end{aligned}$$

However, a faster solution is found by squaring the side of the square using the shortcut $(a + b)^2 = a^2 + 2ab + b^2$.

$$\begin{aligned} A &= (x + 10)^2 \\ &= x^2 + 20x + 100 \end{aligned}$$

79. After cutting four squares with side x inches from each corner of the original piece of cardboard (measuring 12 inches on each side), the dashed edges that will become the eventual edges of the base of the cardboard box now measure $12 - 2x$ inches. The sides are then folded upwards to form a cardboard box with no top.



The volume of the box is found by taking the product of the length, width, and height of the box.

$$V = LWH$$
$$V = (12 - 2x)(12 - 2x)x$$

Changing the order of multiplication and using exponents, this can be written more concisely.

$$V = x(12 - 2x)^2$$

We can use $(a - b)^2 = a^2 - 2ab + b^2$ to square the binomial.

$$V = x(144 - 48x + 4x^2)$$

Finally, we can distribute the x.

$$V = 144x - 48x^2 + 4x^3$$

Using function notation, we can also write $V(x) = 144x - 48x^2 + 4x^3$. To find the volume when the edge of the square cut from each corner measures 1.25 inches, substitute 1.25 for x in the polynomial.

$$V(2) = 144(1.25) - 48(1.25)^2 + 4(1.25)^3$$

Use your calculator to obtain the following result.

$$V(2) = 112.8125$$

Thus, the volume of the box is approximately 113 cubic inches.

Chapter 6

Factoring

6.1 The Greatest Common Factor

1. First, list all possible ways that we can express 42 as a product of two positive integers:

$$42 = 1 \cdot 42 \qquad 42 = 2 \cdot 21 \qquad 42 = 3 \cdot 14$$
$$42 = 6 \cdot 7$$

Therefore, the list of divisors of 42 is:

$$\{1, 2, 3, 6, 7, 14, 21, 42\}$$

3. First, list all possible ways that we can express 44 as a product of two positive integers:

$$44 = 1 \cdot 44 \qquad 44 = 2 \cdot 22 \qquad 44 = 4 \cdot 11$$

Therefore, the list of divisors of 44 is:

$$\{1, 2, 4, 11, 22, 44\}$$

5. First, list all possible ways that we can express 51 as a product of two positive integers:

$$51 = 1 \cdot 51 \qquad 51 = 3 \cdot 17$$

Therefore, the list of divisors of 51 is:

$$\{1, 3, 17, 51\}$$

7. First, list the positive divisors of 36:

$$1, 2, 3, 4, 6, 9, 12, 18, 36$$

Secondly, list the positive divisors of 42:

$$1, 2, 3, 6, 7, 14, 21, 42$$

Finally, list the positive divisors that are in common.

$$1, 2, 3, 6$$

9. First, list the positive divisors of 78:

$$1, 2, 3, 6, 13, 26, 39, 78$$

Secondly, list the positive divisors of 54:

$$1, 2, 3, 6, 9, 18, 27, 54$$

Finally, list the positive divisors that are in common.

$$1, 2, 3, 6$$

11. First, list the positive divisors of 8:

$$1, 2, 4, 8$$

Secondly, list the positive divisors of 76:

$$1, 2, 4, 19, 38, 76$$

Finally, list the positive divisors that are in common.

$$1, 2, 4$$

13. We're asked to find the greatest common divisor of 76 and 8. Therefore, we must try to find the largest number that divides evenly (zero remainder) into both 76 and 8. For some folks, the number 4 just pops into their heads. However, if the number doesn't just "pop into your head," then you can:

i) List the positive divisors of 76:

$$1, 2, 4, 19, 38, 76$$

ii) List the positive divisors of 8:

$$1, 2, 4, 8$$

iii) List the positive divisors that are in common.

$$1, 2, 4$$

The greatest common divisor is therefore 4.

15. We're asked to find the greatest common divisor of 32 and 36. Therefore, we must try to find the largest number that divides evenly (zero remainder) into both 32 and 36. For some folks, the number 4 just pops into their heads. However, if the number doesn't just "pop into your head," then you can:

i) List the positive divisors of 32:

$$1, 2, 4, 8, 16, 32$$

ii) List the positive divisors of 36:

$$1, 2, 3, 4, 6, 9, 12, 18, 36$$

iii) List the positive divisors that are in common.

$$1, 2, 4$$

The greatest common divisor is therefore 4.

17. We're asked to find the greatest common divisor of 24 and 28. Therefore, we must try to find the largest number that divides evenly (zero remainder) into both 24 and 28. For some folks, the number 4 just pops into their heads. However, if the number doesn't just "pop into your head," then you can:

i) List the positive divisors of 24:

$$1, 2, 3, 4, 6, 8, 12, 24$$

ii) List the positive divisors of 28:

$$1, 2, 4, 7, 14, 28$$

iii) List the positive divisors that are in common.

$$1, 2, 4$$

The greatest common divisor is therefore 4.

19. Prime factor each number and place the result in compact form using exponents.

$$\begin{aligned} 600 &= 2^3 \cdot 3^1 \cdot 5^2 \\ 1080 &= 2^3 \cdot 3^3 \cdot 5^1 \end{aligned}$$

Write each prime factor that appears above to the highest power that appears in common.

$$\text{GCD} = 2^3 \cdot 3^1 \cdot 5^1 \quad \text{Raise each factor to highest power that appears in common.}$$

Expand and simplify.

$$\begin{aligned} &= 8 \cdot 3 \cdot 5 && \text{Expand: } 2^3 = 8,\ 3^1 = 3, \text{ and } 5^1 = 5 \\ &= 120 && \text{Multiply.} \end{aligned}$$

Therefore, $\text{GCD}(600, 1080) = 120$.

21. Prime factor each number and place the result in compact form using exponents.

$$\begin{aligned} 1800 &= 2^3 \cdot 3^2 \cdot 5^2 \\ 2250 &= 2^1 \cdot 3^2 \cdot 5^3 \end{aligned}$$

Write each prime factor that appears above to the highest power that appears in common.

$$\text{GCD} = 2^1 \cdot 3^2 \cdot 5^2 \quad \text{Raise each factor to highest power that appears in common.}$$

Expand and simplify.

$$\begin{aligned} &= 2 \cdot 9 \cdot 25 && \text{Expand: } 2^1 = 2,\ 3^2 = 9, \text{ and } 5^2 = 25 \\ &= 450 && \text{Multiply.} \end{aligned}$$

Therefore, $\text{GCD}(1800, 2250) = 450$.

23. Prime factor each number and place the result in compact form using exponents.

$$\begin{aligned} 600 &= 2^3 \cdot 3^1 \cdot 5^2 \\ 450 &= 2^1 \cdot 3^2 \cdot 5^2 \end{aligned}$$

Write each prime factor that appears above to the highest power that appears in common.

$$\text{GCD} = 2^1 \cdot 3^1 \cdot 5^2 \quad \text{Raise each factor to highest power that appears in common.}$$

Expand and simplify.

$$= 2 \cdot 3 \cdot 25 \qquad \text{Expand: } 2^1 = 2,\ 3^1 = 3, \text{ and } 5^2 = 25$$
$$= 150 \qquad \text{Multiply.}$$

Therefore, $\mathrm{GCD}(600, 450) = 150$.

25. To find the GCF of $16b^4$ and $56b^9$, we note that:

1. The greatest common factor (divisor) of 16 and 56 is 8.
2. The monomials $16b^4$ and $56b^9$ have the variable b in common.
3. The highest power of b in common is b^4.

Thus, the greatest common factor is $\mathrm{GCF}(16b^4, 56b^9) = 8b^4$. Note what happens when we write each of the given monomials as a product of the greatest common factor and a second monomial:

$$\begin{aligned} 16b^4 &= 8b^4 \cdot 2 \\ 56b^9 &= 8b^4 \cdot 7b^5 \end{aligned}$$

Note how the set of second monomial factors (2 and $7b^5$) contain no additional common factors.

27. To find the GCF of $35z^2$ and $49z^7$, we note that:

1. The greatest common factor (divisor) of 35 and 49 is 7.
2. The monomials $35z^2$ and $49z^7$ have the variable z in common.
3. The highest power of z in common is z^2.

Thus, the greatest common factor is $\mathrm{GCF}(35z^2, 49z^7) = 7z^2$. Note what happens when we write each of the given monomials as a product of the greatest common factor and a second monomial:

$$\begin{aligned} 35z^2 &= 7z^2 \cdot 5 \\ 49z^7 &= 7z^2 \cdot 7z^5 \end{aligned}$$

Note how the set of second monomial factors (5 and $7z^5$) contain no additional common factors.

29. To find the GCF of $56x^3y^4$ and $16x^2y^5$, we note that:

1. The greatest common factor (divisor) of 56 and 16 is 8.
2. The monomials $56x^3y^4$ and $16x^2y^5$ have the variables x and y in common.
3. The highest power of x in common is x^2. The highest power of y in common is y^4.

Thus, the greatest common factor is $\text{GCF}(56x^3y^4, 16x^2y^5) = 8x^2y^4$. Note what happens when we write each of the given monomials as a product of the greatest common factor and a second monomial:

$$\begin{aligned} 56x^3y^4 &= 8x^2y^4 \cdot 7x \\ 16x^2y^5 &= 8x^2y^4 \cdot 2y \end{aligned}$$

Note how the set of second monomial factors ($7x$ and $2y$) contain no additional common factors.

31. To find the GCF of $24s^4t^5$ and $16s^3t^6$, we note that:

1. The greatest common factor (divisor) of 24 and 16 is 8.
2. The monomials $24s^4t^5$ and $16s^3t^6$ have the variables s and t in common.
3. The highest power of s in common is s^3. The highest power of t in common is t^5.

Thus, the greatest common factor is $\text{GCF}(24s^4t^5, 16s^3t^6) = 8s^3t^5$. Note what happens when we write each of the given monomials as a product of the greatest common factor and a second monomial:

$$\begin{aligned} 24s^4t^5 &= 8s^3t^5 \cdot 3s \\ 16s^3t^6 &= 8s^3t^5 \cdot 2t \end{aligned}$$

Note how the set of second monomial factors ($3s$ and $2t$) contain no additional common factors.

33. To find the GCF of $18y^7$, $45y^6$, and $27y^5$, we note that:

1. The greatest common factor (divisor) of 18, 45, and 27 is 9.
2. The monomials $18y^7$, $45y^6$, and $27y^5$ have the variable y in common.
3. The highest power of y in common is y^5.

Thus, the greatest common factor is $\text{GCF}(18y^7, 45y^6, 27y^5) = 9y^5$. Note what happens when we write each of the given monomials as a product of the greatest common factor and a second monomial:

$$\begin{aligned} 18y^7 &= 9y^5 \cdot 2y^2 \\ 45y^6 &= 9y^5 \cdot 5y \\ 27y^5 &= 9y^5 \cdot 3 \end{aligned}$$

Note how the set of second monomial factors ($2y^2$, $5y$, and 3) contain no additional common factors.

35. To find the GCF of $9a^6$, $6a^5$, and $15a^4$, we note that:

1. The greatest common factor (divisor) of 9, 6, and 15 is 3.
2. The monomials $9a^6$, $6a^5$, and $15a^4$ have the variable a in common.
3. The highest power of a in common is a^4.

Thus, the greatest common factor is $\text{GCF}(9a^6, 6a^5, 15a^4) = 3a^4$. Note what happens when we write each of the given monomials as a product of the greatest common factor and a second monomial:

$$\begin{aligned} 9a^6 &= 3a^4 \cdot 3a^2 \\ 6a^5 &= 3a^4 \cdot 2a \\ 15a^4 &= 3a^4 \cdot 5 \end{aligned}$$

Note how the set of second monomial factors ($3a^2$, $2a$, and 5) contain no additional common factors.

37. The greatest common factor (GCF) of $25a^2$, $10a$ and 20 is 5. Factor out the GCF.

$$\begin{aligned} 25a^2 + 10a + 20 &= 5 \cdot 5a^2 + 5 \cdot 2a + 5 \cdot 4 \\ &= 5(5a^2 + 2a + 4) \end{aligned}$$

Check: Multiply. Distribute the 5.

$$\begin{aligned} 5(5a^2 + 2a + 4) &= 5 \cdot 5a^2 + 5 \cdot 2a + 5 \cdot 4 \\ &= 25a^2 + 10a + 20 \end{aligned}$$

That's the original polynomial, so we factored correctly.

39. The greatest common factor (GCF) of $35s^2$, $25s$ and 45 is 5. Factor out the GCF.

$$\begin{aligned} 35s^2 + 25s + 45 &= 5 \cdot 7s^2 + 5 \cdot 5s + 5 \cdot 9 \\ &= 5(7s^2 + 5s + 9) \end{aligned}$$

Check: Multiply. Distribute the 5.

$$\begin{aligned} 5(7s^2 + 5s + 9) &= 5 \cdot 7s^2 + 5 \cdot 5s + 5 \cdot 9 \\ &= 35s^2 + 25s + 45 \end{aligned}$$

That's the original polynomial, so we factored correctly.

41. The greatest common factor (GCF) of $16c^3$, $32c^2$ and $36c$ is $4c$. Factor out the GCF.

$$\begin{aligned} 16c^3 + 32c^2 + 36c &= 4c \cdot 4c^2 + 4c \cdot 8c + 4c \cdot 9 \\ &= 4c(4c^2 + 8c + 9) \end{aligned}$$

Check: Multiply. Distribute the $4c$.

$$\begin{aligned} 4c(4c^2 + 8c + 9) &= 4c \cdot 4c^2 + 4c \cdot 8c + 4c \cdot 9 \\ &= 16c^3 + 32c^2 + 36c \end{aligned}$$

That's the original polynomial, so we factored correctly.

43. The greatest common factor (GCF) of $42s^3$, $24s^2$ and $18s$ is $6s$. Factor out the GCF.

$$\begin{aligned} 42s^3 + 24s^2 + 18s &= 6s \cdot 7s^2 + 6s \cdot 4s + 6s \cdot 3 \\ &= 6s(7s^2 + 4s + 3) \end{aligned}$$

Check: Multiply. Distribute the $6s$.

$$\begin{aligned} 6s(7s^2 + 4s + 3) &= 6s \cdot 7s^2 + 6s \cdot 4s + 6s \cdot 3 \\ &= 42s^3 + 24s^2 + 18s \end{aligned}$$

That's the original polynomial, so we factored correctly.

45. The greatest common factor (GCF) of $35s^7$, $49s^6$ and $63s^5$ is $7s^5$. Factor out the GCF.

$$\begin{aligned} 35s^7 + 49s^6 + 63s^5 &= 7s^5 \cdot 5s^2 + 7s^5 \cdot 7s + 7s^5 \cdot 9 \\ &= 7s^5(5s^2 + 7s + 9) \end{aligned}$$

Check: Multiply. Distribute the $7s^5$.

$$\begin{aligned} 7s^5(5s^2 + 7s + 9) &= 7s^5 \cdot 5s^2 + 7s^5 \cdot 7s + 7s^5 \cdot 9 \\ &= 35s^7 + 49s^6 + 63s^5 \end{aligned}$$

That's the original polynomial, so we factored correctly.

47. The greatest common factor (GCF) of $14b^7$, $35b^6$ and $56b^5$ is $7b^5$. Factor out the GCF.

$$\begin{aligned} 14b^7 + 35b^6 + 56b^5 &= 7b^5 \cdot 2b^2 + 7b^5 \cdot 5b + 7b^5 \cdot 8 \\ &= 7b^5(2b^2 + 5b + 8) \end{aligned}$$

Check: Multiply. Distribute the $7b^5$.

$$\begin{aligned} 7b^5(2b^2 + 5b + 8) &= 7b^5 \cdot 2b^2 + 7b^5 \cdot 5b + 7b^5 \cdot 8 \\ &= 14b^7 + 35b^6 + 56b^5 \end{aligned}$$

That's the original polynomial, so we factored correctly.

49. The greatest common factor (GCF) of $54y^5z^3$, $30y^4z^4$ and $36y^3z^5$ is $6y^3z^3$. Factor out the GCF.

$$\begin{aligned} 54y^5z^3 + 30y^4z^4 + 36y^3z^5 &= 6y^3z^3 \cdot 9y^2 + 6y^3z^3 \cdot 5yz + 6y^3z^3 \cdot 6z^2 \\ &= 6y^3z^3(9y^2 + 5yz + 6z^2) \end{aligned}$$

Check: Multiply. Distribute the $6y^3z^3$.

$$\begin{aligned} 6y^3z^3(9y^2 + 5yz + 6z^2) &= 6y^3z^3 \cdot 9y^2 + 6y^3z^3 \cdot 5yz + 6y^3z^3 \cdot 6z^2 \\ &= 54y^5z^3 + 30y^4z^4 + 36y^3z^5 \end{aligned}$$

That's the original polynomial, so we factored correctly.

51. The greatest common factor (GCF) of $45s^4t^3$, $40s^3t^4$ and $15s^2t^5$ is $5s^2t^3$. Factor out the GCF.

$$\begin{aligned} 45s^4t^3 + 40s^3t^4 + 15s^2t^5 &= 5s^2t^3 \cdot 9s^2 + 5s^2t^3 \cdot 8st + 5s^2t^3 \cdot 3t^2 \\ &= 5s^2t^3(9s^2 + 8st + 3t^2) \end{aligned}$$

Check: Multiply. Distribute the $5s^2t^3$.

$$\begin{aligned} 5s^2t^3(9s^2 + 8st + 3t^2) &= 5s^2t^3 \cdot 9s^2 + 5s^2t^3 \cdot 8st + 5s^2t^3 \cdot 3t^2 \\ &= 45s^4t^3 + 40s^3t^4 + 15s^2t^5 \end{aligned}$$

That's the original polynomial, so we factored correctly.

53. In this case, the greatest common factor (GCF) is $2w - 3$.

$$\begin{aligned} 7w(2w-3) - 8(2w-3) &= 7w \cdot (2w-3) - 8 \cdot (2w-3) \\ &= (7w-8)(2w-3) \end{aligned}$$

Because of the commutative property of multiplication, it is equally valid to pull the GCF out in front.

$$\begin{aligned} 7w(2w-3) - 8(2w-3) &= (2w-3) \cdot 7w - (2w-3) \cdot 8 \\ &= (2w-3)(7w-8) \end{aligned}$$

Note that the order of factors differs from the first solution, but because of the commutative property of multiplication, the order does not matter. The answers are the same.

55. In this case, the greatest common factor (GCF) is $5r - 1$.

$$\begin{aligned} 9r(5r-1) + 8(5r-1) &= 9r \cdot (5r-1) + 8 \cdot (5r-1) \\ &= (9r+8)(5r-1) \end{aligned}$$

Because of the commutative property of multiplication, it is equally valid to pull the GCF out in front.

$$\begin{aligned} 9r(5r-1) + 8(5r-1) &= (5r-1) \cdot 9r + (5r-1) \cdot 8 \\ &= (5r-1)(9r+8) \end{aligned}$$

Note that the order of factors differs from the first solution, but because of the commutative property of multiplication, the order does not matter. The answers are the same.

57. In this case, the greatest common factor (GCF) is $6(2a + 5)$.

$$\begin{aligned} 48a(2a+5) - 42(2a+5) &= 6(2a+5) \cdot 8a - 6(2a+5) \cdot 7 \\ &= 6(2a+5)(8a-7) \end{aligned}$$

Alternate solution: It is possible that you might fail to notice that 15 and 12 are divisible by 3, factoring out only a common factor $2a + 5$.

$$\begin{aligned} 48a(2a+5) - 42(2a+5) &= 48a \cdot (2a+5) - 42 \cdot (2a+5) \\ &= (48a-42)(2a+5) \end{aligned}$$

However, you now need to notice that you can continue, factoring out a 6 from both $48a$ and -42.

$$= 6(8a-7)(2a+5)$$

Note that the order of factors differs from the first solution, but because of the commutative property of multiplication, the order does not matter. The answers are the same.

59. In this case, the greatest common factor (GCF) is $7(2a-1)$.

$$\begin{aligned} 56a(2a-1)-21(2a-1) &= 7(2a-1)\cdot 8a - 7(2a-1)\cdot 3 \\ &= 7(2a-1)(8a-3) \end{aligned}$$

Alternate solution: It is possible that you might fail to notice that 15 and 12 are divisible by 3, factoring out only a common factor $2a-1$.

$$\begin{aligned} 56a(2a-1)-21(2a-1) &= 56a\cdot(2a-1)-21\cdot(2a-1) \\ &= (56a-21)(2a-1) \end{aligned}$$

However, you now need to notice that you can continue, factoring out a 7 from both $56a$ and -21.

$$= 7(8a-3)(2a-1)$$

Note that the order of factors differs from the first solution, but because of the commutative property of multiplication, the order does not matter. The answers are the same.

61. We "group" the first and second terms, noting that we can factor x out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor -9 out of both of these terms.

$$x^2+2x-9x-18 = x\,(x+2)-9\,(x+2)$$

Note that we can now factor $x+2$ out of both of these terms.

$$= (x-9)(x+2)$$

63. We "group" the first and second terms, noting that we can factor x out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor 6 out of both of these terms.

$$x^2+3x+6x+18 = x\,(x+3)+6\,(x+3)$$

Note that we can now factor $x+3$ out of both of these terms.

$$= (x+6)(x+3)$$

65. We "group" the first and second terms, noting that we can factor x out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor -3 out of both of these terms.

$$x^2 + 6x + 3x + 18 = x\,(x - 6) - 3\,(x - 6)$$

Note that we can now factor $x - 6$ out of both of these terms.

$$= (x - 3)(x - 6)$$

67. We "group" the first and second terms, noting that we can factor x out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor 3 out of both of these terms.

$$x^2 - 9x + 3x - 27 = x\,(x - 9) + 3\,(x - 9)$$

Note that we can now factor $x - 9$ out of both of these terms.

$$= (x + 3)(x - 9)$$

69. We "group" the first and second terms, noting that we can factor x out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor -7 out of both of these terms.

$$8x^2 + 3x - 56x - 21 = x\,(8x + 3) - 7\,(8x + 3)$$

Note that we can now factor $8x + 3$ out of both of these terms.

$$= (x - 7)(8x + 3)$$

71. We "group" the first and second terms, noting that we can factor $9x$ out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor -5 out of both of these terms.

$$9x^2 + 36x - 5x - 20 = 9x\,(x + 4) - 5\,(x + 4)$$

Note that we can now factor $x + 4$ out of both of these terms.

$$= (9x - 5)(x + 4)$$

73. We "group" the first and second terms, noting that we can factor x out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor -8 out of both of these terms.

$$6x^2 - 7x - 48x + 56 = x\,(6x - 7) - 8\,(6x - 7)$$

Note that we can now factor $6x - 7$ out of both of these terms.

$$= (x - 8)(6x - 7)$$

75. We "group" the first and second terms, noting that we can factor $2x$ out of both of these terms. Then we "group" the third and fourth terms, noting that we can factor 7 out of both of these terms.

$$2x^2 + 12x + 7x + 42 = 2x\,(x + 6) + 7\,(x + 6)$$

Note that we can now factor $x + 6$ out of both of these terms.

$$= (2x + 7)(x + 6)$$

6.2 Solving Nonlinear Equations

1. The product of two factors equals zero.

$$(9x + 2)(8x + 3) = 0$$

Hence, at least one of the factors must equal zero, so set each factor equal to zero and solve the resulting equations for x.

$$\begin{aligned} 9x + 2 &= 0 &\quad\text{or}\quad&& 8x + 3 &= 0 \\ 9x &= -2 &&& 8x &= -3 \\ x &= -\frac{2}{9} &&& x &= -\frac{3}{8} \end{aligned}$$

Hence, the solutions are $x = -2/9$ and $x = -3/8$

3. The product of three factors equals zero.

$$x(4x+7)(9x-8)=0$$

Hence, at least one of the factors must equal zero, so set each factor equal to zero and solve the resulting equations for x.

$$\begin{aligned} x&=0 &\quad\text{or}\quad 4x+7&=0 &\quad\text{or}\quad 9x-8&=0\\ && 4x&=-7 && 9x&=8\\ && x&=-\frac{7}{4} && x&=\frac{8}{9} \end{aligned}$$

Hence, the solutions are $x=0$, $x=-7/4$, and $x=8/9$

5. The product of two factors equals zero.

$$-9x(9x+4)=0$$

Hence, at least one of the factors must equal zero, so set each factor equal to zero and solve the resulting equations for x.

$$\begin{aligned} -9x&=0 &\quad\text{or}\quad 9x+4&=0\\ x&=0 &\quad\text{or}\quad 9x&=-4\\ && x&=-\frac{4}{9} \end{aligned}$$

Hence, the solutions are $x=0$ and $x=-4/9$

7. The product of two factors equals zero.

$$(x+1)(x+6)=0$$

Hence, at least one of the factors must equal zero, so set each factor equal to zero and solve the resulting equations for x.

$$\begin{aligned} x+1&=0 &\quad\text{or}\quad x+6&=0\\ x&=-1 && x&=-6 \end{aligned}$$

Hence, the solutions are $x=-1$ and $x=-6$

9. The equation $x^2+7x=9x+63$ contains a power of x higher than one (it contains an x^2). Hence, this equation is *nonlinear*.

11. The highest power of x present in the equation $6x - 2 = 5x - 8$ is one. Hence, this equation is *linear*.

13. The equation $7x^2 = -2x$ contains a power of x higher than one (it contains an x^2). Hence, this equation is *nonlinear*.

15. The equation $3x^2 + 8x = -9$ contains a power of x higher than one (it contains an x^2). Hence, this equation is *nonlinear*.

17. The highest power of x present in the equation $-3x + 6 = -9$ is one. Hence, this equation is *linear*.

19. The equation $3x + 8 = 9$ contains no power of x higher than one. Hence, this equation is *linear*. Move all terms containing x to one side of the equation and all terms not containing x to the other side of the equation.

$$\begin{aligned} 3x + 8 &= 9 && \text{Original equation is linear.} \\ 3x &= 9 - 8 && \text{Subtract 8 from both sides.} \end{aligned}$$

Note that all terms containing x are now on one side of the equation, while all terms that do not contain x are on the other side of the equation.

$$\begin{aligned} 3x &= 1 && \text{Simplify} \\ x &= \frac{1}{3} && \text{Divide both sides by 3.} \end{aligned}$$

21. Because the instruction is "solve for x," and the highest power of x is larger than one, the equation $9x^2 = -x$ is *nonlinear*. Hence, the strategy requires that we move all terms to one side of the equation, making one side zero.

$$\begin{aligned} 9x^2 &= -x && \text{Original equation.} \\ 9x^2 + x &= 0 && \text{Add } x \text{ to both sides.} \end{aligned}$$

Note how we have succeeded in moving all terms to one side of the equation, making one side equal to zero. To finish the solution, we factor out the GCF on the left-hand side.

$$x(9x + 1) = 0 \qquad \text{Factor out the GCF.}$$

Note that we now have a product of two factors that equals zero. By the zero product property, either the first factor is zero or the second factor is zero.

$$x = 0 \qquad \text{or} \qquad \begin{aligned} 9x + 1 &= 0 \\ 9x &= -1 \\ x &= -\frac{1}{9} \end{aligned}$$

Hence, the solutions are $x = 0$ and $x = -1/9$.

23. The equation $3x + 9 = 8x + 7$ contains no power of x higher than one. Hence, this equation is *linear*. Move all terms containing x to one side of the equation and all terms not containing x to the other side of the equation.

$$\begin{aligned} 3x + 9 &= 8x + 7 && \text{Original equation is linear.} \\ 3x + 9 - 8x &= 7 && \text{Subtract } 8x \text{ from both sides.} \\ 3x - 8x &= 7 - 9 && \text{Subtract 9 from both sides.} \end{aligned}$$

Note that all terms containing x are now on one side of the equation, while all terms that do not contain x are on the other side of the equation.

$$\begin{aligned} -5x &= -2 && \text{Simplify} \\ x &= \frac{2}{5} && \text{Divide both sides by } -5. \end{aligned}$$

25. Because the instruction is "solve for x," and the highest power of x is larger than one, the equation $8x^2 = -2x$ is *nonlinear*. Hence, the strategy requires that we move all terms to one side of the equation, making one side zero.

$$\begin{aligned} 8x^2 &= -2x && \text{Original equation.} \\ 8x^2 + 2x &= 0 && \text{Add } 2x \text{ to both sides.} \end{aligned}$$

Note how we have succeeded in moving all terms to one side of the equation, making one side equal to zero. To finish the solution, we factor out the GCF on the left-hand side.

$$2x(4x + 1) = 0 \qquad \text{Factor out the GCF.}$$

Note that we now have a product of two factors that equals zero. By the zero product property, either the first factor is zero or the second factor is zero.

$$\begin{aligned} 2x &= 0 \\ x &= 0 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 4x + 1 &= 0 \\ 4x &= -1 \\ x &= -\frac{1}{4} \end{aligned}$$

Hence, the solutions are $x = 0$ and $x = -1/4$.

27. The equation $9x + 2 = 7$ contains no power of x higher than one. Hence, this equation is *linear*. Move all terms containing x to one side of the equation and all terms not containing x to the other side of the equation.

$$\begin{aligned} 9x + 2 &= 7 && \text{Original equation is linear.} \\ 9x &= 7 - 2 && \text{Subtract 2 from both sides.} \end{aligned}$$

Note that all terms containing x are now on one side of the equation, while all terms that do not contain x are on the other side of the equation.

$$\begin{aligned} 9x &= 5 && \text{Simplify} \\ x &= \frac{5}{9} && \text{Divide both sides by 9.} \end{aligned}$$

29. Because the instruction is "solve for x," and the highest power of x is larger than one, the equation $9x^2 = 6x$ is *nonlinear*. Hence, the strategy requires that we move all terms to one side of the equation, making one side zero.

$$\begin{aligned} 9x^2 &= 6x && \text{Original equation.} \\ 9x^2 - 6x &= 0 && \text{Subtract } 6x \text{ from both sides.} \end{aligned}$$

Note how we have succeeded in moving all terms to one side of the equation, making one side equal to zero. To finish the solution, we factor out the GCF on the left-hand side.

$$3x(3x - 2) = 0 \qquad \text{Factor out the GCF.}$$

Note that we now have a product of two factors that equals zero. By the zero product property, either the first factor is zero or the second factor is zero.

$$\begin{aligned} 3x &= 0 &\quad \text{or} \quad&& 3x - 2 &= 0 \\ x &= 0 &&& 3x &= 2 \\ &&&& x &= \frac{2}{3} \end{aligned}$$

Hence, the solutions are $x = 0$ and $x = 2/3$.

31. Because the instruction is "solve for x," and the highest power of x is larger than one, the equation $7x^2 = -4x$ is *nonlinear*. Hence, the strategy requires that we move all terms to one side of the equation, making one side zero.

$$\begin{aligned} 7x^2 &= -4x && \text{Original equation.} \\ 7x^2 + 4x &= 0 && \text{Add } 4x \text{ to both sides.} \end{aligned}$$

Note how we have succeeded in moving all terms to one side of the equation, making one side equal to zero. To finish the solution, we factor out the GCF on the left-hand side.

$$x(7x+4)=0 \qquad \text{Factor out the GCF.}$$

Note that we now have a product of two factors that equals zero. By the zero product property, either the first factor is zero or the second factor is zero.

$$\begin{aligned} x = 0 \quad \text{or} \quad 7x+4 &= 0 \\ 7x &= -4 \\ x &= -\frac{4}{7} \end{aligned}$$

Hence, the solutions are $x = 0$ and $x = -4/7$.

33. The equation $7x + 2 = 4x + 7$ contains no power of x higher than one. Hence, this equation is *linear*. Move all terms containing x to one side of the equation and all terms not containing x to the other side of the equation.

$$\begin{aligned} 7x+2 &= 4x+7 && \text{Original equation is linear.} \\ 7x+2-4x &= 7 && \text{Subtract } 4x \text{ from both sides.} \\ 7x-4x &= 7-2 && \text{Subtract 2 from both sides.} \end{aligned}$$

Note that all terms containing x are now on one side of the equation, while all terms that do not contain x are on the other side of the equation.

$$\begin{aligned} 3x &= 5 && \text{Simplify} \\ x &= \frac{5}{3} && \text{Divide both sides by 3.} \end{aligned}$$

35. The equation $63x^2 + 56x + 54x + 48 = 0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor $7x$ out of the first two terms and 6 out of the second two terms.

$$\begin{aligned} 63x^2+56x+54x+48 &= 0 \\ 7x\,(9x+8)+6\,(9x+8) &= 0 \end{aligned}$$

Note that we can now factor $9x + 8$ out of both of these terms.

$$(7x+6)(9x+8)=0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} 7x+6&=0 &\quad\text{or}\quad&& 9x+8&=0\\ 7x&=-6 &&& 9x&=-8\\ x&=-\frac{6}{7} &&& x&=-\frac{8}{9} \end{aligned}$$

Hence, the solutions are $x=-6/7$ and $x=-8/9$.

37. The equation $16x^2-18x+40x-45=0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor $2x$ out of the first two terms and 5 out of the second two terms.

$$\begin{aligned} 16x^2-18x+40x-45&=0\\ 2x\,(8x-9)+5\,(8x-9)&=0 \end{aligned}$$

Note that we can now factor $8x-9$ out of both of these terms.

$$(2x+5)(8x-9)=0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} 2x+5&=0 &\quad\text{or}\quad&& 8x-9&=0\\ 2x&=-5 &&& 8x&=9\\ x&=-\frac{5}{2} &&& x&=\frac{9}{8} \end{aligned}$$

Hence, the solutions are $x=-5/2$ and $x=9/8$.

39. The equation $45x^2+18x+20x+8=0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor $9x$ out of the first two terms and 4 out of the second two terms.

$$\begin{aligned} 45x^2+18x+20x+8&=0\\ 9x\,(5x+2)+4\,(5x+2)&=0 \end{aligned}$$

Note that we can now factor $5x+2$ out of both of these terms.

$$(9x+4)(5x+2)=0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} 9x+4&=0 &\quad\text{or}\quad&& 5x+2&=0\\ 9x&=-4 &&& 5x&=-2\\ x&=-\frac{4}{9} &&& x&=-\frac{2}{5}\end{aligned}$$

Hence, the solutions are $x=-4/9$ and $x=-2/5$.

41. The equation $x^2+10x+4x+40=0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor x out of the first two terms and 4 out of the second two terms.

$$\begin{aligned} x^2+10x+4x+40&=0\\ x(x+10)+4(x+10)&=0\end{aligned}$$

Note that we can now factor $x+10$ out of both of these terms.

$$(x+4)(x+10)=0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} x+4&=0 &\quad\text{or}\quad&& x+10&=0\\ x&=-4 &&& x&=-10\end{aligned}$$

Hence, the solutions are $x=-4$ and $x=-10$.

43. The equation $x^2+6x-11x-66=0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor x out of the first two terms and -11 out of the second two terms.

$$\begin{aligned} x^2 + 6x - 11x - 66 &= 0 \\ x\,(x+6) - 11\,(x+6) &= 0 \end{aligned}$$

Note that we can now factor $x + 6$ out of both of these terms.

$$(x - 11)(x + 6) = 0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} x - 11 &= 0 & \quad\text{or}\quad && x + 6 &= 0 \\ x &= 11 & && x &= -6 \end{aligned}$$

Hence, the solutions are $x = 11$ and $x = -6$.

45. The equation $15x^2 - 24x + 35x - 56 = 0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor $3x$ out of the first two terms and 7 out of the second two terms.

$$\begin{aligned} 15x^2 - 24x + 35x - 56 &= 0 \\ 3x\,(5x-8) + 7\,(5x-8) &= 0 \end{aligned}$$

Note that we can now factor $5x - 8$ out of both of these terms.

$$(3x + 7)(5x - 8) = 0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} 3x + 7 &= 0 & \quad\text{or}\quad && 5x - 8 &= 0 \\ 3x &= -7 & && 5x &= 8 \\ x &= -\frac{7}{3} & && x &= \frac{8}{5} \end{aligned}$$

Hence, the solutions are $x = -7/3$ and $x = 8/5$.

47. The equation $x^2+2x+9x+18=0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor x out of the first two terms and 9 out of the second two terms.

$$\begin{aligned} x^2+2x+9x+18&=0 \\ x\,(x+2)+9\,(x+2)&=0 \end{aligned}$$

Note that we can now factor $x+2$ out of both of these terms.

$$(x+9)(x+2)=0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} x+9&=0 &\quad\text{or}\quad&& x+2&=0 \\ x&=-9 &&& x&=-2 \end{aligned}$$

Hence, the solutions are $x=-9$ and $x=-2$.

49. The equation $x^2+4x-8x-32=0$ contains a power of x higher than one. Hence, this equation is *nonlinear* so we must start by moving all terms to one side of the equation, making one side equal to zero. However, this is already done, so let's proceed by factoring by grouping. We can factor x out of the first two terms and -8 out of the second two terms.

$$\begin{aligned} x^2+4x-8x-32&=0 \\ x\,(x+4)-8\,(x+4)&=0 \end{aligned}$$

Note that we can now factor $x+4$ out of both of these terms.

$$(x-8)(x+4)=0$$

Use the zero product property to set each factor equal to zero. Solve the resulting equations for x.

$$\begin{aligned} x-8&=0 &\quad\text{or}\quad&& x+4&=0 \\ x&=8 &&& x&=-4 \end{aligned}$$

Hence, the solutions are $x=8$ and $x=-4$.

51. Algebraic solution. The equation is nonlinear, so make one side zero by moving all terms containing x to one side of the equation.

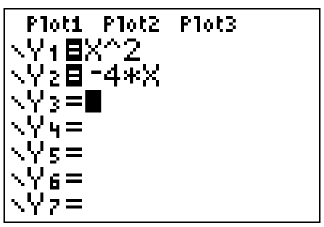
$$x^2 = -4x \qquad \text{The equation is nonlinear, so make one side zero.}$$
$$x^2 + 4x = 0 \qquad \text{Add } 4x \text{ to both sides.}$$
$$x(x+4) = 0 \qquad \text{Factor out GCF.}$$

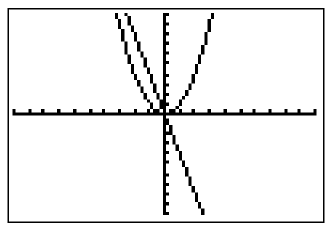
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -4$$

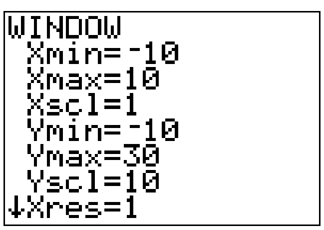
Hence, the solutions are $x = 0$ and $x = -4$.

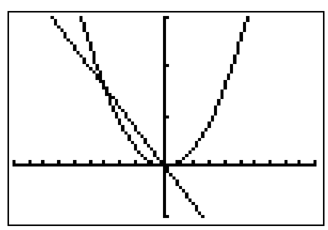
Calculator solution. Load each side of the equation $x^2 = -4x$ into **Y1** and **Y2** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



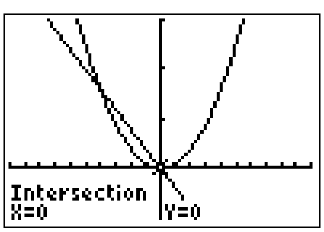


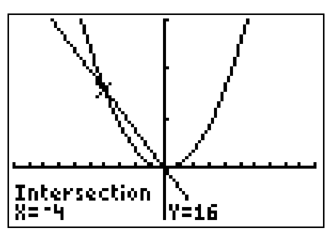
We need both points of intersection to be visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



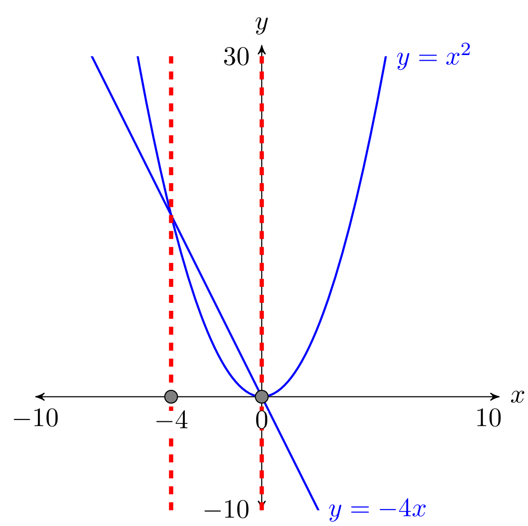


Next, use the **5:intersect** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 = -4x$ are $x = -4$ and $x = 0$. Note how these agree with the algebraic solution.

53. Algebraic solution. The equation is nonlinear, so make one side zero by moving all terms containing x to one side of the equation.

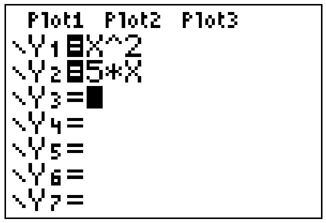
$x^2 = 5x$	The equation is nonlinear, so make one side zero.
$x^2 - 5x = 0$	Subtract $5x$ from both sides.
$x(x - 5) = 0$	Factor out GCF.

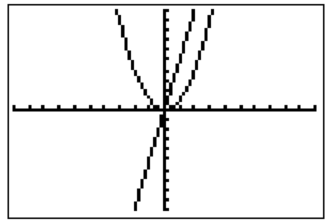
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x = 0 \quad \text{or} \quad x - 5 = 0$$
$$x = 5$$

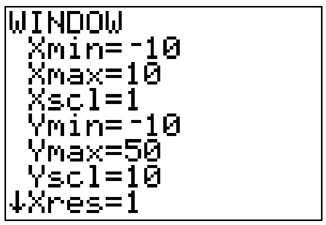
Hence, the solutions are $x = 0$ and $x = 5$.

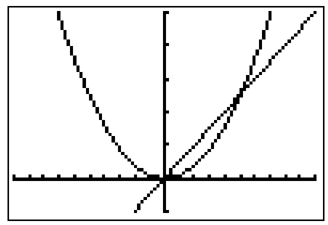
Calculator solution. Load each side of the equation $x^2 = 5x$ into **Y1** and **Y2** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



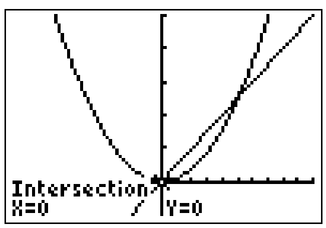


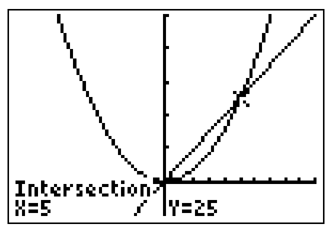
We need both points of intersection to be visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



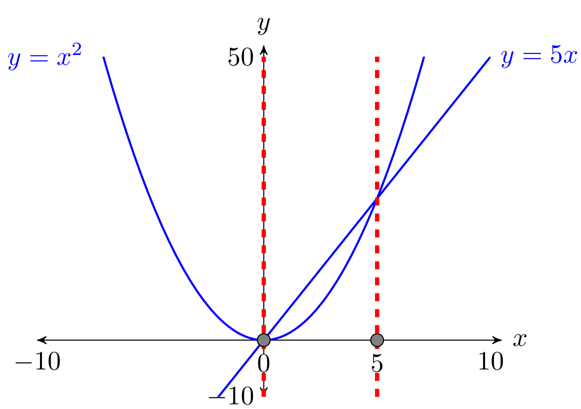


Next, use the **5:intersect** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 = 5x$ are $x = 0$ and $x = 5$. Note how these agree with the algebraic solution.

55. Algebraic solution. The equation is nonlinear, so make one side zero. This is already done, so factor out the GCF.

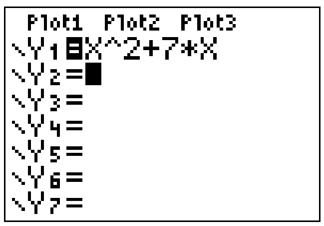
$$\begin{aligned} x^2 + 7x &= 0 && \text{Original equation is nonlinear.} \\ x(x+7) &= 0 && \text{Factor out GCF.} \end{aligned}$$

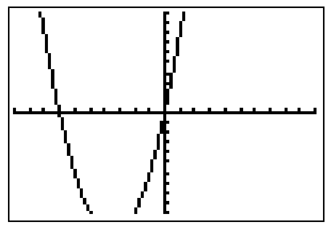
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x = 0 \quad \text{or} \quad \begin{aligned} x + 7 &= 0 \\ x &= -7 \end{aligned}$$

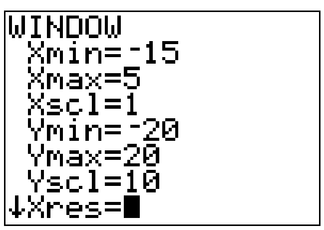
Hence, the solutions are $x = 0$ and $x = -7$.

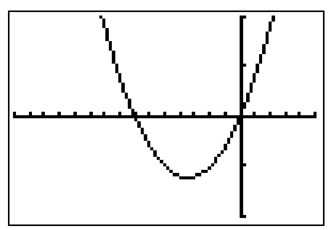
Calculator solution. Load the left-hand side of the equation $x^2 + 7x = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



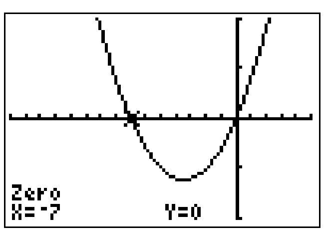


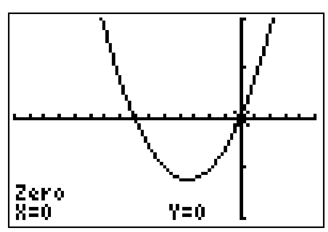
Both x-intercepts are visible, but we really should adjust the viewing window so that the vertex of the parabola is visible in the viewing window as well. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



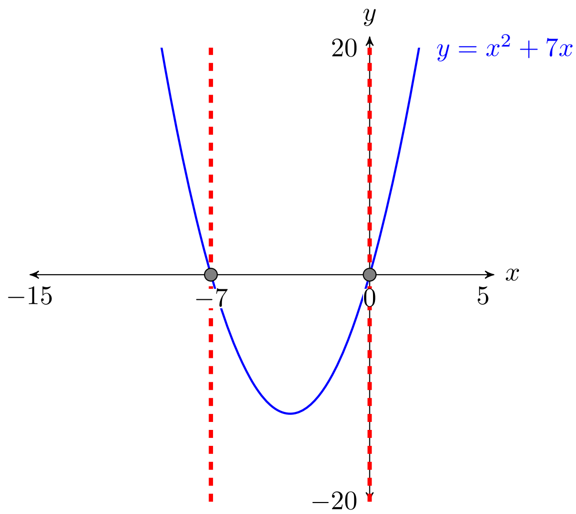


Next, use the **2:zero** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 + 7x = 0$ are $x = -7$ and $x = 0$. Note how these agree with the algebraic solution.

57. Algebraic solution. The equation is nonlinear, so make one side zero. This is already done, so factor out the GCF.

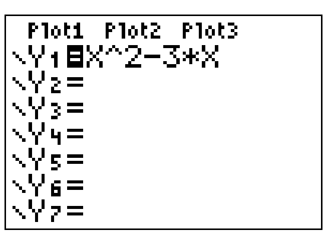
$$\begin{aligned} x^2 - 3x &= 0 && \text{Original equation is nonlinear.} \\ x(x-3) &= 0 && \text{Factor out GCF.} \end{aligned}$$

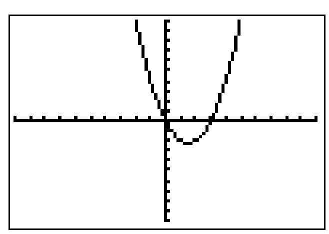
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x = 0 \quad \text{or} \quad \begin{aligned} x - 3 &= 0 \\ x &= 3 \end{aligned}$$

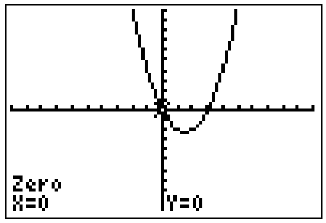
Hence, the solutions are $x = 0$ and $x = 3$.

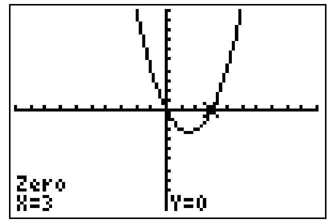
Calculator solution. Load the left-hand side of the equation $x^2 - 3x = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



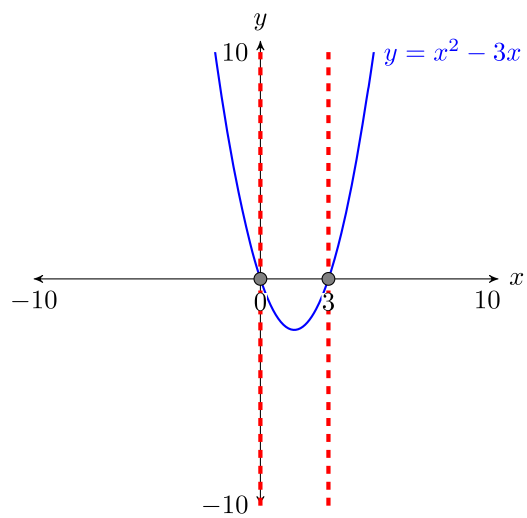


Next, use the **2:zero** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 - 3x = 0$ are $x = 0$ and $x = 3$. Note how these agree with the algebraic solution.

6.3 Factoring Trinomials I

1. We proceed as follows:

i) Compare $x^2 + 7x - 18$ with $ax^2 + bx + c$ and identify $a = 1$, $b = 7$, and $c = -18$. Note that the leading coefficient is $a = 1$.

ii) Calculate ac. Note that $ac = (1)(-18)$, so $ac = -18$.

iii) List all integer pairs whose product is $ac = -18$.

$$\begin{array}{ll} 1, -18 & -1, 18 \\ 2, -9 & -2, 9 \\ 3, -6 & -3, 6 \end{array}$$

iv) Circle the ordered pair whose sum is $b = 7$.

$$\begin{array}{ll} 1, -18 & -1, 18 \\ 2, -9 & \boxed{-2, 9} \\ 3, -6 & -3, 6 \end{array}$$

v) Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 + 7x - 18 &= x^2 - 2x + 9x - 18 \\ &= x(x - 2) + 9(x - 2) \\ &= (x + 9)(x - 2) \end{aligned}$$

3. We proceed as follows:

i) Compare $x^2 - 10x + 9$ with $ax^2 + bx + c$ and identify $a = 1$, $b = -10$, and $c = 9$. Note that the leading coefficient is $a = 1$.

ii) Calculate ac. Note that $ac = (1)(9)$, so $ac = 9$.

iii) List all integer pairs whose product is $ac = 9$.

$$\begin{array}{ll} 1, 9 & -1, -9 \\ 3, 3 & -3, -3 \end{array}$$

iv) Circle the ordered pair whose sum is $b = -10$.

$$\begin{array}{ll} 1, 9 & \boxed{-1, -9} \\ 3, 3 & -3, -3 \end{array}$$

v) Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 10x + 9 &= x^2 - x - 9x + 9 \\ &= x(x - 1) - 9(x - 1) \\ &= (x - 9)(x - 1) \end{aligned}$$

5. We proceed as follows:

i) Compare $x^2 + 14x + 45$ with $ax^2 + bx + c$ and identify $a = 1$, $b = 14$, and $c = 45$. Note that the leading coefficient is $a = 1$.

ii) Calculate ac. Note that $ac = (1)(45)$, so $ac = 45$.

iii) List all integer pairs whose product is $ac = 45$.

$$\begin{array}{ll} 1, 45 & -1, -45 \\ 3, 15 & -3, -15 \\ 5, 9 & -5, -9 \end{array}$$

iv) Circle the ordered pair whose sum is $b = 14$.

$$\begin{array}{cc} 1, 45 & -1, -45 \\ 3, 15 & -3, -15 \\ \boxed{5, 9} & -5, -9 \end{array}$$

v) Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 + 14x + 45 &= x^2 + 5x + 9x + 45 \\ &= x(x+5) + 9(x+5) \\ &= (x+9)(x+5) \end{aligned}$$

7. Compare $x^2 - 16x + 39$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -16$, and $c = 39$. Calculate $ac = (1)(39)$, so $ac = 39$. Start listing the integer pairs whose product is $ac = 39$, but be mindful that you need an integer pair whose sum is $b = -16$.

$$\begin{array}{cc} 1, 39 & -1, -39 \\ 3, 13 & \boxed{-3, -13} \end{array}$$

Note how we ceased listing ordered pairs the moment we found the pair we needed. Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 16x + 39 &= x^2 - 3x - 13x + 39 \\ &= x(x-3) - 13(x-3) \\ &= (x-13)(x-3) \end{aligned}$$

9. Compare $x^2 - 26x + 69$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -26$, and $c = 69$. Calculate $ac = (1)(69)$, so $ac = 69$. Start listing the integer pairs whose product is $ac = 69$, but be mindful that you need an integer pair whose sum is $b = -26$.

$$\begin{array}{cc} 1, 69 & -1, -69 \\ 3, 23 & \boxed{-3, -23} \end{array}$$

Note how we ceased listing ordered pairs the moment we found the pair we needed. Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 26x + 69 &= x^2 - 3x - 23x + 69 \\ &= x(x-3) - 23(x-3) \\ &= (x-23)(x-3) \end{aligned}$$

11. Compare $x^2 - 25x + 84$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -25$, and $c = 84$. Calculate $ac = (1)(84)$, so $ac = 84$. Start listing the integer pairs whose product is $ac = 84$, but be mindful that you need an integer pair whose sum is $b = -25$.

$$\begin{array}{ll} 1, 84 & -1, -84 \\ 2, 42 & -2, -42 \\ 3, 28 & -3, -28 \\ 4, 21 & \boxed{-4, -21} \end{array}$$

Note how we ceased listing ordered pairs the moment we found the pair we needed. Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 25x + 84 &= x^2 - 4x - 21x + 84 \\ &= x(x-4) - 21(x-4) \\ &= (x-21)(x-4) \end{aligned}$$

13. Compare $x^2 - 13x + 36$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -13$, and $c = 36$. Calculate $ac = (1)(36)$, so $ac = 36$. Now can you think of an integer pair whose product is $ac = 36$ and whose sum is $b = -13$? For some, the pair just pops into their head: -4 and -9. "Drop" the pair in place and you are done.

$$x^2 - 13x + 36 = (x-4)(x-9)$$

15. Compare $x^2 + 10x + 21$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 10$, and $c = 21$. Calculate $ac = (1)(21)$, so $ac = 21$. Now can you think of an integer pair whose product is $ac = 21$ and whose sum is $b = 10$? For some, the pair just pops into their head: 3 and 7. "Drop" the pair in place and you are done.

$$x^2 + 10x + 21 = (x+3)(x+7)$$

17. Compare $x^2 - 4x - 5$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -4$, and $c = -5$. Calculate $ac = (1)(-5)$, so $ac = -5$. Now can you think of an integer pair whose product is $ac = -5$ and whose sum is $b = -4$? For some, the pair just pops into their head: 1 and -5. "Drop" the pair in place and you are done.

$$x^2 - 4x - 5 = (x+1)(x-5)$$

19. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} x^2 &= -7x + 30 && \text{Original equation.} \\ x^2 + 7x &= 30 && \text{Add } 7x \text{ to both sides.} \\ x^2 + 7x - 30 &= 0 && \text{Subtract 30 from both sides.} \end{aligned}$$

Compare $x^2+7x-30$ with ax^2+bx+c and note that $a=1$, $b=7$ and $c=-30$. Note that the leading coefficient is a 1. Calculate $ac=(1)(-30)$ and list all integer pairs whose product is $ac=-30$.

$$\begin{array}{ll} 1,-30 & -1,30 \\ 2,-15 & -2,15 \\ 3,-10 & -3,10 \\ 5,-6 & -5,6 \end{array}$$

Circle the ordered pair whose sum is $b=7$.

$$\begin{array}{ll} 1,-30 & -1,30 \\ 2,-15 & -2,15 \\ 3,-10 & \boxed{-3,10} \\ 5,-6 & -5,6 \end{array}$$

Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 3x + 10x - 30 &= 0 \\ x(x-3) + 10(x-3) &= 0 \\ (x-3)(x+10) &= 0 \end{aligned}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x - 3 &= 0 & \quad\text{or}\quad && x + 10 &= 0 \\ x &= 3 & && x &= -10 \end{aligned}$$

Thus, the solutions of $x^2+7x-30$ are $x=3$ and $x=-10$.

21. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} x^2 &= -11x - 10 && \text{Original equation.} \\ x^2 + 11x &= -10 && \text{Add } 11x \text{ to both sides.} \\ x^2 + 11x + 10 &= 0 && \text{Add 10 to both sides.} \end{aligned}$$

Compare $x^2 + 11x + 10$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 11$ and $c = 10$. Note that the leading coefficient is a 1. Calculate $ac = (1)(10)$ and list all integer pairs whose product is $ac = 10$.

$$\begin{array}{ll} 1, 10 & -1, -10 \\ 2, 5 & -2, -5 \end{array}$$

Circle the ordered pair whose sum is $b = 11$.

$$\begin{array}{ll} \boxed{1, 10} & -1, -10 \\ 2, 5 & -2, -5 \end{array}$$

Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 + x + 10x + 10 &= 0 \\ x(x + 1) + 10(x + 1) &= 0 \\ (x + 1)(x + 10) &= 0 \end{aligned}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x + 1 &= 0 & \quad \text{or} \quad && x + 10 &= 0 \\ x &= -1 & && x &= -10 \end{aligned}$$

Thus, the solutions of $x^2 + 11x + 10$ are $x = -1$ and $x = -10$.

23. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} x^2 &= -15x - 50 && \text{Original equation.} \\ x^2 + 15x &= -50 && \text{Add } 15x \text{ to both sides.} \\ x^2 + 15x + 50 &= 0 && \text{Add 50 to both sides.} \end{aligned}$$

Compare $x^2 + 15x + 50$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 15$ and $c = 50$. Note that the leading coefficient is a 1. Calculate $ac = (1)(50)$ and list all integer pairs whose product is $ac = 50$.

$$\begin{array}{ll} 1, 50 & -1, -50 \\ 2, 25 & -2, -25 \\ 5, 10 & -5, -10 \end{array}$$

Circle the ordered pair whose sum is $b = 15$.

$$\begin{array}{ll} 1, 50 & -1, -50 \\ 2, 25 & -2, -25 \\ \boxed{5, 10} & -5, -10 \end{array}$$

Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 + 5x + 10x + 50 &= 0 \\ x(x+5) + 10(x+5) &= 0 \\ (x+5)(x+10) &= 0 \end{aligned}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x + 5 &= 0 &\quad \text{or} \quad && x + 10 &= 0 \\ x &= -5 &&& x &= -10 \end{aligned}$$

Thus, the solutions of $x^2 + 15x + 50$ are $x = -5$ and $x = -10$.

25. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 60 &= x^2 + 11x && \text{Original equation.} \\ 0 &= x^2 + 11x - 60 && \text{Subtract 60 from both sides.} \end{aligned}$$

Compare $x^2 + 11x - 60$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 11$ and $c = -60$. Note that the leading coefficient is a 1. Calculate $ac = (1)(-60)$ and begin listing all integer pairs whose product is $ac = -60$. Stop listing integer pairs when you find and circle a pair whose sum is $b = 11$.

$$\begin{array}{cc} 1, -60 & -1, 60 \\ 2, -30 & -2, 30 \\ 3, -20 & -3, 20 \\ 4, -15 & \boxed{-4, 15} \end{array}$$

Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 4x + 15x - 60 &= 0 \\ x(x-4) + 15(x-4) &= 0 \\ (x-4)(x+15) &= 0 \end{aligned}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x - 4 &= 0 &\quad \text{or} \quad && x + 15 &= 0 \\ x &= 4 &&& x &= -15 \end{aligned}$$

Thus, the solutions of $60 = x^2 + 11x$ are $x = 4$ and $x = -15$.

27. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} -11 &= x^2 - 12x && \text{Original equation.} \\ 0 &= x^2 - 12x + 11 && \text{Add 11 to both sides.} \end{aligned}$$

Compare $x^2 - 12x + 11$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -12$ and $c = 11$. Note that the leading coefficient is a 1. Calculate $ac = (1)(11)$ and begin listing all integer pairs whose product is $ac = 11$. Stop listing integer pairs when you find and circle a pair whose sum is $b = -12$.

$$1, 11 \qquad \boxed{-1, -11}$$

Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - x - 11x + 11 &= 0 \\ x(x-1) - 11(x-1) &= 0 \\ (x-1)(x-11) &= 0 \end{aligned}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x - 1 &= 0 &\quad \text{or} \quad&& x - 11 &= 0 \\ x &= 1 &&& x &= 11 \end{aligned}$$

Thus, the solutions of $-11 = x^2 - 12x$ are $x = 1$ and $x = 11$.

29. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 56 &= x^2 + 10x && \text{Original equation.} \\ 0 &= x^2 + 10x - 56 && \text{Subtract 56 from both sides.} \end{aligned}$$

Compare $x^2 + 10x - 56$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 10$ and $c = -56$. Note that the leading coefficient is a 1. Calculate $ac = (1)(-56)$ and begin listing all integer pairs whose product is $ac = -56$. Stop listing integer pairs when you find and circle a pair whose sum is $b = 10$.

$$\begin{array}{ll} 1, -56 & -1, 56 \\ 2, -28 & -2, 28 \\ 4, -14 & \boxed{-4, 14} \end{array}$$

Write the middle term as a sum of like terms using the boxed ordered pair. Then factor by grouping.

$$\begin{aligned} x^2 - 4x + 14x - 56 &= 0 \\ x(x-4) + 14(x-4) &= 0 \\ (x-4)(x+14) &= 0 \end{aligned}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x - 4 &= 0 & \quad\text{or}\quad && x + 14 &= 0 \\ x &= 4 & && x &= -14 \end{aligned}$$

Thus, the solutions of $56 = x^2 + 10x$ are $x = 4$ and $x = -14$.

31. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} x^2 + 20 &= -12x && \text{Original equation.} \\ x^2 + 12x + 20 &= 0 && \text{Add } 12x \text{ to both sides.} \end{aligned}$$

Compare $x^2 + 12x + 20$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 12$, and $c = 20$. Calculate $ac = (1)(20)$, so $ac = 20$. We need an integer pair whose product is $ac = 20$ and whose sum is $b = 12$. The integer pair 2 and 10 comes to mind. Drop this ordered pair in place.

$$(x+2)(x+10) = 0 \qquad \text{Factor.}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x + 2 &= 0 & \quad\text{or}\quad && x + 10 &= 0 \\ x &= -2 & && x &= -10 \end{aligned}$$

Thus, the solutions of $x^2 + 20 = -12x$ are $x = -2$ and $x = -10$.

33. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} x^2 - 36 &= 9x && \text{Original equation.} \\ x^2 - 9x - 36 &= 0 && \text{Subtract } 9x \text{ from both sides.} \end{aligned}$$

Compare $x^2 - 9x - 36$ with $ax^2 + bx + c$ and note that $a = 1$, $b = -9$, and $c = -36$. Calculate $ac = (1)(-36)$, so $ac = -36$. We need an integer pair whose product is $ac = -36$ and whose sum is $b = -9$. The integer pair 3 and -12 comes to mind. Drop this ordered pair in place.

$$(x+3)(x-12) = 0 \qquad \text{Factor.}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x + 3 &= 0 & \quad \text{or} \quad && x - 12 &= 0 \\ x &= -3 &&& x &= 12 \end{aligned}$$

Thus, the solutions of $x^2 - 36 = 9x$ are $x = -3$ and $x = 12$.

35. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} x^2 + 8 &= -6x && \text{Original equation.} \\ x^2 + 6x + 8 &= 0 && \text{Add } 6x \text{ to both sides.} \end{aligned}$$

Compare $x^2 + 6x + 8$ with $ax^2 + bx + c$ and note that $a = 1$, $b = 6$, and $c = 8$. Calculate $ac = (1)(8)$, so $ac = 8$. We need an integer pair whose product is $ac = 8$ and whose sum is $b = 6$. The integer pair 2 and 4 comes to mind. Drop this ordered pair in place.

$$(x+2)(x+4) = 0 \qquad \text{Factor.}$$

We have a product that equals zero. The zero product property tells us that at least one of the factors is zero. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x + 2 &= 0 & \quad \text{or} \quad && x + 4 &= 0 \\ x &= -2 &&& x &= -4 \end{aligned}$$

Thus, the solutions of $x^2 + 8 = -6x$ are $x = -2$ and $x = -4$.

37. Algebraic solution. The equation is nonlinear, so make one side zero by moving all terms containing x to one side of the equation.

$$\begin{aligned} x^2 &= x + 12 && \text{The equation is nonlinear, so make one side zero.} \\ x^2 - x &= 12 && \text{Subtract } x \text{ from both sides.} \\ x^2 - x - 12 &= 0 && \text{Subtract 12 from both sides.} \end{aligned}$$

Note that $ac = (1)(-12) = -12$. The integer pair 3 and -4 gives a product of -12 and sums to -1, the coefficient of x. Hence, we can "drop these numbers in place" to factor.

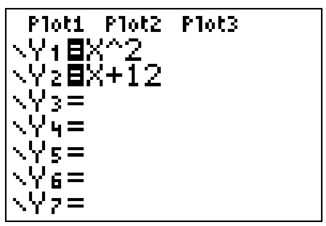
$$(x+3)(x-4) = 0 \qquad \text{Factor.}$$

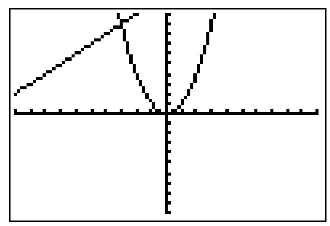
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x + 3 = 0 \quad \text{or} \quad x - 4 = 0$$
$$x = -3 \qquad\qquad x = -4$$

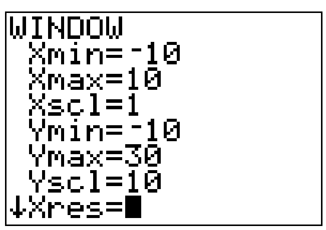
Hence, the solutions are $x = -3$ and $x = 4$.

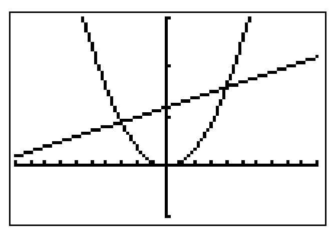
Calculator solution. Load each side of the equation $x^2 = x + 12$ into **Y1** and **Y2** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



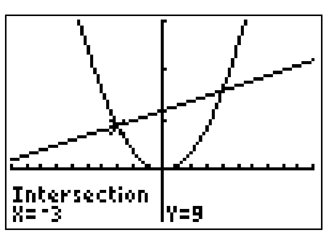


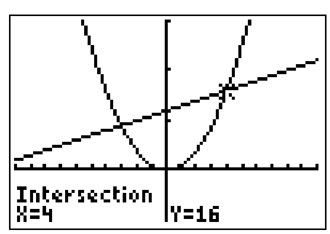
We need both points of intersection to be visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



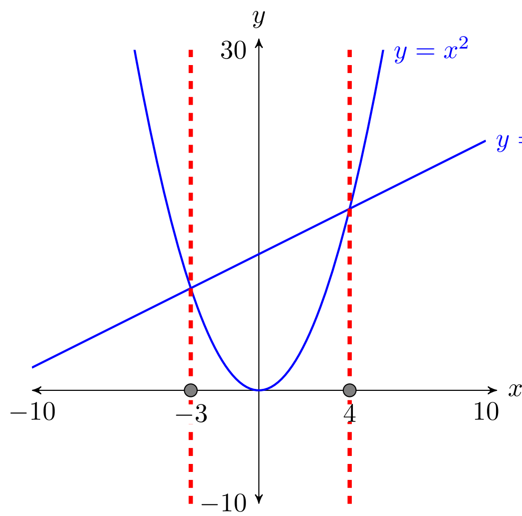


Next, use the **5:intersect** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 = x + 12$ are $x = -3$ and $x = 4$. Note how these agree with the algebraic solution.

39. Algebraic solution. The equation is nonlinear, so make one side zero by moving all terms containing x to one side of the equation.

$x^2 + 12 = 8x$	The equation is nonlinear, so make one side zero.
$x^2 - 8x + 12 = 0$	Subtract $8x$ from both sides.

Note that $ac = (1)(12) = 12$. The integer pair -6 and -2 gives a product of 12 and sums to -8, the coefficient of x. Hence, we can "drop these numbers in place" to factor.

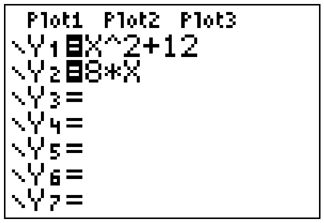
$(x - 6)(x - 2) = 0$	Factor.

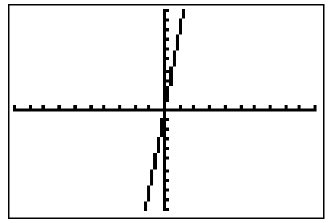
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x - 6 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = 6 \qquad\qquad x = 2$$

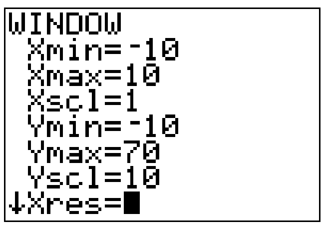
Hence, the solutions are $x = 6$ and $x = 2$.

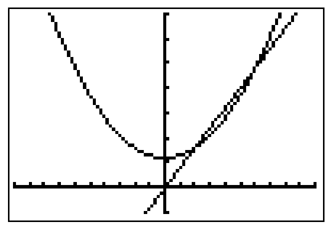
Calculator solution. Load each side of the equation $x^2 + 12 = 8x$ into **Y1** and **Y2** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



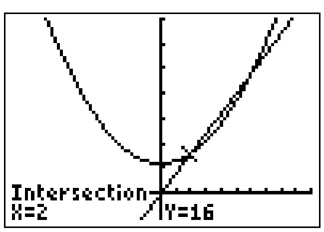


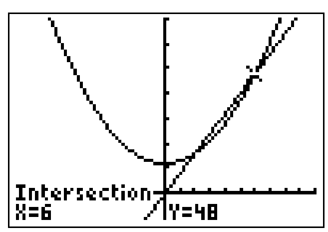
We need both points of intersection to be visible in the viewing window. We know the graph of $y = x^2 + 12$ is a parabola and if we substitute $x = 0$, we get $y = 12$, putting the y-intercept at $(0, 12)$, so we need to elevate the top of the window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



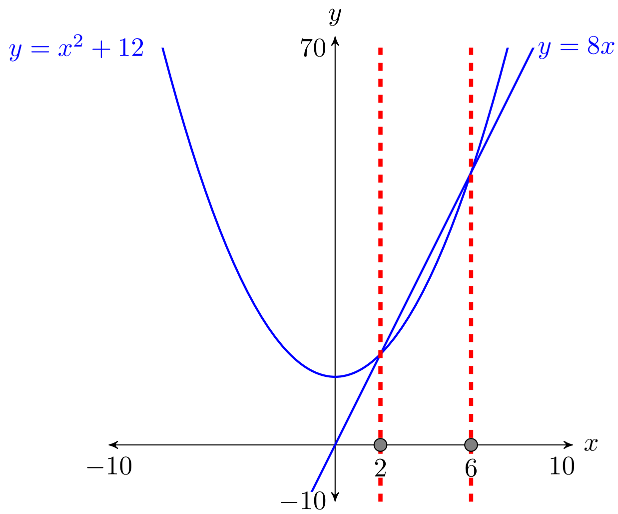


Next, use the **5:intersect** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 + 12 = 8x$ are $x = 2$ and $x = 6$. Note how these agree with the algebraic solution.

41. Algebraic solution. The equation is nonlinear, so make one side zero. This is already done, so use the ac-method to factor. Note that $ac = (1)(-16) = -16$ and the integer pair -8 and 2 has product -16 and adds to -6, the coefficient of x. Hence, we can "drop" this pair in place to factor.

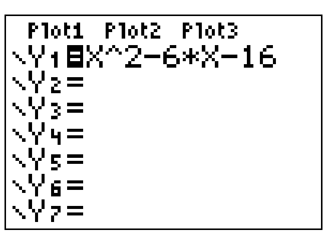
$$\begin{aligned} x^2 - 6x - 16 &= 0 \\ (x - 8)(x + 2) &= 0 \end{aligned}$$

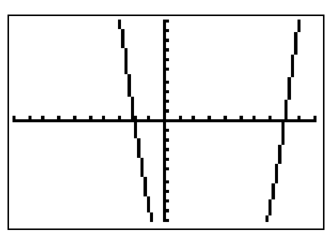
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$\begin{aligned} x - 8 &= 0 & \quad \text{or} \quad && x + 2 &= 0 \\ x &= 8 & && x &= -2 \end{aligned}$$

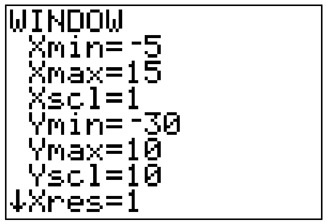
Hence, the solutions are $x = 8$ and $x = -2$.

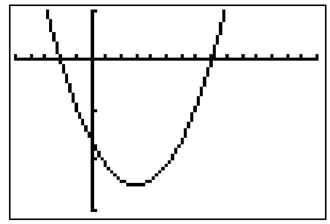
Calculator solution. Load the left-hand side of the equation $x^2 - 6x - 16 = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



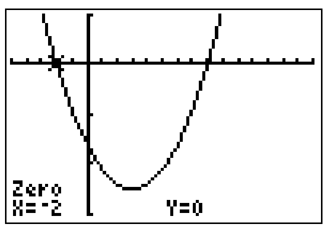


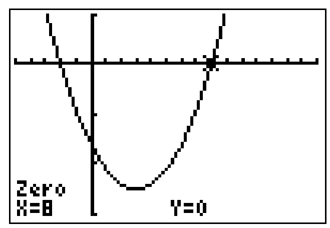
Both x-intercepts are visible, but we really should adjust the viewing window so that the vertex of the parabola is visible in the viewing window as well. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



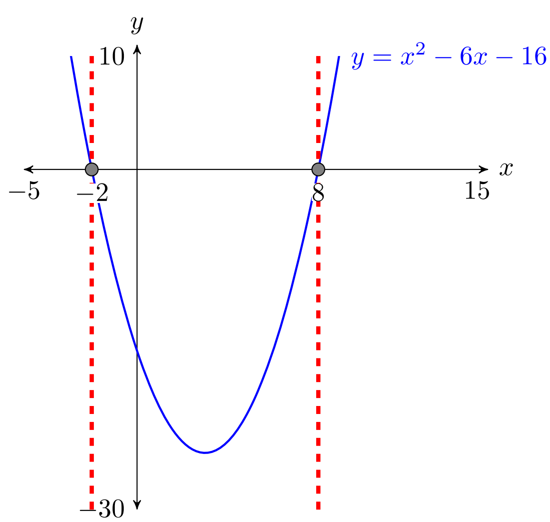


Next, use the **2:zero** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 - 6x - 16 = 0$ are $x = -2$ and $x = 8$. Note how these agree with the algebraic solution.

43. Algebraic solution. The equation is nonlinear, so make one side zero. This is already done, so use the ac-method to factor. Note that $ac = (1)(-24) = -24$ and the integer pair 12 and -2 has product -24 and adds to 10, the coefficient of x. Hence, we can "drop" this pair in place to factor.

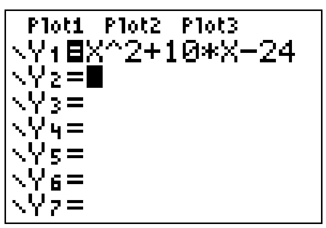
$$\begin{aligned} x^2 + 10x - 24 &= 0 \\ (x+12)(x-2) &= 0 \end{aligned}$$

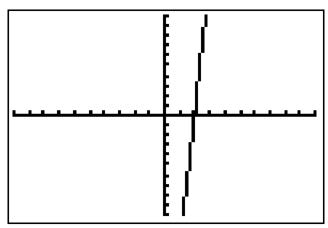
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$\begin{aligned} x + 12 &= 0 & \quad\text{or}\quad && x - 2 &= 0 \\ x &= -12 & && x &= 2 \end{aligned}$$

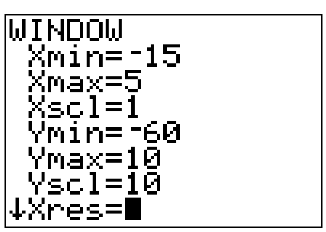
Hence, the solutions are $x = -12$ and $x = 2$.

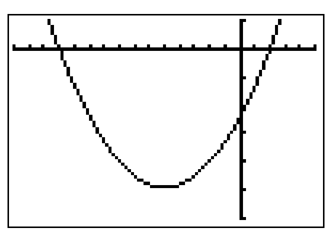
Calculator solution. Load the left-hand side of the equation $x^2+10x-24 = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



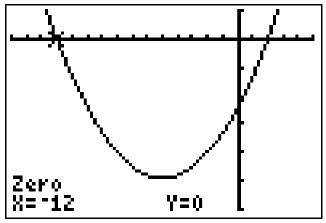


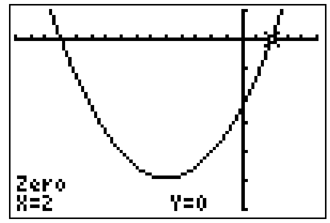
We know that $y = x^2 + 10x - 24 = 0$ is a parabola that opens upward, but only one x-intercept is visible. We need to move the viewing window a bit to the left to see the second x-intercept, and also move it downward so that the vertex of the parabola is visible in the viewing window as well. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



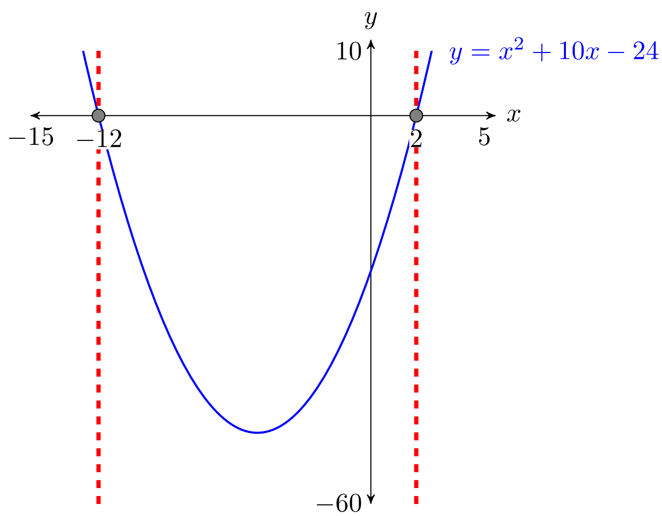


Next, use the **2:zero** utility from the CALC menu to find the points of intersection.





Report the results on your homework as follows.



Hence, the solutions of $x^2 + 10x - 24 = 0$ are $x = -12$ and $x = 2$. Note how these agree with the algebraic solution.

6.4 Factoring Trinomials II

1. We proceed as follows:

i) Compare $6x^2 + 13x - 5$ with $ax^2 + bx + c$ and identify $a = 6$, $b = 13$, and $c = -5$. Note that the leading coefficient is $a = 6$, so this case is **not** a "drop in place" situation.

ii) Calculate ac. Note that $ac = (6)(-5)$, so $ac = -30$.

iii) List all integer pairs whose product is $ac = -30$.

$$\begin{array}{ll} 1, -30 & -1, 30 \\ 2, -15 & -2, 15 \\ 3, -10 & -3, 10 \\ 5, -6 & -5, 6 \end{array}$$

iv) Circle the ordered pair whose sum is $b = 13$.

$$\begin{array}{ll} 1, -30 & -1, 30 \\ 2, -15 & \boxed{-2, 15} \\ 3, -10 & -3, 10 \\ 5, -6 & -5, 6 \end{array}$$

v) If we try to "drop in place", $(x - 2)(x + 15) \neq 6x^2 + 13x - 5$. Right off the bat, the product of the terms in the "First" position does not equal $6x^2$. Instead, we must break up the middle term of $6x^2 + 13x - 5$ into a sum of like terms using our circled pair of integers -2 and 15.

$$6x^2 + 13x - 5 = 6x^2 - 2x + 15x - 5$$

Now we factor by grouping. Factor $2x$ out of the first two terms and 5 out of the second two terms.

$$= 2x\,(3x - 1) + 5\,(3x - 1)$$

Now we can factor out $(3x - 1)$.

$$= (2x + 5)(3x - 1)$$

3. We proceed as follows:

i) Compare $4x^2 - x - 3$ with $ax^2 + bx + c$ and identify $a = 4$, $b = -1$, and $c = -3$. Note that the leading coefficient is $a = 4$, so this case is **not** a "drop in place" situation.

ii) Calculate ac. Note that $ac = (4)(-3)$, so $ac = -12$.

iii) List all integer pairs whose product is $ac = -12$.

$$\begin{array}{ll} 1, -12 & -1, 12 \\ 2, -6 & -2, 6 \\ 3, -4 & -3, 4 \end{array}$$

iv) Circle the ordered pair whose sum is $b = -1$.

$$\begin{array}{ll} 1, -12 & -1, 12 \\ 2, -6 & -2, 6 \\ \boxed{3, -4} & -3, 4 \end{array}$$

v) If we try to "drop in place", $(x + 3)(x - 4) \neq 4x^2 - x - 3$. Right off the bat, the product of the terms in the "First" position does not equal $4x^2$.

Instead, we must break up the middle term of $4x^2 - x - 3$ into a sum of like terms using our circled pair of integers 3 and -4.

$$4x^2 - x - 3 = 4x^2 + 3x - 4x - 3$$

Now we factor by grouping. Factor x out of the first two terms and -1 out of the second two terms.

$$= x\,(4x+3) - 1\,(4x+3)$$

Now we can factor out $(4x+3)$.

$$= (x-1)(4x+3)$$

5. We proceed as follows:

i) Compare $3x^2 + 19x + 28$ with $ax^2 + bx + c$ and identify $a = 3$, $b = 19$, and $c = 28$. Note that the leading coefficient is $a = 3$, so this case is **not** a "drop in place" situation.

ii) Calculate ac. Note that $ac = (3)(28)$, so $ac = 84$.

iii) List all integer pairs whose product is $ac = 84$.

$1, 84$	$-1, -84$
$2, 42$	$-2, -42$
$3, 28$	$-3, -28$
$4, 21$	$-4, -21$
$6, 14$	$-6, -14$
$7, 12$	$-7, -12$

iv) Circle the ordered pair whose sum is $b = 19$.

$1, 84$	$-1, -84$
$2, 42$	$-2, -42$
$3, 28$	$-3, -28$
$4, 21$	$-4, -21$
$6, 14$	$-6, -14$
$\boxed{7, 12}$	$-7, -12$

v) If we try to "drop in place", $(x+7)(x+12) \neq 3x^2 + 19x + 28$. Right off the bat, the product of the terms in the "First" position does not equal $3x^2$. Instead, we must break up the middle term of $3x^2 + 19x + 28$ into a sum of like terms using our circled pair of integers 7 and 12.

$$3x^2 + 19x + 28 = 3x^2 + 7x + 12x + 28$$

Now we factor by grouping. Factor x out of the first two terms and 4 out of the second two terms.

$$= x\,(3x + 7) + 4\,(3x + 7)$$

Now we can factor out $(3x + 7)$.

$$= (x + 4)(3x + 7)$$

7. Compare $12x^2 - 23x + 5$ with $ax^2 + bx + c$ and note that $a = 12$, $b = -23$, and $c = 5$. Calculate $ac = (12)(5)$, so $ac = 60$. Start listing the integer pairs whose product is $ac = 60$, but be mindful that you need an integer pair whose sum is $b = -23$. Note how we ceased listing ordered pairs the moment we found the pair we needed.

$$\begin{array}{ll} 1, 60 & -1, -60 \\ 2, 30 & -2, -30 \\ 3, 20 & \boxed{-3, -20} \end{array}$$

If we try to "drop in place", $(x-3)(x-20) \neq 12x^2-23x+5$. Right off the bat, the product of the terms in the "First" position does not equal $12x^2$. Instead, we must break up the middle term of $12x^2 - 23x + 5$ into a sum of like terms using our circled pair of integers -3 and -20.

$$12x^2 - 23x + 5 = 12x^2 - 3x - 20x + 5$$

Now we factor by grouping. Factor $3x$ out of the first two terms and -5 out of the second two terms.

$$= 3x\,(4x - 1) - 5\,(4x - 1)$$

Now we can factor out $(4x - 1)$.

$$= (3x - 5)(4x - 1)$$

9. Compare $6x^2 + 17x + 7$ with $ax^2 + bx + c$ and note that $a = 6$, $b = 17$, and $c = 7$. Calculate $ac = (6)(7)$, so $ac = 42$. Start listing the integer pairs whose product is $ac = 42$, but be mindful that you need an integer pair whose sum is $b = 17$. Note how we ceased listing ordered pairs the moment we found the pair we needed.

$$\begin{array}{ll} 1, 42 & -1, -42 \\ 2, 21 & -2, -21 \\ \boxed{3, 14} & \end{array}$$

If we try to "drop in place", $(x+3)(x+14) \neq 6x^2+17x+7$. Right off the bat, the product of the terms in the "First" position does not equal $6x^2$. Instead,

we must break up the middle term of $6x^2 + 17x + 7$ into a sum of like terms using our circled pair of integers 3 and 14.

$$6x^2 + 17x + 7 = 6x^2 + 14x + 3x + 7$$

Now we factor by grouping. Factor $2x$ out of the first two terms and 1 out of the second two terms.

$$= 2x\,(3x + 7) + 1\,(3x + 7)$$

Now we can factor out $(3x + 7)$.

$$= (2x + 1)(3x + 7)$$

11. Compare $3x^2 + 4x - 32$ with $ax^2 + bx + c$ and note that $a = 3$, $b = 4$, and $c = -32$. Calculate $ac = (3)(-32)$, so $ac = -96$. Start listing the integer pairs whose product is $ac = -96$, but be mindful that you need an integer pair whose sum is $b = 4$. Note how we ceased listing ordered pairs the moment we found the pair we needed.

$1, -96$	$-1, 96$
$2, -48$	$-2, 48$
$3, -32$	$-3, 32$
$4, -24$	$-4, 24$
$6, -16$	$-6, 16$
$8, -12$	$\boxed{-8, 12}$

If we try to "drop in place", $(x - 8)(x + 12) \neq 3x^2 + 4x - 32$. Right off the bat, the product of the terms in the "First" position does not equal $3x^2$. Instead, we must break up the middle term of $3x^2 + 4x - 32$ into a sum of like terms using our circled pair of integers -8 and 12.

$$3x^2 + 4x - 32 = 3x^2 - 8x + 12x - 32$$

Now we factor by grouping. Factor x out of the first two terms and 4 out of the second two terms.

$$= x\,(3x - 8) + 4\,(3x - 8)$$

Now we can factor out $(3x - 8)$.

$$= (x + 4)(3x - 8)$$

13. Compare $3x^2 + 28x + 9$ with $ax^2 + bx + c$ and note that $a = 3$, $b = 28$, and $c = 9$. Calculate $ac = (3)(9)$, so $ac = 27$. We need an integer pair whose product is $ac = 27$ and whose sum is $b = 28$. The integer pair 1 and 27 comes to mind.

If we try to "drop in place", $(x + 1)(x + 27) \neq 3x^2 + 28x + 9$. Right off the bat, the product of the terms in the "First" position does not equal $3x^2$. Instead, we must break up the middle term of $3x^2 + 28x + 9$ into a sum of like terms using the integer pair 1 and 27.

$$3x^2 + 28x + 9 = 3x^2 + x + 27x + 9$$

Now we factor by grouping. Factor x out of the first two terms and 9 out of the second two terms.

$$= x\,(3x + 1) + 9\,(3x + 1)$$

Now we can factor out $(3x + 1)$.

$$= (x + 9)(3x + 1)$$

15. Compare $4x^2 - 21x + 5$ with $ax^2 + bx + c$ and note that $a = 4$, $b = -21$, and $c = 5$. Calculate $ac = (4)(5)$, so $ac = 20$. We need an integer pair whose product is $ac = 20$ and whose sum is $b = -21$. The integer pair -1 and -20 comes to mind.

If we try to "drop in place", $(x - 1)(x - 20) \neq 4x^2 - 21x + 5$. Right off the bat, the product of the terms in the "First" position does not equal $4x^2$. Instead, we must break up the middle term of $4x^2 - 21x + 5$ into a sum of like terms using the integer pair -1 and -20.

$$4x^2 - 21x + 5 = 4x^2 - x - 20x + 5$$

Now we factor by grouping. Factor x out of the first two terms and -5 out of the second two terms.

$$= x\,(4x - 1) - 5\,(4x - 1)$$

Now we can factor out $(4x - 1)$.

$$= (x - 5)(4x - 1)$$

17. Compare $6x^2 - 11x - 7$ with $ax^2 + bx + c$ and note that $a = 6$, $b = -11$, and $c = -7$. Calculate $ac = (6)(-7)$, so $ac = -42$. We need an integer pair whose product is $ac = -42$ and whose sum is $b = -11$. The integer pair 3 and -14 comes to mind.

If we try to “drop in place”, $(x+3)(x-14) \neq 6x^2 - 11x - 7$. Right off the bat, the product of the terms in the “First” position does not equal $6x^2$. Instead, we must break up the middle term of $6x^2 - 11x - 7$ into a sum of like terms using the integer pair 3 and -14.

$$6x^2 - 11x - 7 = 6x^2 + 3x - 14x - 7$$

Now we factor by grouping. Factor $3x$ out of the first two terms and -7 out of the second two terms.

$$= 3x\,(2x+1) - 7\,(2x+1)$$

Now we can factor out $(2x+1)$.

$$= (3x-7)(2x+1)$$

19. Note that the GCF of $16x^5$, $-36x^4$, and $14x^3$ is $2x^3$. Factor out this GCF.

$$\begin{aligned} 16x^5 - 36x^4 + 14x^3 &= 2x^3 \cdot 8x^2 - 2x^3 \cdot 18x + 2x^3 \cdot 7 \\ &= 2x^3(8x^2 - 18x + 7) \end{aligned}$$

Next, compare $8x^2 - 18x + 7$ with $ax^2 + bx + c$ and note that $a = 8$, $b = -18$, and $c = 7$. Start listing the integer pairs whose product is $ac = (8)(7)$, or $ac = 56$, but be mindful that you need an integer pair whose sum is $b = -18$.

$$\begin{array}{ll} 1, 56 & -1, -56 \\ 2, 28 & -2, -28 \\ 4, 14 & \boxed{-4, -14} \end{array}$$

Break up the middle term of $8x^2 - 18x + 7$ into a sum of like terms using our circled pair of integers -4 and -14.

$$2x^3(8x^2 - 18x + 7) = 2x^3(8x^2 - 14x - 4x + 7)$$

Now we factor by grouping. Factor $2x$ out of the first two terms and -1 out of the second two terms.

$$= 2x^3\left[2x\,(4x-7) - 1\,(4x-7)\right]$$

Now we can factor out $(4x-7)$.

$$= 2x^3(2x-1)(4x-7)$$

21. Note that the GCF of $36x^4$, $-75x^3$, and $21x^2$ is $3x^2$. Factor out this GCF.

$$\begin{aligned} 36x^4 - 75x^3 + 21x^2 &= 3x^2 \cdot 12x^2 - 3x^2 \cdot 25x + 3x^2 \cdot 7 \\ &= 3x^2(12x^2 - 25x + 7) \end{aligned}$$

Next, compare $12x^2 - 25x + 7$ with $ax^2 + bx + c$ and note that $a = 12$, $b = -25$, and $c = 7$. Start listing the integer pairs whose product is $ac = (12)(7)$, or $ac = 84$, but be mindful that you need an integer pair whose sum is $b = -25$.

$1, 84$	$-1, -84$
$2, 42$	$-2, -42$
$3, 28$	$-3, -28$
$4, 21$	$\boxed{-4, -21}$

Break up the middle term of $12x^2 - 25x + 7$ into a sum of like terms using our circled pair of integers -4 and -21.

$$3x^2(12x^2 - 25x + 7) = 3x^2(12x^2 - 21x - 4x + 7)$$

Now we factor by grouping. Factor $3x$ out of the first two terms and -1 out of the second two terms.

$$= 3x^2\left[3x\,(4x - 7) - 1\,(4x - 7)\right]$$

Now we can factor out $(4x - 7)$.

$$= 3x^2(3x - 1)(4x - 7)$$

23. Note that the GCF of $6x^4$, $-33x^3$, and $42x^2$ is $3x^2$. Factor out this GCF.

$$\begin{aligned} 6x^4 - 33x^3 + 42x^2 &= 3x^2 \cdot 2x^2 - 3x^2 \cdot 11x + 3x^2 \cdot 14 \\ &= 3x^2(2x^2 - 11x + 14) \end{aligned}$$

Next, compare $2x^2 - 11x + 14$ with $ax^2 + bx + c$ and note that $a = 2$, $b = -11$, and $c = 14$. Start listing the integer pairs whose product is $ac = (2)(14)$, or $ac = 28$, but be mindful that you need an integer pair whose sum is $b = -11$.

$1, 28$	$-1, -28$
$2, 14$	$-2, -14$
$4, 7$	$\boxed{-4, -7}$

Break up the middle term of $2x^2 - 11x + 14$ into a sum of like terms using our circled pair of integers -4 and -7.

$$3x^2(2x^2 - 11x + 14) = 3x^2(2x^2 - 7x - 4x + 14)$$

Now we factor by grouping. Factor x out of the first two terms and -2 out of the second two terms.

$$= 3x^2\big[x\,(2x-7) - 2\,(2x-7)\big]$$

Now we can factor out $(2x-7)$.

$$= 3x^2(x-2)(2x-7)$$

25. Note that the GCF of $16x^4$, $-36x^3$, and $-36x^2$ is $4x^2$. Factor out this GCF.

$$\begin{aligned} 16x^4 - 36x^3 - 36x^2 &= 4x^2 \cdot 4x^2 - 4x^2 \cdot 9x - 4x^2 \cdot 9 \\ &= 4x^2(4x^2 - 9x - 9) \end{aligned}$$

Next, compare $4x^2 - 9x - 9$ with $ax^2 + bx + c$ and note that $a = 4$, $b = -9$, and $c = -9$. Start listing the integer pairs whose product is $ac = (4)(-9)$, or $ac = -36$, but be mindful that you need an integer pair whose sum is $b = -9$.

$$\begin{array}{ll} 1, -36 & -1, 36 \\ 2, -18 & -2, 18 \\ \boxed{3, -12} & \end{array}$$

Break up the middle term of $4x^2 - 9x - 9$ into a sum of like terms using our circled pair of integers 3 and -12.

$$4x^2(4x^2 - 9x - 9) = 4x^2(4x^2 + 3x - 12x - 9)$$

Now we factor by grouping. Factor x out of the first two terms and -3 out of the second two terms.

$$= 4x^2\big[x\,(4x+3) - 3\,(4x+3)\big]$$

Now we can factor out $(4x+3)$.

$$= 4x^2(x-3)(4x+3)$$

27. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 4x^2 &= -x + 18 && \text{Original equation.} \\ 4x^2 + x &= 18 && \text{Add } x \text{ to both sides.} \\ 4x^2 + x - 18 &= 0 && \text{Subtract 18 from both sides.} \end{aligned}$$

Compare $4x^2 + x - 18$ with $ax^2 + bx + c$ and note that $a = 4$, $b = 1$, and $c = -18$. Calculate $ac = (4)(-18)$, so $ac = -72$. We need an integer pair whose product is $ac = -72$ and whose sum is $b = 1$. The integer pair -8 and 9 comes to mind. Break up the middle term of $4x^2 + x - 18$ into a sum of like terms using the integer pair -8 and 9.

$$\begin{aligned} 4x^2 + 9x - 8x - 18 &= 0 && x = 9x - 8x \\ x(4x + 9) - 2(4x + 9) &= 0 && \text{Factor by grouping.} \\ (x - 2)(4x + 9) &= 0 && \text{Factor out } 4x + 9. \end{aligned}$$

We have a product that equals zero. Use the zero product property to complete the solution.

$$\begin{aligned} x - 2 &= 0 & \quad \text{or} \quad & & 4x + 9 &= 0 \\ x &= 2 & & & 4x &= -9 \\ & & & & x &= -\frac{9}{4} \end{aligned}$$

Thus, the solutions of $4x^2 = -x + 18$ are $x = 2$ and $x = -9/4$.

29. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 3x^2 + 16 &= -14x && \text{Original equation.} \\ 3x^2 + 14x + 16 &= 0 && \text{Add } 14x \text{ to both sides.} \end{aligned}$$

Compare $3x^2 + 14x + 16$ with $ax^2 + bx + c$ and note that $a = 3$, $b = 14$, and $c = 16$. Calculate $ac = (3)(16)$, so $ac = 48$. We need an integer pair whose product is $ac = 48$ and whose sum is $b = 14$. The integer pair 6 and 8 comes to mind. Break up the middle term of $3x^2 + 14x + 16$ into a sum of like terms using the integer pair 6 and 8.

$$\begin{aligned} 3x^2 + 8x + 6x + 16 &= 0 && 14x = 8x + 6x \\ x(3x + 8) + 2(3x + 8) &= 0 && \text{Factor by grouping.} \\ (x + 2)(3x + 8) &= 0 && \text{Factor out } 3x + 8. \end{aligned}$$

We have a product that equals zero. Use the zero product property to complete the solution.

$$\begin{aligned} x + 2 &= 0 & \quad \text{or} \quad & & 3x + 8 &= 0 \\ x &= -2 & & & 3x &= -8 \\ & & & & x &= -\frac{8}{3} \end{aligned}$$

Thus, the solutions of $3x^2 + 16 = -14x$ are $x = -2$ and $x = -8/3$.

31. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 3x^2 + 30 &= 23x && \text{Original equation.} \\ 3x^2 - 23x + 30 &= 0 && \text{Subtract } 23x \text{ from both sides.} \end{aligned}$$

Compare $3x^2 - 23x + 30$ with $ax^2 + bx + c$ and note that $a = 3$, $b = -23$, and $c = 30$. Calculate $ac = (3)(30)$, so $ac = 90$. We need an integer pair whose product is $ac = 90$ and whose sum is $b = -23$. The integer pair -5 and -18 comes to mind. Break up the middle term of $3x^2 - 23x + 30$ into a sum of like terms using the integer pair -5 and -18.

$$\begin{aligned} 3x^2 - 5x - 18x + 30 &= 0 && -23x = -5x - 18x \\ x(3x - 5) - 6(3x - 5) &= 0 && \text{Factor by grouping.} \\ (x - 6)(3x - 5) &= 0 && \text{Factor out } 3x - 5. \end{aligned}$$

We have a product that equals zero. Use the zero product property to complete the solution.

$$\begin{aligned} x - 6 &= 0 & \quad \text{or} \quad && 3x - 5 &= 0 \\ x &= 6 & && 3x &= 5 \\ & & && x &= \frac{5}{3} \end{aligned}$$

Thus, the solutions of $3x^2 + 30 = 23x$ are $x = 6$ and $x = 5/3$.

33. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} -7x - 3 &= -6x^2 && \text{Original equation.} \\ 6x^2 - 7x - 3 &= 0 && \text{Add } 6x^2 \text{ to both sides.} \end{aligned}$$

Compare $6x^2 - 7x - 3$ with $ax^2 + bx + c$ and note that $a = 6$, $b = -7$, and $c = -3$. Calculate $ac = (6)(-3)$, so $ac = -18$. We need an integer pair whose product is $ac = -18$ and whose sum is $b = -7$. The integer pair 2 and -9 comes to mind. Break up the middle term of $6x^2 - 7x - 3$ into a sum of like terms using the integer pair 2 and -9.

$$\begin{aligned} 6x^2 - 9x + 2x - 3 &= 0 && -7x = -9x + 2x \\ 3x(2x - 3) + 1(2x - 3) &= 0 && \text{Factor by grouping.} \\ (3x + 1)(2x - 3) &= 0 && \text{Factor out } 2x - 3. \end{aligned}$$

We have a product that equals zero. Use the zero product property to complete the solution.

$$\begin{aligned} 3x + 1 &= 0 & \quad \text{or} \quad && 2x - 3 &= 0 \\ 3x &= -1 & && 2x &= 3 \\ x &= -\frac{1}{3} & && x &= \frac{3}{2} \end{aligned}$$

Thus, the solutions of $-7x - 3 = -6x^2$ are $x = -1/3$ and $x = 3/2$.

35. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 26x - 9 &= -3x^2 && \text{Original equation.} \\ 3x^2 + 26x - 9 &= 0 && \text{Add } 3x^2 \text{ to both sides.} \end{aligned}$$

Compare $3x^2 + 26x - 9$ with $ax^2 + bx + c$ and note that $a = 3$, $b = 26$, and $c = -9$. Calculate $ac = (3)(-9)$, so $ac = -27$. We need an integer pair whose product is $ac = -27$ and whose sum is $b = 26$. The integer pair -1 and 27 comes to mind. Break up the middle term of $3x^2 + 26x - 9$ into a sum of like terms using the integer pair -1 and 27.

$$\begin{aligned} 3x^2 - x + 27x - 9 &= 0 && 26x = -x + 27x \\ x\,(3x - 1) + 9\,(3x - 1) &= 0 && \text{Factor by grouping.} \\ (x + 9)(3x - 1) &= 0 && \text{Factor out } 3x - 1. \end{aligned}$$

We have a product that equals zero. Use the zero product property to complete the solution.

$$\begin{aligned} x + 9 &= 0 & \quad\text{or}\quad && 3x - 1 &= 0 \\ x &= -9 & && 3x &= 1 \\ & & && x &= \frac{1}{3} \end{aligned}$$

Thus, the solutions of $26x - 9 = -3x^2$ are $x = -9$ and $x = 1/3$.

37. Because there is a power of x larger than one, the equation is nonlinear. Make one side zero.

$$\begin{aligned} 6x^2 &= -25x + 9 && \text{Original equation.} \\ 6x^2 + 25x &= 9 && \text{Add } 25x \text{ to both sides.} \\ 6x^2 + 25x - 9 &= 0 && \text{Subtract 9 from both sides.} \end{aligned}$$

Compare $6x^2 + 25x - 9$ with $ax^2 + bx + c$ and note that $a = 6$, $b = 25$, and $c = -9$. Calculate $ac = (6)(-9)$, so $ac = -54$. We need an integer pair whose product is $ac = -54$ and whose sum is $b = 25$. The integer pair -2 and 27 comes to mind. Break up the middle term of $6x^2 + 25x - 9$ into a sum of like terms using the integer pair -2 and 27.

$$\begin{aligned} 6x^2 + 27x - 2x - 9 &= 0 && 25x = 27x - 2x \\ 3x\,(2x + 9) - 1\,(2x + 9) &= 0 && \text{Factor by grouping.} \\ (3x - 1)(2x + 9) &= 0 && \text{Factor out } 2x + 9. \end{aligned}$$

We have a product that equals zero. Use the zero product property to complete the solution.

$$\begin{aligned} 3x - 1 &= 0 &\quad \text{or} \quad&& 2x + 9 &= 0 \\ 3x &= 1 &&& 2x &= -9 \\ x &= \frac{1}{3} &&& x &= -\frac{9}{2} \end{aligned}$$

Thus, the solutions of $6x^2 = -25x + 9$ are $x = 1/3$ and $x = -9/2$.

39. Algebraic solution. The equation is nonlinear, so make one side zero. That's already done, so use the ac-method to factor. Note that $ac = (2)(-5) = -10$ and the integer pair 1 and -10 has product -10 and sum -9, the coefficient of x. Split the middle term up using this pair.

$$\begin{aligned} 2x^2 - 9x - 5 &= 0 \\ 2x^2 + x - 10x - 5 &= 0 \end{aligned}$$

Factor by grouping.

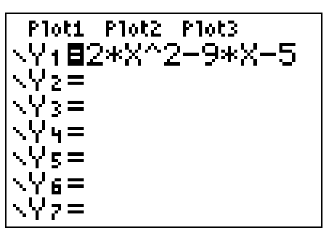
$$\begin{aligned} x(2x + 1) - 5(2x + 1) &= 0 \\ (x - 5)(2x + 1) &= 0 \end{aligned}$$

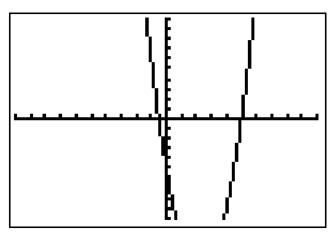
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$\begin{aligned} x - 5 &= 0 &\quad \text{or} \quad&& 2x + 1 &= 0 \\ x &= 5 &&& x &= -\frac{1}{2} \end{aligned}$$

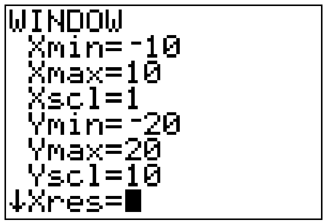
Hence, the solutions are $x = 5$ and $x = -1/2$.

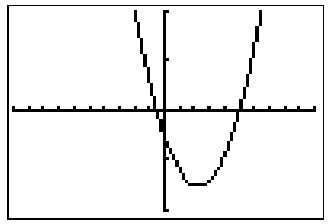
Calculator solution. Load the left-hand side of the equation $2x^2 - 9x - 5 = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



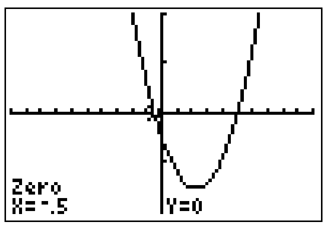


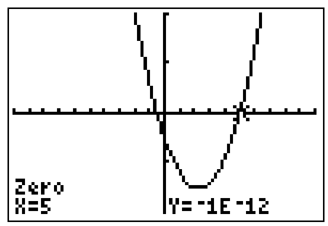
Both x-intercepts are visible in the viewing window, but we really should adjust the WINDOW parameters so that the vertex of the parabola is visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



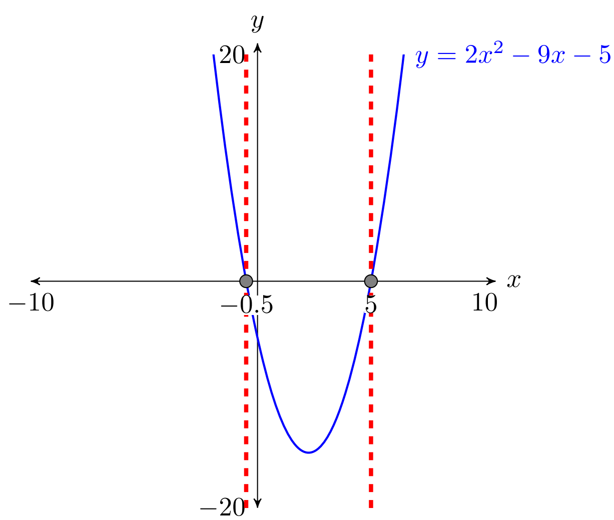


Next, use the **2:zero** utility from the CALC menu to find the x-intercepts of the graph.





Report the results on your homework as follows.



Hence, the solutions of $2x^2 - 9x - 5 = 0$ are $x = -0.5$ and $x = 5$. Note how these agree with the algebraic solution, especially when you note that $-0.5 = -1/2$.

41. Algebraic solution. The equation is nonlinear, so make one side zero. That's already done, so use the ac-method to factor. Note that $ac = (4)(-15) =$

-60 and the integer pair 3 and -20 has product -60 and sum -17, the coefficient of x. Split the middle term up using this pair.

$$\begin{aligned} 4x^2 - 17x - 15 &= 0 \\ 4x^2 + 3x - 20x - 15 &= 0 \end{aligned}$$

Factor by grouping.

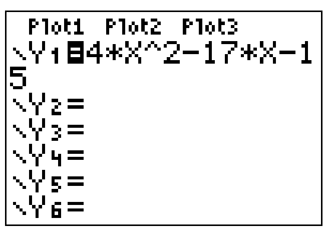
$$\begin{aligned} x(4x + 3) - 5(4x + 3) &= 0 \\ (x - 5)(4x + 3) &= 0 \end{aligned}$$

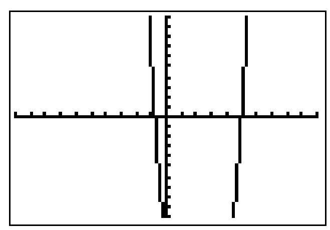
Use the zero product property to set both factors equal to zero, then solve the resulting equations.

$$x - 5 = 0 \quad \text{or} \quad 4x + 3 = 0$$
$$x = 5 \qquad\qquad x = -\frac{3}{4}$$

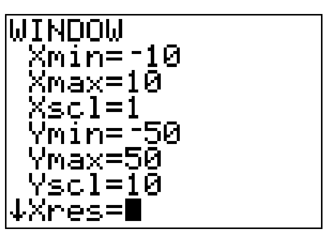
Hence, the solutions are $x = 5$ and $x = -3/4$.

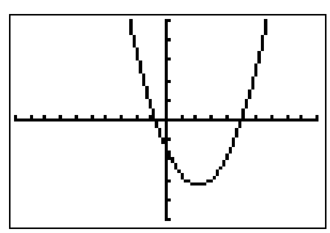
Calculator solution. Load the left-hand side of the equation $4x^2 - 17x - 15 = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



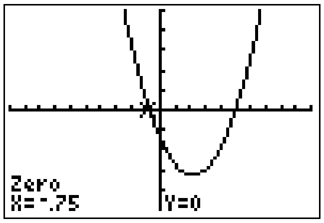


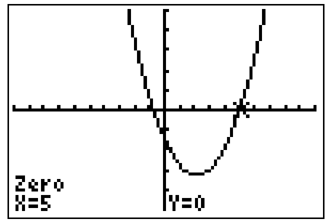
Both x-intercepts are visible in the viewing window, but we really should adjust the WINDOW parameters so that the vertex of the parabola is visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.



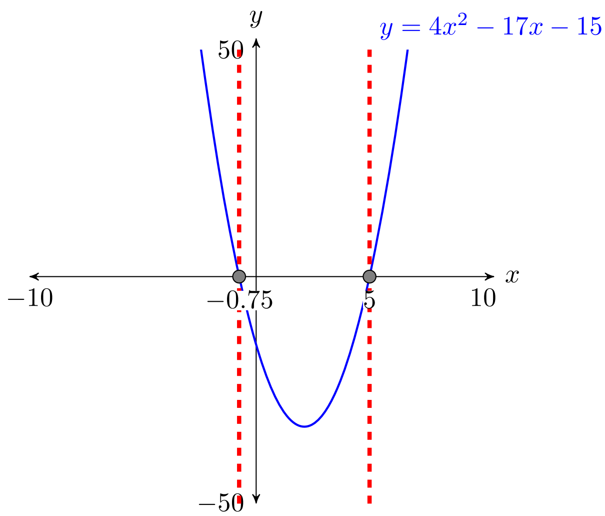


Next, use the **2:zero** utility from the CALC menu to find the x-intercepts of the graph.





Report the results on your homework as follows.



Hence, the solutions of $4x^2 - 17x - 15 = 0$ are $x = -0.75$ and $x = 5$. Note how these agree with the algebraic solution, especially when you note that $-0.75 = -3/4$.

43. Algebraic solution. The equation is nonlinear, so make one side zero. Subtract $3x^2$ and $20x$ from both sides.

$$2x^3 = 3x^2 + 20x$$
$$2x^3 - 3x^2 - 20x = 0$$

Factor out the GCF.

$$x(2x^2 - 3x - 20) = 0$$

Note that $ac = (2)(-20) = -40$ and the integer pair 5 and -8 have product -40 and sum -3, the coefficient of x. Use this pair to break up the middle term.

$$x(2x^2 + 5x - 8x - 20) = 0$$

Factor by grouping.

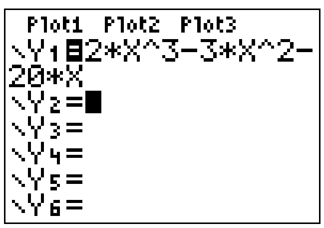
$$\begin{aligned} x(x(2x+5) - 4(2x+5)) &= 0 \\ x(x-4)(2x+5) &= 0 \end{aligned}$$

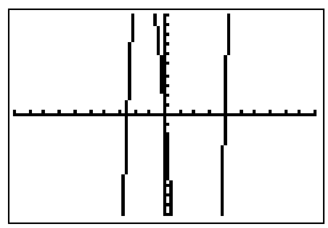
Use the zero product property to set all three factors equal to zero, then solve the resulting equations.

$$\begin{aligned} x = 0 \qquad \text{or} \qquad x - 4 &= 0 \qquad \text{or} \qquad 2x + 5 = 0 \\ x &= 4 \qquad\qquad\qquad\quad x = -\frac{5}{2} \end{aligned}$$

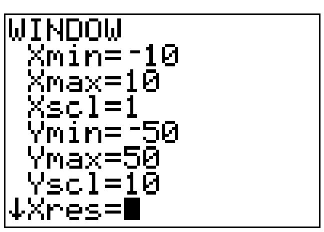
Hence, the solutions are $x = 0$, $x = 4$, and $x = -5/2$.

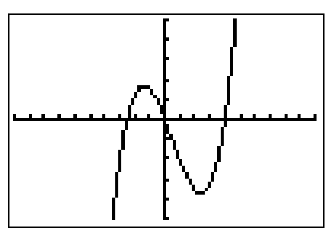
Calculator solution. Load the left-hand side of the equation $2x^3 - 3x^2 - 20x = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



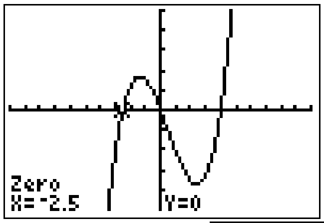


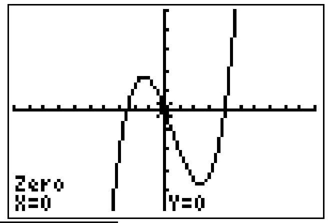
All three x-intercepts are visible in the viewing window, but we really should adjust the WINDOW parameters so that the turning points of the parabola are visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.

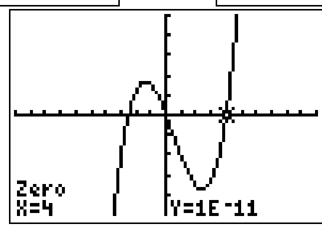




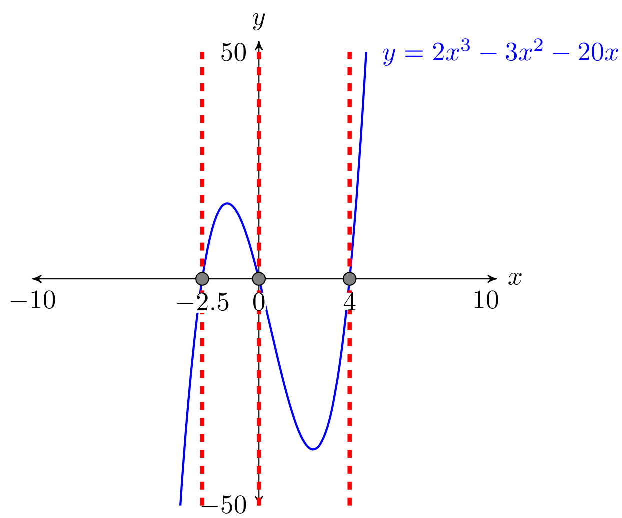
Next, use the **2:zero** utility from the CALC menu to find the x-intercepts of the graph.







Report the results on your homework as follows.



Hence, the solutions of $2x^3 = 3x^2 + 20x$ are $x = -2.5$, $x = 0$, and $x = 4$. Note how these agree with the algebraic solution, especially when you note that $-2.5 = -5/2$.

45. Algebraic solution. The equation is nonlinear, so make one side zero. Subtract $24x$ from both sides.

$$10x^3 + 34x^2 = 24x$$
$$10x^3 + 34x^2 - 24x = 0$$

Factor out the GCF.

$$2x(5x^2 + 17x - 12) = 0$$

Note that $ac = (5)(-12) = -60$ and the integer pair -3 and 20 have product -60 and sum 17, the coefficient of x. Use this pair to break up the middle term.

$$2x(5x^2 - 3x + 20x - 12) = 0$$

Factor by grouping.

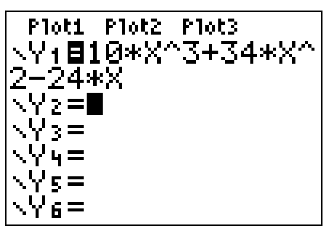
$$\begin{aligned} 2x(x(5x-3) + 4(5x-3)) &= 0 \\ 2x(x+4)(5x-3) &= 0 \end{aligned}$$

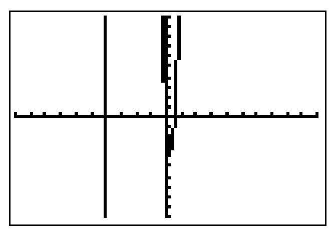
Use the zero product property to set all three factors equal to zero, then solve the resulting equations.

$$\begin{array}{ccccc} 2x = 0 & \text{or} & x + 4 = 0 & \text{or} & 5x - 3 = 0 \\ x = 0 & & x = -4 & & x = \dfrac{3}{5} \end{array}$$

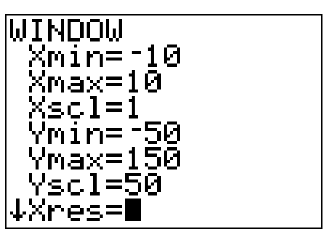
Hence, the solutions are $x = 0$, $x = -4$, and $x = 3/5$.

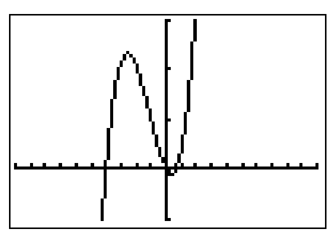
Calculator solution. Load the left-hand side of the equation $10x^3 + 34x^2 - 24x = 0$ into **Y1** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



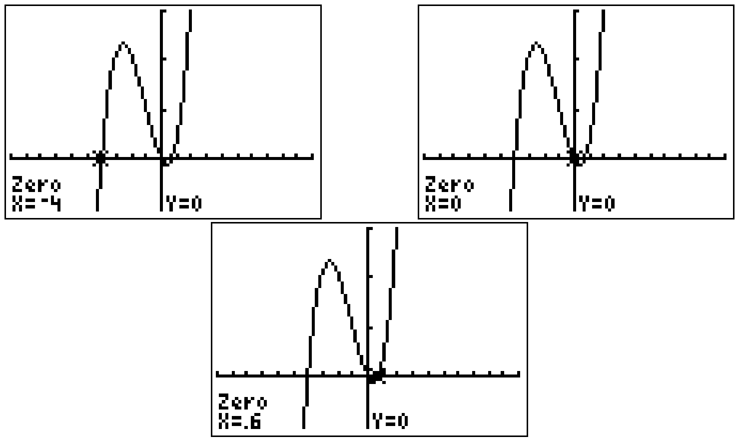


All three x-intercepts are visible in the viewing window, but we really should adjust the WINDOW parameters so that the turning points of the parabola are visible in the viewing window. Adjust the WINDOW parameters as shown, then push the GRAPH button to produce the accompanying graph.

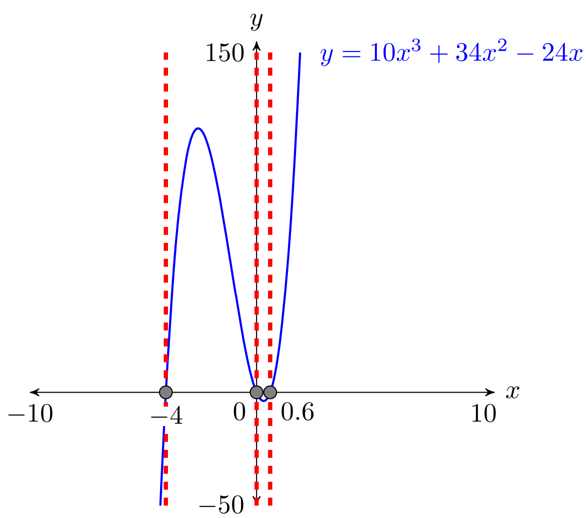




Next, use the **2:zero** utility from the CALC menu to find the x-intercepts of the graph.



Report the results on your homework as follows.



Hence, the solutions of $10x^3 + 34x^2 = 24x$ are $x = -4$, $x = 0$, and $x = 0.6$. Note how these agree with the algebraic solution, especially when you note that $0.6 = 3/5$.

6.5 Special Forms

1. Using the pattern $(a - b)^2 = a^2 - 2ab + b^2$, we can expand $(8r - 3t)^2$ as follows:

$$\begin{aligned}(8r - 3t)^2 &= (8r)^2 - 2(8r)(3t) + (3t)^2 \\ &= 64r^2 - 48rt + 9t^2\end{aligned}$$

Note how we square the first and second terms, then produce the middle term of our answer by multiplying the first and second terms and doubling.

3. Using the pattern $(a+b)^2 = a^2 + 2ab + b^2$, we can expand $(4a+7b)^2$ as follows:

$$\begin{aligned}(4a+7b)^2 &= (4a)^2 + 2(4a)(7b) + (7b)^2 \\ &= 16a^2 + 56ab + 49b^2\end{aligned}$$

Note how we square the first and second terms, then produce the middle term of our answer by multiplying the first and second terms and doubling.

5. Using the pattern $(a-b)^2 = a^2 - 2ab + b^2$, we can expand $(s^3-9)^2$ as follows:

$$\begin{aligned}(s^3-9)^2 &= (s^3)^2 - 2(s^3)(9) + (9)^2 \\ &= s^6 - 18s^3 + 81\end{aligned}$$

Note how we square the first and second terms, then produce the middle term of our answer by multiplying the first and second terms and doubling.

7. Using the pattern $(a+b)^2 = a^2 + 2ab + b^2$, we can expand $(s^2+6t^2)^2$ as follows:

$$\begin{aligned}(s^2+6t^2)^2 &= (s^2)^2 + 2(s^2)(6t^2) + (6t^2)^2 \\ &= s^4 + 12s^2t^2 + 36t^4\end{aligned}$$

Note how we square the first and second terms, then produce the middle term of our answer by multiplying the first and second terms and doubling.

9. In the trinomial $25s^2+60st+36t^2$, note that $(5s)^2 = 25s^2$ and $(6t)^2 = 36t^2$. Hence, the first and last terms are perfect squares. Taking the square roots, we suspect that $25s^2+60st+36t^2$ factors as follows:

$$25s^2 + 60st + 36t^2 = (5s+6t)^2$$

However, we must check to see if the middle term is correct. Multiply $5s$ and $6t$, then double: $2(5s)(6t) = 60st$. Thus, the middle term is correct and we have the correct factorization of $25s^2+60st+36t^2$.

11. In the trinomial $36v^2 - 60vw + 25w^2$, note that $(6v)^2 = 36v^2$ and $(5w)^2 = 25w^2$. Hence, the first and last terms are perfect squares. Taking the square roots, we suspect that $36v^2 - 60vw + 25w^2$ factors as follows:

$$36v^2 - 60vw + 25w^2 = (6v - 5w)^2$$

However, we must check to see if the middle term is correct. Multiply $6v$ and $5w$, then double: $2(6v)(5w) = 60vw$. Thus, the middle term is correct and we have the correct factorization of $36v^2 - 60vw + 25w^2$.

13. In the trinomial $a^4 + 18a^2b^2 + 81b^4$, note that $(a^2)^2 = a^4$ and $(9b^2)^2 = 81b^4$. Hence, the first and last terms are perfect squares. Taking the square roots, we suspect that $a^4 + 18a^2b^2 + 81b^4$ factors as follows:

$$a^4 + 18a^2b^2 + 81b^4 = (a^2 + 9b^2)^2$$

However, we must check to see if the middle term is correct. Multiply a^2 and $9b^2$, then double: $2(a^2)(9b^2) = 18a^2b^2$. Thus, the middle term is correct and we have the correct factorization of $a^4 + 18a^2b^2 + 81b^4$.

15. In the trinomial $49s^4 - 28s^2t^2 + 4t^4$, note that $(7s^2)^2 = 49s^4$ and $(2t^2)^2 = 4t^4$. Hence, the first and last terms are perfect squares. Taking the square roots, we suspect that $49s^4 - 28s^2t^2 + 4t^4$ factors as follows:

$$49s^4 - 28s^2t^2 + 4t^4 = (7s^2 - 2t^2)^2$$

However, we must check to see if the middle term is correct. Multiply $7s^2$ and $2t^2$, then double: $2(7s^2)(2t^2) = 28s^2t^2$. Thus, the middle term is correct and we have the correct factorization of $49s^4 - 28s^2t^2 + 4t^4$.

17. In the trinomial $49b^6 - 112b^3 + 64$, note that $(7b^3)^2 = 49b^6$ and $(8)^2 = 64$. Hence, the first and last terms are perfect squares. Taking the square roots, we suspect that $49b^6 - 112b^3 + 64$ factors as follows:

$$49b^6 - 112b^3 + 64 = (7b^3 - 8)^2$$

However, we must check to see if the middle term is correct. Multiply $7b^3$ and 8, then double: $2(7b^3)(8) = 112b^3$. Thus, the middle term is correct and we have the correct factorization of $49b^6 - 112b^3 + 64$.

19. In the trinomial $49r^6 + 112r^3 + 64$, note that $(7r^3)^2 = 49r^6$ and $(8)^2 = 64$. Hence, the first and last terms are perfect squares. Taking the square roots, we suspect that $49r^6 + 112r^3 + 64$ factors as follows:

$$49r^6 + 112r^3 + 64 = (7r^3 + 8)^2$$

However, we must check to see if the middle term is correct. Multiply $7r^3$ and 8, then double: $2(7r^3)(8) = 112r^3$. Thus, the middle term is correct and we have the correct factorization of $49r^6 + 112r^3 + 64$.

21.

23.

25. In the trinomial $-48b^3 + 120b^2 - 75b$, we note that the GCF of $48b^3$, $120b^2$, and $75b$ is $3b$. We first factor out $3b$.

$$-48b^3 + 120b^2 - 75b = 3b(-16b^2 + 40b - 25)$$

However, the first and third terms of $-16b^2 + 40b - 25$ are negative, and thus are not perfect squares. Let's begin again, this time factoring out $-3b$.

$$-48b^3 + 120b^2 - 75b = -3b(16b^2 - 40b + 25)$$

This time the first and third terms of $16b^2 - 40b + 25$ are perfect squares. We take their square roots and write:

$$= -3b(4b - 5)^2$$

We must check that our middle term is correct. Because $2(4b)(5) = 40b$, we do have a perfect square trinomial and our result is correct.

27. In the trinomial $-5u^5 - 30u^4 - 45u^3$, we note that the GCF of $5u^5$, $30u^4$, and $45u^3$ is $5u^3$. We first factor out $5u^3$.

$$-5u^5 - 30u^4 - 45u^3 = 5u^3(-u^2 - 6u - 9)$$

However, the first and third terms of $-u^2 - 6u - 9$ are negative, and thus are not perfect squares. Let's begin again, this time factoring out $-5u^3$.

$$-5u^5 - 30u^4 - 45u^3 = -5u^3(u^2 + 6u + 9)$$

This time the first and third terms of $u^2 + 6u + 9$ are perfect squares. We take their square roots and write:

$$= -5u^3(u + 3)^2$$

We must check that our middle term is correct. Because $2(u)(3) = 6u$, we do have a perfect square trinomial and our result is correct.

29. In $(21c + 16)(21c - 16)$, we have the exact same terms in the "First" and "Last" positions, with the first set separated by a plus sign and the second set separated by a minus sign. Using the difference of squares pattern $(a + b)(a - b) = a^2 - b^2$, we square the "First" and "Last" positions, then place a minus sign between the results. Hence:

$$\begin{aligned}(21c + 16)(21c - 16) &= (21c)^2 - (16)^2 \\ &= 441c^2 - 256\end{aligned}$$

31. In $(5x + 19z)(5x - 19z)$, we have the exact same terms in the "First" and "Last" positions, with the first set separated by a plus sign and the second set separated by a minus sign. Using the difference of squares pattern $(a + b)(a - b) = a^2 - b^2$, we square the "First" and "Last" positions, then place a minus sign between the results. Hence:

$$\begin{aligned}(5x + 19z)(5x - 19z) &= (5x)^2 - (19z)^2 \\ &= 25x^2 - 361z^2\end{aligned}$$

33. In $(3y^4 + 23z^4)(3y^4 - 23z^4)$, we have the exact same terms in the "First" and "Last" positions, with the first set separated by a plus sign and the second set separated by a minus sign. Using the difference of squares pattern $(a + b)(a - b) = a^2 - b^2$, we square the "First" and "Last" positions, then place a minus sign between the results. Hence:

$$\begin{aligned}(3y^4 + 23z^4)(3y^4 - 23z^4) &= (3y^4)^2 - (23z^4)^2 \\ &= 9y^8 - 529z^8\end{aligned}$$

35. In $(8r^5 + 19s^5)(8r^5 - 19s^5)$, we have the exact same terms in the "First" and "Last" positions, with the first set separated by a plus sign and the second set separated by a minus sign. Using the difference of squares pattern $(a + b)(a - b) = a^2 - b^2$, we square the "First" and "Last" positions, then place a minus sign between the results. Hence:

$$\begin{aligned}(8r^5 + 19s^5)(8r^5 - 19s^5) &= (8r^5)^2 - (19s^5)^2 \\ &= 64r^{10} - 361s^{10}\end{aligned}$$

37. In $361x^2 - 529$, note that we have two perfect squares separated by a minus sign. Note that $(19x)^2 = 361x^2$ and $(23)^2 = 529$. Using the difference of squares pattern $a^2 - b^2 = (a + b)(a - b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$361x^2 - 529 = (19x + 23)(19x - 23)$$

39. In $16v^2 - 169$, note that we have two perfect squares separated by a minus sign. Note that $(4v)^2 = 16v^2$ and $(13)^2 = 169$. Using the difference of squares pattern $a^2 - b^2 = (a + b)(a - b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$16v^2 - 169 = (4v + 13)(4v - 13)$$

41. In $169x^2 - 576y^2$, note that we have two perfect squares separated by a minus sign. Note that $(13x)^2 = 169x^2$ and $(24y)^2 = 576y^2$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$169x^2 - 576y^2 = (13x + 24y)(13x - 24y)$$

43. In $529r^2 - 289s^2$, note that we have two perfect squares separated by a minus sign. Note that $(23r)^2 = 529r^2$ and $(17s)^2 = 289s^2$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$529r^2 - 289s^2 = (23r + 17s)(23r - 17s)$$

45. In $49r^6 - 256t^6$, note that we have two perfect squares separated by a minus sign. Note that $(7r^3)^2 = 49r^6$ and $(16t^3)^2 = 256t^6$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$49r^6 - 256t^6 = (7r^3 + 16t^3)(7r^3 - 16t^3)$$

47. In $36u^{10} - 25w^{10}$, note that we have two perfect squares separated by a minus sign. Note that $(6u^5)^2 = 36u^{10}$ and $(5w^5)^2 = 25w^{10}$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$36u^{10} - 25w^{10} = (6u^5 + 5w^5)(6u^5 - 5w^5)$$

49. In $72y^5 - 242y^3$, the GCF of $72y^5$ and $242y^3$ is $2y^3$. Factor out $2y^3$.

$$72y^5 - 242y^3 = 2y^3(36y^2 - 121)$$

Note that we have two perfect squares separated by a minus sign. Note that $(6y)^2 = 36y^2$ and $(11)^2 = 121$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$2y^3(36y^2 - 121) = 2y^3(6y + 11)(6y - 11)$$

Thus, $72y^5 - 242y^3 = 2y^3(6y + 11)(6y - 11)$.

51. In $1444a^3b - 324ab^3$, the GCF of $1444a^3b$ and $324ab^3$ is $4ab$. Factor out $4ab$.

$$1444a^3b - 324ab^3 = 4ab(361a^2 - 81b^2)$$

Note that we have two perfect squares separated by a minus sign. Note that $(19a)^2 = 361a^2$ and $(9b)^2 = 81b^2$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$4ab(361a^2 - 81b^2) = 4ab(19a + 9b)(19a - 9b)$$

Thus, $1444a^3b - 324ab^3 = 4ab(19a + 9b)(19a - 9b)$.

53. In $576x^3z - 1156xz^3$, the GCF of $576x^3z$ and $1156xz^3$ is $4xz$. Factor out $4xz$.

$$576x^3z - 1156xz^3 = 4xz(144x^2 - 289z^2)$$

Note that we have two perfect squares separated by a minus sign. Note that $(12x)^2 = 144x^2$ and $(17z)^2 = 289z^2$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$4xz(144x^2 - 289z^2) = 4xz(12x + 17z)(12x - 17z)$$

Thus, $576x^3z - 1156xz^3 = 4xz(12x + 17z)(12x - 17z)$.

55. In $576t^4 - 4t^2$, the GCF of $576t^4$ and $4t^2$ is $4t^2$. Factor out $4t^2$.

$$576t^4 - 4t^2 = 4t^2(144t^2 - 1)$$

Note that we have two perfect squares separated by a minus sign. Note that $(12t)^2 = 144t^2$ and $(1)^2 = 1$. Using the difference of squares pattern $a^2 - b^2 = (a+b)(a-b)$, we take the square roots, separate one pair with a plus sign and one pair with a minus sign. Hence:

$$4t^2(144t^2 - 1) = 4t^2(12t + 1)(12t - 1)$$

Thus, $576t^4 - 4t^2 = 4t^2(12t + 1)(12t - 1)$.

57. In $81x^4 - 256$, we have the difference of two squares: $(9x^2)^2 = 81x^4$ and $(16)^2 = 256$. First, we take the square roots, $9x^2$ and 16, then separate one set with a plus sign and the other set with a minus sign.

$$81x^4 - 256 = (9x^2 + 16)(9x^2 - 16)$$

Note that $9x^2 + 16$ is the *sum* of two squares and does not factor further. However, $9x^2 - 16$ is the difference of two squares: $(3x)^2 = 9x^2$ and $(4)^2 = 16$. Take the square roots, $3x$ and 4, then separate one set with a plus sign and the other set with a minus sign.

$$= (9x^2 + 16)(3x + 4)(3x - 4)$$

Done. We cannot factor further.

59. In $81x^4 - 16$, we have the difference of two squares: $(9x^2)^2 = 81x^4$ and $(4)^2 = 16$. First, we take the square roots, $9x^2$ and 4, then separate one set with a plus sign and the other set with a minus sign.

$$81x^4 - 16 = (9x^2 + 4)(9x^2 - 4)$$

Note that $9x^2 + 4$ is the *sum* of two squares and does not factor further. However, $9x^2 - 4$ is the difference of two squares: $(3x)^2 = 9x^2$ and $(2)^2 = 4$. Take the square roots, $3x$ and 2, then separate one set with a plus sign and the other set with a minus sign.

$$= (9x^2 + 4)(3x + 2)(3x - 2)$$

Done. We cannot factor further.

61. We factor by grouping. Factor an z^2 out of the first two terms and a -9 out of the second two terms.

$$z^3 + z^2 - 9z - 9 = z^2(z + 1) - 9(z + 1)$$

Now we can factor out a $z + 1$.

$$= (z^2 - 9)(z + 1)$$

We're still not done because $z^2 - 9$ is the difference of two squares and can be factored as follows:

$$= (z + 3)(z - 3)(z + 1)$$

63. We factor by grouping. Factor an x^2 out of the first two terms and a $-y^2$ out of the second two terms.

$$x^3 - 2x^2y - xy^2 + 2y^3 = x^2(x - 2y) - y^2(x - 2y)$$

Now we can factor out a $x - 2y$.

$$= (x^2 - y^2)(x - 2y)$$

We're still not done because $x^2 - y^2$ is the difference of two squares and can be factored as follows:

$$= (x + y)(x - y)(x - 2y)$$

65. We factor by grouping. Factor an r^2 out of the first two terms and a $-25t^2$ out of the second two terms.

$$r^3 - 3r^2t - 25rt^2 + 75t^3 = r^2(r - 3t) - 25t^2(r - 3t)$$

Now we can factor out a $r - 3t$.

$$= (r^2 - 25t^2)(r - 3t)$$

We're still not done because $r^2 - 25t^2$ is the difference of two squares and can be factored as follows:

$$= (r + 5t)(r - 5t)(r - 3t)$$

67. We factor by grouping. Factor an x^2 out of the first two terms and a -16 out of the second two terms.

$$2x^3 + x^2 - 32x - 16 = x^2(2x + 1) - 16(2x + 1)$$

Now we can factor out a $2x + 1$.

$$= (x^2 - 16)(2x + 1)$$

We're still not done because $x^2 - 16$ is the difference of two squares and can be factored as follows:

$$= (x + 4)(x - 4)(2x + 1)$$

69. The equation is nonlinear, so start by making one side equal to zero.

$$\begin{aligned} 2x^3 + 7x^2 &= 72x + 252 && \text{Original equation.} \\ 2x^3 + 7x^2 - 72x &= 252 && \text{Subtract } 72x \text{ from both sides.} \\ 2x^3 + 7x^2 - 72x - 252 &= 0 && \text{Subtract 252 from both sides.} \end{aligned}$$

We now have a four-term expression, so we'll try factoring by grouping. Factor x^2 out of the first two terms, and -36 out of the second two terms.

$$\begin{aligned} x^2(2x + 7) - 36(2x + 7) &= 0 && \text{Factor by grouping.} \\ (x^2 - 36)(2x + 7) &= 0 && \text{Factor out } 2x + 7. \end{aligned}$$

The first factor has two perfect squares separated by a minus sign, the difference of squares pattern. Take the square roots of each term, making one factor plus and one factor minus.

$$(x + 6)(x - 6)(2x + 7) = 0 \qquad \text{Factor using difference of squares.}$$

The polynomial is now completely factored. Use the zero product property to set each factor equal to zero, then solve each of the resulting equations.

$$\begin{aligned} x+6=0 &\quad\text{or}\quad & x-6=0 &\quad\text{or}\quad & 2x+7=0 \\ x=-6 & & x=6 & & x=-\frac{7}{2} \end{aligned}$$

Hence, the solutions of $2x^3+7x^2=72x+252$ are $x=-6$, $x=6$, and $x=-7/2$.

71. The equation is nonlinear, so start by making one side equal to zero.

$$\begin{aligned} x^3+5x^2 &= 64x+320 && \text{Original equation.} \\ x^3+5x^2-64x &= 320 && \text{Subtract } 64x \text{ from both sides.} \\ x^3+5x^2-64x-320 &= 0 && \text{Subtract 320 from both sides.} \end{aligned}$$

We now have a four-term expression, so we'll try factoring by grouping. Factor x^2 out of the first two terms, and -64 out of the second two terms.

$$\begin{aligned} x^2(x+5)-64(x+5) &= 0 && \text{Factor by grouping.} \\ (x^2-64)(x+5) &= 0 && \text{Factor out } x+5. \end{aligned}$$

The first factor has two perfect squares separated by a minus sign, the difference of squares pattern. Take the square roots of each term, making one factor plus and one factor minus.

$$(x+8)(x-8)(x+5)=0 \qquad \text{Factor using difference of squares.}$$

The polynomial is now completely factored. Use the zero product property to set each factor equal to zero, then solve each of the resulting equations.

$$\begin{aligned} x+8=0 &\quad\text{or}\quad & x-8=0 &\quad\text{or}\quad & x+5=0 \\ x=-8 & & x=8 & & x=-5 \end{aligned}$$

Hence, the solutions of $x^3+5x^2=64x+320$ are $x=-8$, $x=8$, and $x=-5$.

73. The equation is nonlinear. Start by making one side equal to zero.

$$\begin{aligned} 144x^2+121 &= 264x && \text{Original equation.} \\ 144x^2-264x+121 &= 0 && \text{Subtract } 264x \text{ from both sides.} \end{aligned}$$

Note that the first and last terms of the trinomial are perfect squares. Hence, it make sense to try and factor as a perfect square trinomial, taking the square roots of the first and last terms.

$$(12x-11)^2=0 \qquad \text{Factor.}$$

Of course, be sure to check the middle term. Because $-2(12x)(11) = -264x$, the middle term is correct. We can now use the zero product property to set each factor equal to zero and solve the resulting equations.

$$\begin{aligned} 12x - 11 &= 0 & \quad\text{or}\quad && 12x - 11 &= 0 \\ x &= \frac{11}{12} & && x &= \frac{11}{12} \end{aligned}$$

Hence, the only solution of $144x^2 + 121 = 264x$ is $x = 11/12$. We encourage readers to check this solution.

75. The equation is nonlinear, so start the solution by making one side equal to zero.

$$\begin{aligned} 16x^2 &= 169 && \text{Original equation.} \\ 16x^2 - 169 &= 0 && \text{Subtract 169 from both sides.} \\ (4x + 13)(4x - 13) &= 0 && \text{Factor using difference of squares.} \end{aligned}$$

Use the zero product property to set each factor equal to zero, then solve each equation for x.

$$\begin{aligned} 4x + 13 &= 0 & \quad\text{or}\quad && 4x - 13 &= 0 \\ x &= -\frac{13}{4} & && x &= \frac{13}{4} \end{aligned}$$

Hence, the solutions of $16x^2 = 169$ are $x = -13/4$ and $x = 13/4$. We encourage readers to check each of these solutions.

77. The equation is nonlinear, so start the solution by making one side equal to zero.

$$\begin{aligned} 9x^2 &= 25 && \text{Original equation.} \\ 9x^2 - 25 &= 0 && \text{Subtract 25 from both sides.} \\ (3x + 5)(3x - 5) &= 0 && \text{Factor using difference of squares.} \end{aligned}$$

Use the zero product property to set each factor equal to zero, then solve each equation for x.

$$\begin{aligned} 3x + 5 &= 0 & \quad\text{or}\quad && 3x - 5 &= 0 \\ x &= -\frac{5}{3} & && x &= \frac{5}{3} \end{aligned}$$

Hence, the solutions of $9x^2 = 25$ are $x = -5/3$ and $x = 5/3$. We encourage readers to check each of these solutions.

79. The equation is nonlinear. Start by making one side equal to zero.

$$\begin{aligned} 256x^2 + 361 &= -608x && \text{Original equation.} \\ 256x^2 + 608x + 361 &= 0 && \text{Add } 608x \text{ to both sides.} \end{aligned}$$

Note that the first and last terms of the trinomial are perfect squares. Hence, it make sense to try and factor as a perfect square trinomial, taking the square roots of the first and last terms.

$$(16x + 19)^2 = 0 \qquad \text{Factor.}$$

Of course, be sure to check the middle term. Because $2(16x)(19) = 608x$, the middle term is correct. We can now use the zero product property to set each factor equal to zero and solve the resulting equations.

$$\begin{aligned} 16x + 19 &= 0 & \quad\text{or}\quad && 16x + 19 &= 0 \\ x &= -\frac{19}{16} & && x &= -\frac{19}{16} \end{aligned}$$

Hence, the only solution of $256x^2+361 = -608x$ is $x = -19/16$. We encourage readers to check this solution.

81. Algebraic solution. The equation is nonlinear, so make one side zero. Subtract x from both sides.

$$\begin{aligned} x^3 &= x \\ x^3 - x &= 0 \end{aligned}$$

Factor out the GCF.

$$x(x^2 - 1) = 0$$

Use the difference of squares pattern $a^2 - b^2 = (a + b)(a - b)$ to factor.

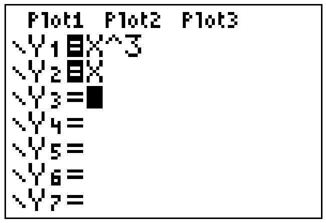
$$x(x + 1)(x - 1) = 0$$

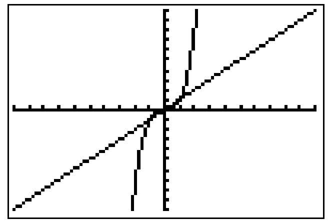
Use the zero product property to set all three factors equal to zero, then solve the resulting equations.

$$\begin{aligned} x = 0 \quad\text{or}\quad x + 1 &= 0 & \quad\text{or}\quad x - 1 &= 0 \\ x &= -1 & x &= 1 \end{aligned}$$

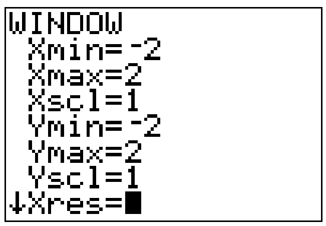
Hence, the solutions are $x = 0$, $x = -1$, and $x = 1$.

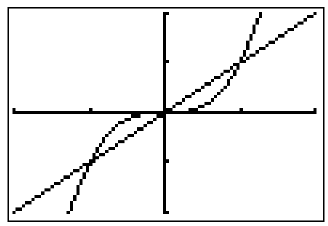
Calculator solution. Load the left- and right-hand sides of the equation $x^3 = x$ into **Y1** and **Y2** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



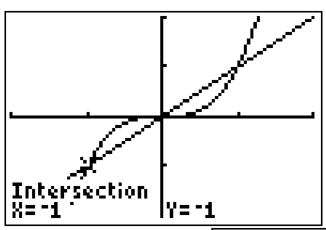


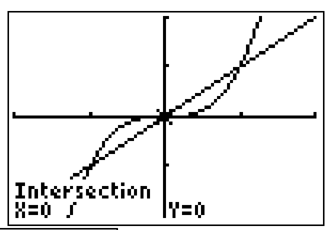
We need to adjust the WINDOW parameters to make the points of intersection more visible. It seems that the points of intersection occur near the origin, so after some experimentation, we decided on the following parameters which produced the accompanying image.

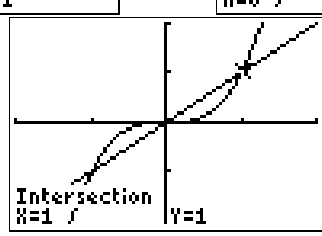




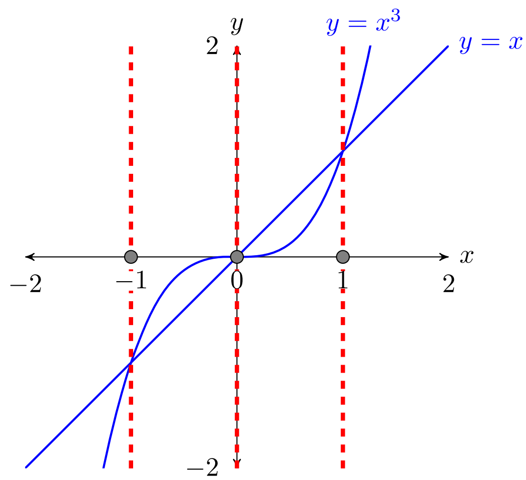
Next, use the **5:intersect** utility from the CALC menu to find the three points of intersection.







Report the results on your homework as follows.



Hence, the solutions of $x^3 = x$ are $x = -1$, $x = 0$, and $x = 1$. Note how these agree with the algebraic solution.

83. Algebraic solution. The equation is nonlinear, so make one side zero. Subtract x from both sides.

$$\begin{aligned} 4x^3 &= x \\ 4x^3 - x &= 0 \end{aligned}$$

Factor out the GCF.

$$x(4x^2 - 1) = 0$$

Use the difference of squares pattern $a^2 - b^2 = (a + b)(a - b)$ to factor.

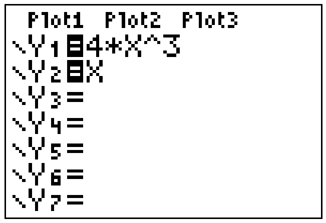
$$x(2x + 1)(2x - 1) = 0$$

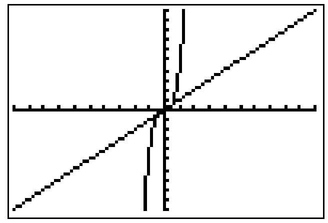
Use the zero product property to set all three factors equal to zero, then solve the resulting equations.

$$x = 0 \quad \text{or} \quad 2x + 1 = 0 \quad \text{or} \quad 2x - 1 = 0$$
$$x = -\frac{1}{2} \qquad\qquad x = \frac{1}{2}$$

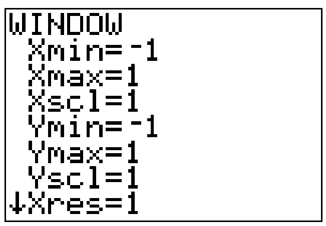
Hence, the solutions are $x = 0$, $x = -1/2$, and $x = 1/2$.

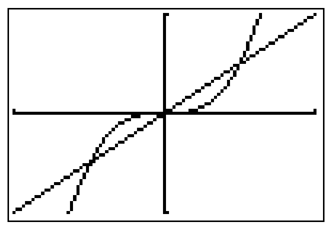
Calculator solution. Load the left- and right-hand sides of the equation $4x^3 = x$ into **Y1** and **Y2** in the Y= menu, then select **6:ZStandard** from the ZOOM menu to produce the following graph.



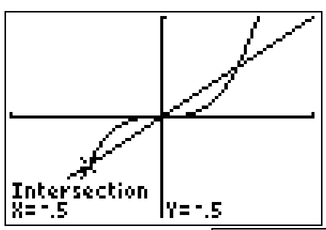


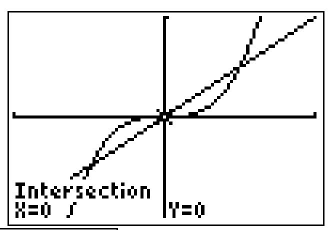
We need to adjust the WINDOW parameters to make visible the points of intersection. It seems that the points of intersection occur near the origin. After some experimentation, we decided on the following parameters which produced the accompanying image.

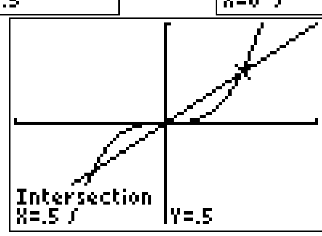




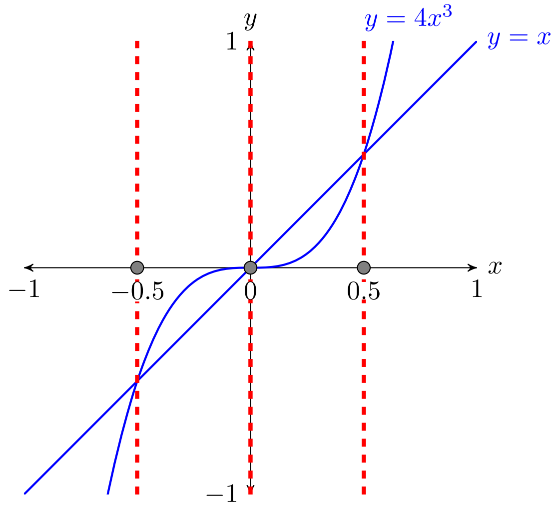
Next, use the **5:intersect** utility from the CALC menu to find the three points of intersection.







Report the results on your homework as follows.



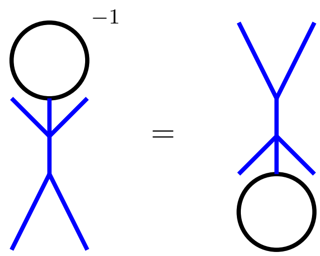
Hence, the solutions of $4x^3 = x$ are $x = -0.5$, $x = 0$, and $x = 0.5$. Note how these agree with the algebraic solution, particularly since $-0.5 = -1/2$ and $0.5 = 1/2$.

Chapter 7

Rational Functions

7.1 Negative Exponents

1. Recall what it means to raise a number to a power of -1.



Thus:

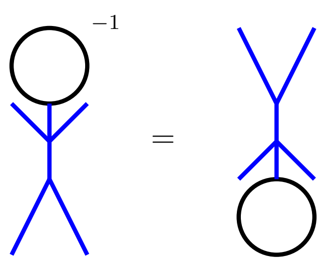
$$\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$$

An exponent of -1 has nothing to do with the sign of the answer. Invert a positive number, you get a positive result. Invert a negative number, you get a negative result.

Thus, to raise a number to a power of -1, simply invert the number.

$$\left(\frac{1}{7}\right)^{-1}=7$$

3. Recall what it means to raise a number to a power of -1.



Thus:

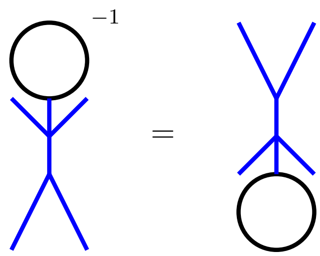
$$\left(\frac{a}{b}\right)^{-1}=\frac{b}{a}$$

An exponent of -1 has nothing to do with the sign of the answer. Invert a positive number, you get a positive result. Invert a negative number, you get a negative result.

Thus, to raise a number to a power of -1, simply invert the number.

$$\left(-\frac{8}{9}\right)^{-1}=-\frac{9}{8}$$

5. Recall what it means to raise a number to a power of -1.



Thus:

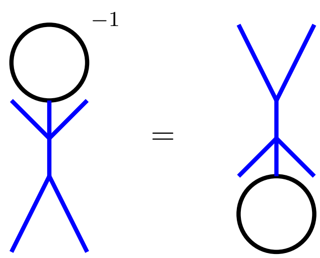
$$a^{-1} = \frac{1}{a}$$

An exponent of -1 has nothing to do with the sign of the answer. Invert a positive number, you get a positive result. Invert a negative number, you get a negative result.

Thus, to raise a number to a power of -1, simply invert the number.

$$(18)^{-1} = \frac{1}{18}$$

7. Recall what it means to raise a number to a power of -1.



Thus:

$$a^{-1} = \frac{1}{a}$$

An exponent of -1 has nothing to do with the sign of the answer. Invert a positive number, you get a positive result. Invert a negative number, you get a negative result.

Thus, to raise a number to a power of -1, simply invert the number.

$$(16)^{-1} = \frac{1}{16}$$

9. In this case, we are multiplying like bases, so we'll use the following law of exponents:

$$a^m a^n = a^{m+n}$$

. That is, we'll repeat the base, then add the exponents.

$$\begin{aligned} a^{-9} \cdot a^3 &= a^{-9+3} && \text{Repeat base, add exponents.} \\ &= a^{-6} && \text{Simplify.} \end{aligned}$$

11. In this case, we are multiplying like bases, so we'll use the following law of exponents:

$$a^m a^n = a^{m+n}$$

. That is, we'll repeat the base, then add the exponents.

$$\begin{aligned} b^{-9} \cdot b^8 &= b^{-9+8} && \text{Repeat base, add exponents.} \\ &= b^{-1} && \text{Simplify.} \end{aligned}$$

13. In this case, we are multiplying like bases, so we'll use the following law of exponents:

$$a^m a^n = a^{m+n}$$

. That is, we'll repeat the base, then add the exponents.

$$\begin{aligned} 2^9 \cdot 2^{-4} &= 2^{9+(-4)} && \text{Repeat base, add exponents.} \\ &= 2^5 && \text{Simplify.} \end{aligned}$$

15. In this case, we are multiplying like bases, so we'll use the following law of exponents:

$$a^m a^n = a^{m+n}$$

. That is, we'll repeat the base, then add the exponents.

$$\begin{aligned} 9^{-6} \cdot 9^{-5} &= 9^{-6+(-5)} && \text{Repeat base, add exponents.} \\ &= 9^{-11} && \text{Simplify.} \end{aligned}$$

17. In this case, we are dividing like bases, so we'll use the following law of exponents:

$$\frac{a^m}{a^n} = a^{m-n}$$

That is, we'll repeat the base, then subtract the exponents.

$$\begin{aligned} \frac{2^6}{2^{-8}} &= 2^{6-(-8)} && \text{Repeat base, subtract exponents.} \\ &= 2^{6+8} && \text{Add the oopposite.} \\ &= 2^{14} && \text{Simplify.} \end{aligned}$$

19. In this case, we are dividing like bases, so we'll use the following law of exponents:

$$\frac{a^m}{a^n} = a^{m-n}$$

That is, we'll repeat the base, then subtract the exponents.

$$\begin{aligned} \frac{z^{-1}}{z^9} &= z^{-1-9} && \text{Repeat base, subtract exponents.} \\ &= z^{-1+(-9)} && \text{Add the opposite.} \\ &= z^{-10} && \text{Simplify.} \end{aligned}$$

21. In this case, we are dividing like bases, so we'll use the following law of exponents:

$$\frac{a^m}{a^n} = a^{m-n}$$

That is, we'll repeat the base, then subtract the exponents.

$$\begin{aligned} \frac{w^{-9}}{w^7} &= w^{-9-7} && \text{Repeat base, subtract exponents.} \\ &= w^{-9+(-7)} && \text{Add the opposite.} \\ &= w^{-16} && \text{Simplify.} \end{aligned}$$

23. In this case, we are dividing like bases, so we'll use the following law of exponents:

$$\frac{a^m}{a^n} = a^{m-n}$$

That is, we'll repeat the base, then subtract the exponents.

$$\begin{aligned} \frac{7^{-3}}{7^{-1}} &= 7^{-3-(-1)} && \text{Repeat base, subtract exponents.} \\ &= 7^{-3+1} && \text{Add the oopposite.} \\ &= 7^{-2} && \text{Simplify.} \end{aligned}$$

25. In this case, we are raising a power to a power, so we'll use the following law of exponents:

$$(a^m)^n = a^{mn}$$

That is, we'll repeat the base, then multiply the exponents.

$$\begin{aligned} \left(t^{-1}\right)^4 &= t^{-1(4)} && \text{Repeat base, multiply exponents.} \\ &= t^{-4} && \text{Simplify.} \end{aligned}$$

27. In this case, we are raising a power to a power, so we'll use the following law of exponents:

$$(a^m)^n = a^{mn}$$

That is, we'll repeat the base, then multiply the exponents.

$$\begin{aligned} \left(6^{-6}\right)^7 &= 6^{-6(7)} && \text{Repeat base, multiply exponents.} \\ &= 6^{-42} && \text{Simplify.} \end{aligned}$$

29. In this case, we are raising a power to a power, so we'll use the following law of exponents:

$$(a^m)^n = a^{mn}$$

That is, we'll repeat the base, then multiply the exponents.

$$\begin{aligned} \left(z^{-9}\right)^{-9} &= z^{-9(-9)} && \text{Repeat base, multiply exponents.} \\ &= z^{81} && \text{Simplify.} \end{aligned}$$

31. In this case, we are raising a power to a power, so we'll use the following law of exponents:

$$(a^m)^n = a^{mn}$$

That is, we'll repeat the base, then multiply the exponents.

$$\begin{aligned} \left(3^{-2}\right)^{3} &= 3^{-2(3)} && \text{Repeat base, multiply exponents.} \\ &= 3^{-6} && \text{Simplify.} \end{aligned}$$

33. Note that 4^{-3} is equivalent to $\left(4^3\right)^{-1}$. They are equivalent because the laws of exponents instruct us to multiply the exponents when raising a power to another power. To evaluate $\left(4^3\right)^{-1}$, we first cube, then invert the result.

$$\begin{aligned} 4^{-3} &= \left(4^3\right)^{-1} && \text{Repeat base and multiply exponents.} \\ &= 64^{-1} && \text{Cube: } 4^3 = 64. \\ &= \frac{1}{64} && \text{Invert.} \end{aligned}$$

Mental approach. It is much easier to simplify this expression mentally. In the expression 4^{-3}, the 3 means cube and the minus sign in front of the 3 means "invert." To do this problem in our head, start with 4 and cube to get 64, then invert to get 1/64.

35. Note that 2^{-4} is equivalent to $\left(2^4\right)^{-1}$. They are equivalent because the laws of exponents instruct us to multiply the exponents when raising a power to another power. To evaluate $\left(2^4\right)^{-1}$, we first raise to the fourth power, then invert the result.

$$\begin{aligned} 2^{-4} &= \left(2^4\right)^{-1} && \text{Repeat base and multiply exponents.} \\ &= 16^{-1} && \text{Raise to the fourth power: } 2^4 = 16. \\ &= \frac{1}{16} && \text{Invert.} \end{aligned}$$

Mental approach. It is much easier to simplify this expression mentally. In the expression 2^{-4}, the 4 means raise to the fourth power and the minus sign in front of the 4 means "invert." To do this problem in our head, start with 2 and raise to the fourth power to get 16, then invert to get 1/16.

37. Note:

$$\left(\frac{1}{2}\right)^{-5} \qquad \text{is equivalent to} \qquad \left[\left(\frac{1}{2}\right)^{5}\right]^{-1}$$

They are equivalent because the laws of exponents instruct us to multiply the exponents when raising a power to another power. To evaluate $[(1/2)^5]^{-1}$, we first raise to the fifth power, then invert the result.

$$\begin{aligned}
\left(\frac{1}{2}\right)^{-5} &= \left[\left(\frac{1}{2}\right)^{5}\right]^{-1} && \text{Repeat base and multiply exponents.} \\
&= \left(\frac{1}{32}\right)^{-1} && \text{Raise to the fifth power: } (1/2)^5 = 1/32. \\
&= 32 && \text{Invert.}
\end{aligned}$$

Mental approach. It is much easier to simplify this expression mentally. In the expression $(1/2)^{-5}$, the 5 means raise to the fifth power and the minus sign in front of the 5 means "invert." To do this problem in our head, start with $1/2$ and raise to the fifth power to get $1/32$, then invert to get 32.

39. Note:

$$\left(-\frac{1}{2}\right)^{-5} \qquad \text{is equivalent to} \qquad \left[\left(-\frac{1}{2}\right)^{5}\right]^{-1}$$

They are equivalent because the laws of exponents instruct us to multiply the exponents when raising a power to another power. To evaluate $[(-1/2)^5]^{-1}$, we first raise to the fifth power, then invert the result.

$$\begin{aligned}
\left(-\frac{1}{2}\right)^{-5} &= \left[\left(-\frac{1}{2}\right)^{5}\right]^{-1} && \text{Repeat base and multiply exponents.} \\
&= \left(-\frac{1}{32}\right)^{-1} && \text{Raise to the fifth power: } (-1/2)^5 = -1/32. \\
&= -32 && \text{Invert.}
\end{aligned}$$

Mental approach. It is much easier to simplify this expression mentally. In the expression $(-1/2)^{-5}$, the 5 means raise to the fifth power and the minus sign in front of the 5 means "invert." To do this problem in our head, start with $-1/2$ and raise to the fifth power to get $-1/32$, then invert to get -32.

41. All the operators involved are multiplication, so the commutative and associative properties of multiplication allow us to change the order and grouping. We'll show this regrouping here, but this step can be done mentally.

$$\left(4u^{-6}v^{-9}\right)\left(5u^{8}v^{-8}\right) = [(4)(5)](u^{-6}u^{8})(v^{-9}v^{-8})$$

Multiply 4 and 5 to get 20, then repeat the bases and add the exponents.

$$\begin{aligned} &= 20u^{-6+8}v^{-9+(-8)} \\ &= 20u^{2}v^{-17} \end{aligned}$$

In the solution above, we've probably shown way too much work. It's far easier to perform all of these steps mentally, multiplying the 4 and the 5 to get 20, then repeating bases and adding exponents, as in:

$$\left(4u^{-6}v^{-9}\right)\left(5u^{8}v^{-8}\right) = 20u^{2}v^{-17}$$

43. All the operators involved are multiplication, so the commutative and associative properties of multiplication allow us to change the order and grouping. We'll show this regrouping here, but this step can be done mentally.

$$\left(6x^{-6}y^{-5}\right)\left(-4x^{4}y^{-2}\right) = [(6)(-4)](x^{-6}x^{4})(y^{-5}y^{-2})$$

Multiply 6 and -4 to get -24, then repeat the bases and add the exponents.

$$\begin{aligned} &= -24x^{-6+4}y^{-5+(-2)} \\ &= -24x^{-2}y^{-7} \end{aligned}$$

In the solution above, we've probably shown way too much work. It's far easier to perform all of these steps mentally, multiplying the 6 and the -4 to get -24, then repeating bases and adding exponents, as in:

$$\left(6x^{-6}y^{-5}\right)\left(-4x^{4}y^{-2}\right) = -24x^{-2}y^{-7}$$

45. The simplest approach is to first write the expression as a product.

$$\frac{-6\,x^{7}\,z^{9}}{4\,x^{-9}\,z^{-2}} = \frac{-6}{4}\cdot\frac{x^{7}}{x^{-9}}\cdot\frac{z^{9}}{z^{-2}}$$

Reduce $-6/4$ to lowest terms. Because we are dividing like bases, we repeat the bases and subtract the exponents.

$$\begin{aligned} &= -\frac{3}{2}\,x^{7-(-9)}\,z^{9-(-2)} \\ &= -\frac{3}{2}\,x^{7+9}\,z^{9+2} \\ &= -\frac{3}{2}\,x^{16}\,z^{11} \end{aligned}$$

In the solution above, we've shown way too much work. It's far easier to imagine writing the expression as a product, reducing $-6/4$, then repeating bases and subtracting exponents, as in:

$$\frac{-6\,x^7\,z^9}{4\,x^{-9}\,z^{-2}} = -\frac{3}{2}\,x^{16}\,z^{11}$$

47. The simplest approach is to first write the expression as a product.

$$\frac{-6\,a^9\,c^6}{-4\,a^{-5}\,c^{-7}} = \frac{-6}{-4} \cdot \frac{a^9}{a^{-5}} \cdot \frac{c^6}{c^{-7}}$$

Reduce $-6/(-4)$ to lowest terms. Because we are dividing like bases, we repeat the bases and subtract the exponents.

$$\begin{aligned} &= \frac{3}{2}\,a^{9-(-5)}\,c^{6-(-7)} \\ &= \frac{3}{2}\,a^{9+5}\,c^{6+7} \\ &= \frac{3}{2}\,a^{14}\,c^{13} \end{aligned}$$

In the solution above, we've shown way too much work. It's far easier to imagine writing the expression as a product, reducing $-6/(-4)$, then repeating bases and subtracting exponents, as in:

$$\frac{-6\,a^9\,c^6}{-4\,a^{-5}\,c^{-7}} = \frac{3}{2}\,a^{14}\,c^{13}$$

49. The law of exponents $(ab)^n = a^n b^n$ says that when you raise a product to a power, you must raise each factor to that power. So we begin by raising each factor to the power -5.

$$\left(2\,v^{-2}\,w^4\right)^{-5} = 2^{-5}\,\left(v^{-2}\right)^{-5}\,\left(w^4\right)^{-5}$$

To raise 2 to the -5, first raise 2 to the fifth power, then invert: $2^{-5} = 1/32$. Next, raising a power to a power requires that we repeat the base and multiply exponents.

$$\begin{aligned} &= 1/32\,v^{-2(-5)}\,w^{4(-5)} \\ &= 32\,v^{10}\,w^{-20} \end{aligned}$$

In the solution above , we've shown way too much work. It's far easier to raise each factor to the fifth power mentally: $2^{-5} = 1/32$, then multiply each exponent on the remaining factors by 5, as in

$$\left(2\,v^{-2}\,w^4\right)^{-5} = 1/32\,v^{10}\,w^{-20}$$

51. The law of exponents $(ab)^n = a^n b^n$ says that when you raise a product to a power, you must raise each factor to that power. So we begin by raising each factor to the fourth power.

$$\left(3\,x^{-1}\,y^7\right)^4 = 3^4\,\left(x^{-1}\right)^4\,\left(y^7\right)^4$$

Note that $3^4 = 81$. Next, raising a power to a power requires that we repeat the base and multiply exponents.

$$\begin{aligned} &= 81\,x^{-1(4)}\,y^{7(4)} \\ &= 81\,x^{-4}\,y^{28} \end{aligned}$$

In the solution above , we've shown way too much work. It's far easier to raise each factor to the fourth power mentally: $3^4 = 81$, then multiply each exponent on the remaining factors by 4, as in

$$\left(3\,x^{-1}\,y^7\right)^4 = 81\,x^{-4}\,y^{28}$$

53. The law of exponents $(ab)^n = a^n b^n$ says that when you raise a product to a power, you must raise each factor to that power. So we begin by raising each factor to the fifth power.

$$\left(2\,x^6\,z^{-7}\right)^5 = 2^5\,\left(x^6\right)^5\,\left(z^{-7}\right)^5$$

Note that $2^5 = 32$. Next, raising a power to a power requires that we repeat the base and multiply exponents.

$$\begin{aligned} &= 32\,x^{6(5)}\,z^{-7(5)} \\ &= 32\,x^{30}\,z^{-35} \end{aligned}$$

In the solution above , we've shown way too much work. It's far easier to raise each factor to the fifth power mentally: $2^5 = 32$, then multiply each exponent on the remaining factors by 5, as in

$$\left(2\,x^6\,z^{-7}\right)^5 = 32\,x^{30}\,z^{-35}$$

55. The law of exponents $(ab)^n = a^n b^n$ says that when you raise a product to a power, you must raise each factor to that power. So we begin by raising each factor to the power -4.

$$\left(2\,a^{-4}\,c^8\right)^{-4} = 2^{-4}\,\left(a^{-4}\right)^{-4}\,\left(c^8\right)^{-4}$$

To raise 2 to the -4, first raise 2 to the fourth power, then invert: $2^{-4} = 1/16$. Next, raising a power to a power requires that we repeat the base and multiply exponents.

$$\begin{aligned} &= 1/16\, a^{-4(-4)}\, c^{8(-4)} \\ &= 16\, a^{16}\, c^{-32} \end{aligned}$$

In the solution above , we've shown way too much work. It's far easier to raise each factor to the fourth power mentally: $2^{-4} = 1/16$, then multiply each exponent on the remaining factors by 4, as in

$$\left(2\, a^{-4}\, c^{8}\right)^{-4} = 1/16\, a^{16}\, c^{-32}$$

57. Multiply numerator and denominator by y^2.

$$\begin{aligned} \frac{x^5y^{-2}}{z^3} &= \frac{x^5y^{-2}}{z^3} \cdot \frac{y^2}{y^2} && \text{Multiply numerator and denominator by } y^2 \\ &= \frac{x^5y^0}{y^2z^3} && \text{Simplify: } y^{-2}\, y^2 = y^0 \\ &= \frac{x^5}{y^2z^3} && \text{Simplify: } y^0 = 1 \end{aligned}$$

59. Multiply numerator and denominator by s^2.

$$\begin{aligned} \frac{r^9s^{-2}}{t^3} &= \frac{r^9s^{-2}}{t^3} \cdot \frac{s^2}{s^2} && \text{Multiply numerator and denominator by } s^2 \\ &= \frac{r^9s^0}{s^2t^3} && \text{Simplify: } s^{-2}\, s^2 = s^0 \\ &= \frac{r^9}{s^2t^3} && \text{Simplify: } s^0 = 1 \end{aligned}$$

61. Multiply numerator and denominator by y^8.

$$\begin{aligned} \frac{x^3}{y^{-8}z^5} &= \frac{x^3}{y^{-8}z^5} \cdot \frac{y^8}{y^8} && \text{Multiply numerator and denominator by } y^8 \\ &= \frac{x^3y^8}{y^0z^5} && \text{Simplify: } y^{-8}\, y^8 = y^0 \\ &= \frac{x^3y^8}{z^5} && \text{Simplify: } y^0 = 1 \end{aligned}$$

63. Multiply numerator and denominator by v^4.

$$\begin{aligned}
\frac{u^9}{v^{-4}w^7} &= \frac{u^9}{v^{-4}w^7} \cdot \frac{v^4}{v^4} && \text{Multiply numerator and denominator by } v^4 \\
&= \frac{u^9v^4}{v^0w^7} && \text{Simplify: } v^{-4}\,v^4 = v^0 \\
&= \frac{u^9v^4}{w^7} && \text{Simplify: } v^0 = 1
\end{aligned}$$

65. Multiply 7 and -7 to get -49, then repeat the base and add the exponents.

$$(7x^{-1})(-7x^{-1}) = -49x^{-2}$$

The negative exponent means invert, so we can replace x^{-2} with $1/x^2$, then multiply numerators and denominators.

$$\begin{aligned}
&= \frac{-49}{1} \cdot \frac{1}{x^2} \\
&= \frac{-49}{x^2}
\end{aligned}$$

67. Multiply 8 and 7 to get 56, then repeat the base and add the exponents.

$$(8a^{-8})(7a^{-7}) = 56a^{-15}$$

The negative exponent means invert, so we can replace a^{-15} with $1/a^{15}$, then multiply numerators and denominators.

$$\begin{aligned}
&= \frac{56}{1} \cdot \frac{1}{a^{15}} \\
&= \frac{56}{a^{15}}
\end{aligned}$$

69. Write the expression as a product.

$$\frac{4x^{-9}}{8x^3} = \frac{4}{8} \cdot \frac{x^{-9}}{x^3}$$

Reduce 4/8 to lowest terms, then repeat the base and subtract the exponents.

$$= \frac{1}{2} \cdot x^{-12}$$

The negative exponent means invert, so we can replace x^{-12} with $1/x^{12}$, then multiply numerators and denominators.

$$\begin{aligned}
&= \frac{1}{2} \cdot \frac{1}{x^{12}} \\
&= \frac{1}{2x^{12}}
\end{aligned}$$

71. Write the expression as a product.

$$\frac{6c^2}{-4c^7} = \frac{6}{-4} \cdot \frac{c^2}{c^7}$$

Reduce $6/(-4)$ to lowest terms, then repeat the base and subtract the exponents.

$$= -\frac{3}{2} \cdot c^{-5}$$

The negative exponent means invert, so we can replace c^{-5} with $1/c^5$, then multiply numerators and denominators.

$$\begin{aligned} &= -\frac{3}{2} \cdot \frac{1}{c^5} \\ &= -\frac{3}{2c^5} \end{aligned}$$

73. First raise each factor to the -4 power.

$$(-3s^9)^{-4} = (-3)^{-4}(s^9)^{-4}$$

Now, raise -3 to the fourth power, then invert to get $1/81$. Next, because we are raising a power to a power, repeat the base and multiply the exponents.

$$= \frac{1}{81}\, s^{-36}$$

The negative exponent means invert, so we can replace s^{-36} with $1/s^{36}$, then multiply numerators and denominators.

$$\begin{aligned} &= \frac{1}{81} \cdot \frac{1}{s^{36}} \\ &= \frac{1}{81s^{36}} \end{aligned}$$

75. First raise each factor to the -5 power.

$$(2y^4)^{-5} = 2^{-5}(y^4)^{-5}$$

Now, raise 2 to the fifth power, then invert to get $1/32$. Next, because we are raising a power to a power, repeat the base and multiply the exponents.

$$= \frac{1}{32}\, y^{-20}$$

The negative exponent means invert, so we can replace y^{-20} with $1/y^{20}$, then multiply numerators and denominators.

$$
\begin{aligned}
&= \frac{1}{32} \cdot \frac{1}{y^{20}} \\
&= \frac{1}{32y^{20}}
\end{aligned}
$$

7.2 Scientific Notation

1. When the power of ten is a negative integer, it dictates the total number of decimal places to use in expressing the number in decimal form. In the case of 10^{-4}, the exponent -4 tells us to use 4 decimal places. This requires that we write a decimal point, 3 zeros, then the number 1.

$$10^{-4} = 0.0001$$

3. When the power of ten is a negative integer, it dictates the total number of decimal places to use in expressing the number in decimal form. In the case of 10^{-8}, the exponent -8 tells us to use 8 decimal places. This requires that we write a decimal point, 7 zeros, then the number 1.

$$10^{-8} = 0.00000001$$

5. When the power of ten is a whole number, it dictates the number of zeros that you should add after the number 1. In the case of 10^8, the exponent 8 tells us to write 8 zeros after the number 1.

$$10^8 = 100000000$$

Next, we delimit our answer with commas in the appropriate places.

$$10^8 = 100,000,000$$

7. When the power of ten is a whole number, it dictates the number of zeros that you should add after the number 1. In the case of 10^7, the exponent 7 tells us to write 7 zeros after the number 1.

$$10^7 = 10000000$$

Next, we delimit our answer with commas in the appropriate places.

$$10^7 = 10,000,000$$

9. When multiplying by a power of ten, such as 10^n, the exponent tells us how many places to move the decimal point. If n is greater than or equal to zero (nonegative), then we move the decimal point n places to the right. If n is less than zero (negative), then we move the decimal point n places to the left. In the case of 6506399.9×10^{-4}, the exponent is negative, so we move the decimal point 4 places to the left. Hence:

$$6506399.9 \times 10^{-4} = 650.63999$$

11. When multiplying by a power of ten, such as 10^n, the exponent tells us how many places to move the decimal point. If n is greater than or equal to zero (nonegative), then we move the decimal point n places to the right. If n is less than zero (negative), then we move the decimal point n places to the left. In the case of 3959.430928×10^2, the exponent is nonnegative, so we move the decimal point 2 places to the right. Hence:

$$3959.430928 \times 10^2 = 395943.0928$$

13. When multiplying by a power of ten, such as 10^n, the exponent tells us how many places to move the decimal point. If n is greater than or equal to zero (nonegative), then we move the decimal point n places to the right. If n is less than zero (negative), then we move the decimal point n places to the left. In the case of 440906.28×10^{-4}, the exponent is negative, so we move the decimal point 4 places to the left. Hence:

$$440906.28 \times 10^{-4} = 44.090628$$

15. When multiplying by a power of ten, such as 10^n, the exponent tells us how many places to move the decimal point. If n is greater than or equal to zero (nonegative), then we move the decimal point n places to the right. If n is less than zero (negative), then we move the decimal point n places to the left. In the case of 849.855115×10^4, the exponent is nonnegative, so we move the decimal point 4 places to the right. Hence:

$$849.855115 \times 10^4 = 8498551.15$$

17. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 390000, we must move the decimal point in the number 390000 five

places to the left, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$390000 = 3.9 \times 10^5$$

Check: To check the solution, recall that multiplying by 10^5 moves the decimal point five places to the right. Hence:

$$3.9 \times 10^5 = 390000$$

Thus, the solution checks.

19. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 0.202, we must move the decimal point in the number 0.202 one place to the right, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$0.202 = 2.02 \times 10^{-1}$$

Check: To check the solution, recall that multiplying by 10^{-1} moves the decimal point one place to the left. Hence:

$$2.02 \times 10^{-1} = 0.202$$

Thus, the solution checks.

21. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 0.81, we must move the decimal point in the number 0.81 one place to the right, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$0.81 = 8.1 \times 10^{-1}$$

Check: To check the solution, recall that multiplying by 10^{-1} moves the decimal point one place to the left. Hence:

$$8.1 \times 10^{-1} = 0.81$$

Thus, the solution checks.

23. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 0.0007264, we must move the decimal point in the number 0.0007264 four places to the right, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$0.0007264 = 7.264 \times 10^{-4}$$

Check: To check the solution, recall that multiplying by 10^{-4} moves the decimal point four places to the left. Hence:

$$7.264 \times 10^{-4} = 0.0007264$$

Thus, the solution checks.

25. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 0.04264×10^{-4}, we'll first convert the number 0.04264 into scientific notation, ignoring 10^{-4} for a moment. To do that, we must move the decimal point in the number 0.04264 two places to the right, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$0.04264 \times 10^{-4} = 4.264 \times 10^{-2} \times 10^{-4}$$

To convert to a single power of 10, repeat the base and add the exponents.

$$\begin{aligned} &= 4.264 \times 10^{-2+(-4)} \\ &= 4.264 \times 10^{-6} \end{aligned}$$

Thus, $0.04264 \times 10^{-4} = 4.264 \times 10^{-6}$.

27. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 130000×10^3, we'll first convert the number 130000 into

scientific notation, ignoring 10^3 for a moment. To do that, we must move the decimal point in the number 130000 five places to the left, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$130000 \times 10^3 = 1.3 \times 10^5 \times 10^3$$

To convert to a single power of 10, repeat the base and add the exponents.

$$\begin{aligned} &= 1.3 \times 10^{5+3} \\ &= 1.3 \times 10^8 \end{aligned}$$

Thus, $130000 \times 10^3 = 1.3 \times 10^8$.

29. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 30.04×10^5, we'll first convert the number 30.04 into scientific notation, ignoring 10^5 for a moment. To do that, we must move the decimal point in the number 30.04 one place to the left, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$30.04 \times 10^5 = 3.004 \times 10^1 \times 10^5$$

To convert to a single power of 10, repeat the base and add the exponents.

$$\begin{aligned} &= 3.004 \times 10^{1+5} \\ &= 3.004 \times 10^6 \end{aligned}$$

Thus, $30.04 \times 10^5 = 3.004 \times 10^6$.

31. Converting a number into scientifc notation requires that we convert the given number into the form $a \times 10^k$, where k is an integer and $1 \leq |a| < 10$. The requirement $1 \leq |a| < 10$ says that the magnitude of the number a must be greater than or equal to 1, but strictly less than 10. This means that there must be a single nonzero digit to the left of the decimal point. In the case of the number 0.011×10^1, we'll first convert the number 0.011 into scientific notation, ignoring 10^1 for a moment. To do that, we must move the decimal point in the number 0.011 two places to the right, then compensate by multiplying by multiplying by a power of ten so that the result is still identical to the original number. That is:

$$0.011 \times 10^1 = 1.1 \times 10^{-2} \times 10^1$$

To convert to a single power of 10, repeat the base and add the exponents.

$$\begin{aligned} &= 1.1 \times 10^{-2+1} \\ &= 1.1 \times 10^{-1} \end{aligned}$$

Thus, $0.011 \times 10^1 = 1.1 \times 10^{-1}$.

33. The notation 1.134E-1 is the calculator's way of expressing scientific notation. That is, the notation 1.134E-1 is equivalent to the symbolism 1.134×10^{-1}. Because the power of ten is negative, we move the decimal point 1 place to the left. Thus:

$$\begin{aligned} 1.134\text{E-}1 &= 1.134 \times 10^{-1} \\ &= 0.1134 \end{aligned}$$

35. The notation 1.556E-2 is the calculator's way of expressing scientific notation. That is, the notation 1.556E-2 is equivalent to the symbolism 1.556×10^{-2}. Because the power of ten is negative, we move the decimal point 2 places to the left. Thus:

$$\begin{aligned} 1.556\text{E-}2 &= 1.556 \times 10^{-2} \\ &= 0.01556 \end{aligned}$$

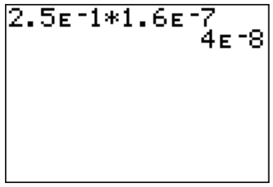
37. The notation 1.748E-4 is the calculator's way of expressing scientific notation. That is, the notation 1.748E-4 is equivalent to the symbolism 1.748×10^{-4}. Because the power of ten is negative, we move the decimal point 4 places to the left. Thus:

$$\begin{aligned} 1.748\text{E-}4 &= 1.748 \times 10^{-4} \\ &= 0.0001748 \end{aligned}$$

39. We'll use the approximations $2.5 \approx 3$ and $1.6 \approx 2$, which enable us to write:

$$\begin{aligned} (2.5 \times 10^{-1})(1.6 \times 10^{-7}) &\approx (3 \times 10^{-1})(2 \times 10^{-7}) \\ &\approx 6 \times 10^{-1+(-7)} \\ &\approx 6 \times 10^{-8} \end{aligned}$$

Next, enter $(2.5 \times 10^{-1})(1.6 \times 10^{-7})$ as **2.5E-1*1.6E-7** on your calculator, yielding the result shown in the following window.

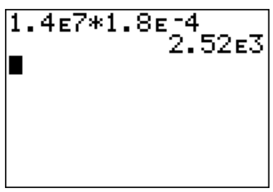


Hence, $(2.5 \times 10^{-1})(1.6 \times 10^{-7}) = 4 \times 10^{-8}$.

41. We'll use the approximations $1.4 \approx 1$ and $1.8 \approx 2$, which enable us to write:

$$\begin{aligned}(1.4 \times 10^{7})(1.8 \times 10^{-4}) &\approx (1 \times 10^{7})(2 \times 10^{-4})\\ &\approx 2 \times 10^{7+(-4)}\\ &\approx 2 \times 10^{3}\end{aligned}$$

Next, enter $(1.4\times 10^{7})(1.8\times 10^{-4})$ as **1.4E7*1.8E-4** on your calculator, yielding the result shown in the following window.

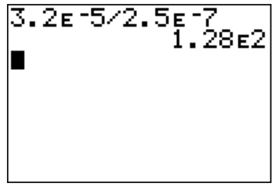


Hence, $(1.4 \times 10^{7})(1.8 \times 10^{-4}) = 2.52 \times 10^{3}$.

43. We'll use the approximations $3.2 \approx 3$ and $2.5 \approx 3$, which enables us to write:

$$\begin{aligned}\frac{3.2 \times 10^{-5}}{2.5 \times 10^{-7}} &\approx \frac{3 \times 10^{-5}}{3 \times 10^{-7}}\\ &\approx \frac{3}{3} \cdot \frac{10^{-5}}{10^{-7}}\\ &\approx 1 \cdot 10^{-5-(-7)}\\ &\approx 1 \times 10^{2}\end{aligned}$$

Push the MODE button, then highlight **SCI** mode and press ENTER. Move your cursor to the same row containing the **FLOAT** command, then highlight the number **2** and press ENTER. Press 2ND MODE to quit the MODE menu. Next, enter $(3.2 \times 10^{-5})/(2.5 \times 10^{-7})$ as **3.2E-5/2.5E-7** on your calculator, yielding the result shown in the following window.

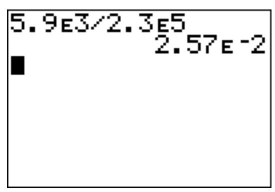


Hence, $(3.2 \times 10^{-5})/(2.5 \times 10^{-7}) \approx 1.28 \times 10^{2}$. Don't forget to return your calculator to its original mode by selecting **NORMAL** and **FLOAT** in the MODE menu.

45. We'll use the approximations $5.9 \approx 6$ and $2.3 \approx 2$, which enables us to write:

$$\begin{aligned}\frac{5.9 \times 10^3}{2.3 \times 10^5} &\approx \frac{6 \times 10^3}{2 \times 10^5} \\ &\approx \frac{6}{2} \cdot \frac{10^3}{10^5} \\ &\approx 3 \cdot 10^{3-5} \\ &\approx 3 \times 10^{-2}\end{aligned}$$

Push the MODE button, then highlight **SCI** mode and press ENTER. Move your cursor to the same row containing the **FLOAT** command, then highlight the number **2** and press ENTER. Press 2ND MODE to quit the MODE menu. Next, enter $(5.9 \times 10^3)/(2.3 \times 10^5)$ as **5.9E3/2.3E5** on your calculator, yielding the result shown in the following window.



Hence, $(5.9 \times 10^3)/(2.3 \times 10^5) \approx 2.57 \times 10^{-2}$. Don't forget to return your calculator to its original mode by selecting **NORMAL** and **FLOAT** in the MODE menu.

47. We need to form the ratio of biomass to the mass of the Earth

$$\begin{aligned}\frac{6.8 \times 10^{13}\,\text{kg}}{5.9736 \times 10^{24}\,\text{kg}} &= \frac{6.8}{5.9736} \cdot \frac{10^{13}}{10^{24}} \\ &\approx 1.14 \times 10^{-11}\end{aligned}$$

Taking our number out of scientific notation we get 0.0000000000114. Changing this to a percent by moving the decimal two places to the right we get that the ratio of biomass to the mass of the Earth is about 0.00000000114%. Not a very large portion of the mass of our planet. Kind of makes you feel a bit insignificant.

49. We are given the distance and the rate. What we want is the time traveled in days and years. To find this we will use the model $D = rt$. Let's solve this for t and we get $t = D/r$. Our distance is given to be $D = 1.43 \times 10^6$ miles and our rate is $r = 65$ mph. Putting these values into our formula yields:

$$\begin{aligned} t &= \frac{1.43 \times 10^6\,\text{miles}}{65\,\text{miles per hour}} \\ &= 0.022 \times 10^6\,\text{hours} \\ &= 2,200\,\text{hours} \end{aligned}$$

We are asked to give our result in days and years.

$$\begin{aligned} 2,200\,\text{hours} &= 2,200\,\text{hours} \times \frac{\text{days}}{24\,\text{hours}} \\ &= 2,200\,\cancel{\text{hours}} \times \frac{\text{days}}{24\,\cancel{\text{hours}}} \\ &\approx 916.7\,\text{days} \\ &= 916.7\,\text{days} \times \frac{\text{years}}{365\,\text{days}} \\ &= 916.7\,\cancel{\text{days}} \times \frac{\text{years}}{365\,\cancel{\text{days}}} \\ &\approx 2.5\,\text{years} \end{aligned}$$

So, you would have to drive nonstop for about 916.7 days or about 2.5 years to cover all the paved roads in the USA. Be sure and bring a lot of coffee.

7.3 Simplifying Rational Expressions

1. Multiply numerators and denominators.

$$\frac{12}{s^2} \cdot \frac{s^5}{9} = \frac{12s^5}{9s^2}$$

Now, there several different ways you can reduce this answer to lowest terms, two of which are shown below.

You can factor numerator and denominator, then cancel common factors.

$$\begin{aligned} \frac{12s^5}{9s^2} &= \frac{2 \cdot 2 \cdot 3 \cdot s \cdot s \cdot s \cdot s \cdot s}{3 \cdot 3 \cdot s \cdot s} \\ &= \frac{2 \cdot 2 \cdot \cancel{3} \cdot \cancel{s} \cdot \cancel{s} \cdot s \cdot s \cdot s}{\cancel{3} \cdot 3 \cdot \cancel{s} \cdot \cancel{s}} \\ &= \frac{4s^3}{3} \end{aligned}$$

Or you can write the answer as a product, repeat the base and subtract exponents.

$$\begin{aligned} \frac{12s^5}{9s^2} &= \frac{12}{9} \cdot \frac{s^5}{s^2} \\ &= \frac{4}{3} \cdot s^3 \\ &= \frac{4s^3}{3} \end{aligned}$$

3. Multiply numerators and denominators.

$$\frac{12}{v^3} \cdot \frac{v^4}{10} = \frac{12v^4}{10v^3}$$

Now, there several different ways you can reduce this answer to lowest terms, two of which are shown below.

You can factor numerator and denominator, then cancel common factors.

$$\begin{aligned} \frac{12v^4}{10v^3} &= \frac{2 \cdot 2 \cdot 3 \cdot v \cdot v \cdot v \cdot v}{2 \cdot 5 \cdot v \cdot v \cdot v} \\ &= \frac{\cancel{2} \cdot 2 \cdot 3 \cdot \cancel{v} \cdot \cancel{v} \cdot \cancel{v} \cdot v}{\cancel{2} \cdot 5 \cdot \cancel{v} \cdot \cancel{v} \cdot \cancel{v}} \\ &= \frac{6v}{5} \end{aligned}$$

Or you can write the answer as a product, repeat the base and subtract exponents.

$$\begin{aligned} \frac{12v^4}{10v^3} &= \frac{12}{10} \cdot \frac{v^4}{v^3} \\ &= \frac{6}{5} \cdot v^1 \\ &= \frac{6v}{5} \end{aligned}$$

5. Invert, then multiply.

$$\begin{aligned} \frac{s^5}{t^4} \div \frac{9s^2}{t^2} &= \frac{s^5}{t^4} \cdot \frac{t^2}{9s^2} \\ &= \frac{s^5t^2}{9s^2t^4} \end{aligned}$$

Now, there several different ways you can reduce this answer to lowest terms, two of which are shown below.

You can factor numerator and denominator, then cancel common factors.

$$\begin{aligned} \frac{s^5t^2}{9s^2t^4} &= \frac{s \cdot s \cdot s \cdot s \cdot s \cdot t \cdot t}{3 \cdot 3 \cdot s \cdot s \cdot t \cdot t \cdot t \cdot t} \\ &= \frac{\cancel{s} \cdot \cancel{s} \cdot s \cdot s \cdot s \cdot \cancel{t} \cdot \cancel{t}}{3 \cdot 3 \cdot \cancel{s} \cdot \cancel{s} \cdot \cancel{t} \cdot \cancel{t} \cdot t \cdot t} \\ &= \frac{s^3}{9t^2} \end{aligned}$$

Or you can write the answer as a product, repeat the base and subtract exponents.

$$\begin{aligned} \frac{s^5t^2}{9s^2t^4} &= \frac{1}{9} \cdot \frac{s^5}{s^2} \cdot \frac{t^2}{t^4} \\ &= \frac{1}{9} \cdot s^3 \cdot t^{-2} \\ &= \frac{s^3}{9t^2} \end{aligned}$$

7. Invert, then multiply.

$$\begin{aligned} \frac{b^4}{c^4} \div \frac{9b^2}{c^2} &= \frac{b^4}{c^4} \cdot \frac{c^2}{9b^2} \\ &= \frac{b^4c^2}{9b^2c^4} \end{aligned}$$

Now, there several different ways you can reduce this answer to lowest terms, two of which are shown below.

You can factor numerator and denominator, then cancel common factors.

$$\begin{aligned}\frac{b^4c^2}{9b^2c^4} &= \frac{b\cdot b\cdot b\cdot b\cdot c\cdot c}{3\cdot 3\cdot b\cdot b\cdot c\cdot c\cdot c\cdot c}\\ &= \frac{\cancel{b}\cdot \cancel{b}\cdot b\cdot b\cdot \cancel{c}\cdot \cancel{c}}{3\cdot 3\cdot \cancel{b}\cdot \cancel{b}\cdot \cancel{c}\cdot \cancel{c}\cdot c\cdot c}\\ &= \frac{b^2}{9c^2}\end{aligned}$$

Or you can write the answer as a product, repeat the base and subtract exponents.

$$\begin{aligned}\frac{b^4c^2}{9b^2c^4} &= \frac{1}{9}\cdot\frac{b^4}{b^2}\cdot\frac{c^2}{c^4}\\ &= \frac{1}{9}\cdot b^2\cdot c^{-2}\\ &= \frac{b^2}{9c^2}\end{aligned}$$

9. Becase we have a common denominator, we can simply add the numerators, placing the answer over the common denominator.

$$\begin{aligned}-\frac{10s}{18}+\frac{19s}{18} &= \frac{-10s+19s}{18} && \text{Add the numerators over the common denominator.}\\ &= \frac{9s}{18} && \text{Simplify: } -10s+19s=9s\\ &= \frac{s}{2} && \text{Reduce.}\end{aligned}$$

11. Becase we have a common denominator, we can simply subtract the numerators, placing the answer over the common denominator.

$$\begin{aligned}\frac{5}{9c}-\frac{17}{9c} &= \frac{5-17}{9c} && \text{Subtract the numerators over the common denominator.}\\ &= \frac{-12}{9c} && \text{Subtract: } 5-17=-12\\ &= -\frac{4}{3c} && \text{Reduce.}\end{aligned}$$

13. Becase we have a common denominator, we can simply subtract the numerators, placing the answer over the common denominator.

$$\begin{aligned}-\frac{8x}{15yz}-\frac{16x}{15yz} &= \frac{-8x-16x}{15yz} && \text{Subtract the numerators over the common denominator.}\\ &= \frac{-24}{15yz} && \text{Subtract: } -8x-16x=-24x\\ &= -\frac{8x}{5yz} && \text{Reduce.}\end{aligned}$$

15. The smallest number divisible by both 10 and 2 is 10; i.e., $\text{LCD}(10, 2) = 10$. We must first make equivalent fractions with a common denominator of 10.

$$\begin{aligned}
\frac{9z}{10} + \frac{5z}{2} &= \frac{9z}{10} \cdot \frac{1}{1} + \frac{5z}{2} \cdot \frac{5}{5} && \text{Make equivalent fractions} \\
&= \frac{9z}{10} + \frac{25z}{10} && \text{with LCD} = 10.
\end{aligned}$$

We can now add the numerators and put the result over the common denominator.

$$\begin{aligned}
&= \frac{34z}{10} && \text{Add: } 9z + 25z = 34z \\
&= \frac{17z}{5} && \text{Reduce.}
\end{aligned}$$

17. The smallest expression divisible by both $10v$ and $5v$ is $10v$; i.e., $\text{LCD}(10v, 5v) = 10v$. We must first make equivalent fractions with a common denominator of $10v$.

$$\begin{aligned}
\frac{3}{10v} - \frac{4}{5v} &= \frac{3}{10v} \cdot \frac{1}{1} - \frac{4}{5v} \cdot \frac{2}{2} && \text{Make equivalent fractions} \\
&= \frac{3}{10v} - \frac{8}{10v} && \text{with LCD} = 10v.
\end{aligned}$$

We can now subtract the numerators and put the result over the common denominator.

$$\begin{aligned}
&= \frac{-5}{10v} && \text{Subtract: } 3 - 8 = -5 \\
&= -\frac{1}{2v} && \text{Reduce.}
\end{aligned}$$

19. The smallest expression divisible by both $5st$ and $10st$ is $10st$; i.e., $\text{LCD}(5st, 10st) = 10st$. We must first make equivalent fractions with a common denominator of $10st$.

$$\begin{aligned}
-\frac{8r}{5st} - \frac{9r}{10st} &= -\frac{8r}{5st} \cdot \frac{2}{2} - \frac{9r}{10st} \cdot \frac{1}{1} && \text{Make equivalent fractions} \\
&= -\frac{16r}{10st} - \frac{9r}{10st} && \text{with LCD} = 10st.
\end{aligned}$$

We can now subtract the numerators and put the result over the common denominator.

$$\begin{aligned}
&= \frac{-25r}{10st} && \text{Subtract: } -16r - 9r = -25r \\
&= -\frac{5r}{2st} && \text{Reduce.}
\end{aligned}$$

21. Prime factor each denominator, placing the results in exponential form.

$$\begin{aligned} 18rs^2 &= 2^1 \cdot 3^2 \cdot r^1 \cdot s^2 \\ 24r^2s &= 2^3 \cdot 3^1 \cdot r^2 \cdot s^1 \end{aligned}$$

To find the LCD, list each factor that appears to the highest power that it appears.

$$\text{LCD} = 2^3 \cdot 3^2 \cdot r^2 \cdot s^2$$

Simplify.

$$\begin{aligned} \text{LCD} &= 8 \cdot 9 \cdot r^2 \cdot s^2 \\ \text{LCD} &= 72r^2s^2 \end{aligned}$$

After making equivalent fractions, place the sum of the numerators over this common denominator.

$$\begin{aligned} \frac{11}{18rs^2} + \frac{5}{24r^2s} &= \frac{11}{18rs^2} \cdot \frac{4r}{4r} + \frac{5}{24r^2s} \cdot \frac{3s}{3s} \\ &= \frac{44r}{72r^2s^2} + \frac{15s}{72r^2s^2} \\ &= \frac{44r + 15s}{72r^2s^2} \end{aligned}$$

23. Prime factor each denominator, placing the results in exponential form.

$$\begin{aligned} 24rs^2 &= 2^3 \cdot 3^1 \cdot r^1 \cdot s^2 \\ 36r^2s &= 2^2 \cdot 3^2 \cdot r^2 \cdot s^1 \end{aligned}$$

To find the LCD, list each factor that appears to the highest power that it appears.

$$\text{LCD} = 2^3 \cdot 3^2 \cdot r^2 \cdot s^2$$

Simplify.

$$\begin{aligned} \text{LCD} &= 8 \cdot 9 \cdot r^2 \cdot s^2 \\ \text{LCD} &= 72r^2s^2 \end{aligned}$$

After making equivalent fractions, place the sum of the numerators over this common denominator.

$$\begin{aligned} \frac{5}{24rs^2} + \frac{17}{36r^2s} &= \frac{5}{24rs^2} \cdot \frac{3r}{3r} + \frac{17}{36r^2s} \cdot \frac{2s}{2s} \\ &= \frac{15r}{72r^2s^2} + \frac{34s}{72r^2s^2} \\ &= \frac{15r + 34s}{72r^2s^2} \end{aligned}$$

25. Prime factor each denominator, placing the results in exponential form.

$$\begin{aligned} 36y^3 &= 2^2 \cdot 3^2 \cdot y^3 \\ 48z^3 &= 2^4 \cdot 3^1 \cdot z^3 \end{aligned}$$

To find the LCD, list each factor that appears to the highest power that it appears.

$$\text{LCD} = 2^4 \cdot 3^2 \cdot y^3 \cdot z^3$$

Simplify.

$$\begin{aligned} \text{LCD} &= 16 \cdot 9 \cdot y^3 \cdot z^3 \\ \text{LCD} &= 144y^3z^3 \end{aligned}$$

After making equivalent fractions, place the sum of the numerators over this common denominator.

$$\begin{aligned} \frac{7}{36y^3} + \frac{11}{48z^3} &= \frac{7}{36y^3} \cdot \frac{4z^3}{4z^3} + \frac{11}{48z^3} \cdot \frac{3y^3}{3y^3} \\ &= \frac{28z^3}{144y^3z^3} + \frac{33y^3}{144y^3z^3} \\ &= \frac{28z^3 + 33y^3}{144y^3z^3} \end{aligned}$$

27. Prime factor each denominator, placing the results in exponential form.

$$\begin{aligned} 48v^3 &= 2^4 \cdot 3^1 \cdot v^3 \\ 36w^3 &= 2^2 \cdot 3^2 \cdot w^3 \end{aligned}$$

To find the LCD, list each factor that appears to the highest power that it appears.

$$\text{LCD} = 2^4 \cdot 3^2 \cdot v^3 \cdot w^3$$

Simplify.

$$\begin{aligned} \text{LCD} &= 16 \cdot 9 \cdot v^3 \cdot w^3 \\ \text{LCD} &= 144v^3w^3 \end{aligned}$$

After making equivalent fractions, place the sum of the numerators over this common denominator.

$$\begin{aligned} \frac{5}{48v^3} + \frac{13}{36w^3} &= \frac{5}{48v^3} \cdot \frac{3w^3}{3w^3} + \frac{13}{36w^3} \cdot \frac{4v^3}{4v^3} \\ &= \frac{15w^3}{144v^3w^3} + \frac{52v^3}{144v^3w^3} \\ &= \frac{15w^3 + 52v^3}{144v^3w^3} \end{aligned}$$

29. Prime factor each denominator, placing the results in exponential form.

$$\begin{aligned} 50xy &= 2^1 \cdot 5^2 \cdot x^1 \cdot y^1 \\ 40yz &= 2^3 \cdot 5^1 \cdot y^1 \cdot z^1 \end{aligned}$$

To find the LCD, list each factor that appears to the highest power that it appears.

$$\text{LCD} = 2^3 \cdot 5^2 \cdot x^1 \cdot y^1 \cdot z^1$$

Simplify.

$$\begin{aligned} \text{LCD} &= 8 \cdot 25 \cdot x \cdot y \cdot z \\ \text{LCD} &= 200xyz \end{aligned}$$

After making equivalent fractions, place the sum of the numerators over this common denominator.

$$\begin{aligned} \frac{11}{50xy} - \frac{9}{40yz} &= \frac{11}{50xy} \cdot \frac{4z}{4z} - \frac{9}{40yz} \cdot \frac{5x}{5x} \\ &= \frac{44z}{200xyz} - \frac{45x}{200xyz} \\ &= \frac{44z - 45x}{200xyz} \end{aligned}$$

31. Prime factor each denominator, placing the results in exponential form.

$$\begin{aligned} 50ab &= 2^1 \cdot 5^2 \cdot a^1 \cdot b^1 \\ 40bc &= 2^3 \cdot 5^1 \cdot b^1 \cdot c^1 \end{aligned}$$

To find the LCD, list each factor that appears to the highest power that it appears.

$$\text{LCD} = 2^3 \cdot 5^2 \cdot a^1 \cdot b^1 \cdot c^1$$

Simplify.

$$\begin{aligned} \text{LCD} &= 8 \cdot 25 \cdot a \cdot b \cdot c \\ \text{LCD} &= 200abc \end{aligned}$$

After making equivalent fractions, place the sum of the numerators over this common denominator.

$$\begin{aligned} \frac{19}{50ab} - \frac{17}{40bc} &= \frac{19}{50ab} \cdot \frac{4c}{4c} - \frac{17}{40bc} \cdot \frac{5a}{5a} \\ &= \frac{76c}{200abc} - \frac{85a}{200abc} \\ &= \frac{76c - 85a}{200abc} \end{aligned}$$

33. We use the distributive property, dividing each term by 3.

$$\begin{aligned} \frac{6v+12}{3} &= \frac{6v}{3} + \frac{12}{3} && \text{Distribute 3.} \\ &= 2v+4 && \text{Simplify: } 6v/3 = 2v \text{ and } 12/3 = 4 \end{aligned}$$

35. We use the distributive property, dividing each term by 5.

$$\begin{aligned} \frac{25u+45}{5} &= \frac{25u}{5} + \frac{45}{5} && \text{Distribute 5.} \\ &= 5u+9 && \text{Simplify: } 25u/5 = 5u \text{ and } 45/5 = 9 \end{aligned}$$

37. We use the distributive property, dividing each term by s.

$$\begin{aligned} \frac{2s-4}{s} &= \frac{2s}{s} - \frac{4}{s} && \text{Distribute } s. \\ &= 2 - \frac{4}{s} && \text{Simplify: } 2s/s = 2 \end{aligned}$$

39. We use the distributive property, dividing each term by r.

$$\begin{aligned} \frac{3r-5}{r} &= \frac{3r}{r} - \frac{5}{r} && \text{Distribute } r. \\ &= 3 - \frac{5}{r} && \text{Simplify: } 3r/r = 3 \end{aligned}$$

41. We use the distributive property, dividing each term by x^2.

$$\begin{aligned} \frac{3x^2-8x-9}{x^2} &= \frac{3x^2}{x^2} - \frac{8x}{x^2} - \frac{9}{x^2} && \text{Distribute } x^2. \\ &= 3 - \frac{8}{x} - \frac{9}{x^2} && \text{Simplify: } 3x^2/x^2 = 3 \\ & && \text{and } 8x/x^2 = 8/x \end{aligned}$$

43. We use the distributive property, dividing each term by x^2.

$$\begin{aligned} \frac{2x^2-3x-6}{x^2} &= \frac{2x^2}{x^2} - \frac{3x}{x^2} - \frac{6}{x^2} && \text{Distribute } x^2. \\ &= 2 - \frac{3}{x} - \frac{6}{x^2} && \text{Simplify: } 2x^2/x^2 = 2 \\ & && \text{and } 3x/x^2 = 3/x \end{aligned}$$

45. We use the distributive property, dividing each term by $12t^2$.

$$\begin{aligned}\frac{12t^2+2t-16}{12t^2} &= \frac{12t^2}{12t^2}+\frac{2t}{12t^2}-\frac{16}{12t^2} && \text{Distribute } 12t^2.\\ &= 1+\frac{1}{6t}-\frac{4}{3t^2} && \text{Simplify: } 12t^2/(12t^2)=1,\\ & && 2t/(12t^2)=1/(6t), \text{ and}\\ & && 16/(12t^2)=4/(3t^2)\end{aligned}$$

47. We use the distributive property, dividing each term by $4s^2$.

$$\begin{aligned}\frac{4s^2+2s-10}{4s^2} &= \frac{4s^2}{4s^2}+\frac{2s}{4s^2}-\frac{10}{4s^2} && \text{Distribute } 4s^2.\\ &= 1+\frac{1}{2s}-\frac{5}{2s^2} && \text{Simplify: } 4s^2/(4s^2)=1,\\ & && 2s/(4s^2)=1/(2s), \text{ and}\\ & && 10/(4s^2)=5/(2s^2)\end{aligned}$$

7.4 Solving Rational Equations

1. The least common denominator (LCD) is x, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} x &= 11+\frac{26}{x} && \text{Original equation.}\\ x[x] &= \left[11+\frac{26}{x}\right]x && \text{Multiply both sides by } x.\\ x[x] &= x\,[11]+x\left[\frac{26}{x}\right] && \text{Distribute } x.\\ x^2 &= 11x+26 && \text{Cancel and simplify.}\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero.

$$\begin{aligned} x^2-11x &= 26 && \text{Subtract } 11x \text{ from both sides.}\\ x^2-11x-26 &= 0 && \text{Subtract 26 from both sides.}\end{aligned}$$

Compare $x^2-11x-26$ with ax^2+bx+c and note that $ac=(1)(-26)$ and $b=-11$. The integer pair 2 and -13 have product $ac=-26$ and sum $b=-11$. Hence, the trinomial factors as follows.

$$(x+2)(x-13)=0 \qquad \text{Use } ac\text{-method to factor.}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x+2&=0 &\quad\text{or}\quad&& x-13&=0\\ x&=-2 &&& x&=13\end{aligned}$$

Hence, the solutions are $x=-2$ and $x=13$.

3. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} 1-\frac{12}{x}&=-\frac{27}{x^2} && \text{Original equation.}\\ x^2\left[1-\frac{12}{x}\right]&=\left[-\frac{27}{x^2}\right]x^2 && \text{Multiply both sides by } x^2.\\ x^2\,[1]-x^2\left[\frac{12}{x}\right]&=\left[-\frac{27}{x^2}\right]x^2 && \text{Distribute } x^2.\\ x^2-12x&=-27 && \text{Cancel and simplify.}\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero, then factor.

$$x^2-12x+27=0 \qquad \text{Add 27 to both sides.}$$

Compare $x^2-12x+27$ with ax^2+bx+c and note that $ac=(1)(27)$ and $b=-12$. The integer pair -3 and -9 have product $ac=27$ and sum $b=-12$. Hence, the trinomial factors as follows.

$$(x-3)(x-9)=0 \qquad \text{Use } ac\text{-method to factor.}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x-3&=0 &\quad\text{or}\quad&& x-9&=0\\ x&=3 &&& x&=9\end{aligned}$$

Hence, the solutions are $x=3$ and $x=9$.

5. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} 1-\frac{10}{x}&=\frac{11}{x^2} && \text{Original equation.}\\ x^2\left[1-\frac{10}{x}\right]&=\left[\frac{11}{x^2}\right]x^2 && \text{Multiply both sides by } x^2.\\ x^2\,[1]-x^2\left[\frac{10}{x}\right]&=\left[\frac{11}{x^2}\right]x^2 && \text{Distribute } x^2.\\ x^2-10x&=11 && \text{Cancel and simplify.}\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero, then factor.

$$x^2 - 10x - 11 = 0 \qquad \text{Subtract 11 from both sides.}$$

Compare $x^2 - 10x - 11$ with $ax^2 + bx + c$ and note that $ac = (1)(-11)$ and $b = -10$. The integer pair -11 and 1 have product $ac = -11$ and sum $b = -10$. Hence, the trinomial factors as follows.

$$(x - 11)(x + 1) = 0 \qquad \text{Use } ac\text{-method to factor.}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x - 11 &= 0 & \quad\text{or}\quad && x + 1 &= 0 \\ x &= 11 & && x &= -1 \end{aligned}$$

Hence, the solutions are $x = 11$ and $x = -1$.

7. The least common denominator (LCD) is x, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} x &= 7 + \frac{44}{x} && \text{Original equation.} \\ x[x] &= \left[7 + \frac{44}{x}\right]x && \text{Multiply both sides by } x. \\ x[x] &= x\,[7] + x\left[\frac{44}{x}\right] && \text{Distribute } x. \\ x^2 &= 7x + 44 && \text{Cancel and simplify.} \end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero.

$$\begin{aligned} x^2 - 7x &= 44 && \text{Subtract } 7x \text{ from both sides.} \\ x^2 - 7x - 44 &= 0 && \text{Subtract 44 from both sides.} \end{aligned}$$

Compare $x^2 - 7x - 44$ with $ax^2 + bx + c$ and note that $ac = (1)(-44)$ and $b = -7$. The integer pair 4 and -11 have product $ac = -44$ and sum $b = -7$. Hence, the trinomial factors as follows.

$$(x + 4)(x - 11) = 0 \qquad \text{Use } ac\text{-method to factor.}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} x + 4 &= 0 & \quad\text{or}\quad && x - 11 &= 0 \\ x &= -4 & && x &= 11 \end{aligned}$$

Hence, the solutions are $x = -4$ and $x = 11$.

9. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned}
12x &= 97 - \frac{8}{x} && \text{Original equation.}\\
x\,[12x] &= \left[97 - \frac{8}{x}\right]x && \text{Multiply both sides by } x.\\
x\,[12x] &= x\,[97] - x\left[\frac{8}{x}\right] && \text{Distribute } x.\\
12x^2 &= 97x - 8 && \text{Cancel and simplify.}
\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$\begin{aligned}
12x^2 - 97x &= -8 && \text{Subtract } 97x \text{ from both sides.}\\
12x^2 - 97x + 8 &= 0 && \text{Add 8 to both sides.}
\end{aligned}$$

Compare $12x^2 - 97x + 8$ with $ax^2 + bx + c$ and note that $ac = (12)(8)$ and $b = -97$. The integer pair -1 and -96 have product $ac = 96$ and sum $b = -97$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

$$\begin{aligned}
12x^2 - x - 96x + 8 &= 0 && -97x = -x - 96x\\
x(12x - 1) - 8(12x - 1) &= 0 && \text{Factor by grouping.}\\
(x - 8)(12x - 1) &= 0 && \text{Factor out } 12x - 1.
\end{aligned}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned}
x - 8 &= 0 \qquad \text{or} \qquad & 12x - 1 &= 0\\
x &= 8 & x &= \frac{1}{12}
\end{aligned}$$

Hence, the solutions are $x = 8$ and $x = 1/12$.

11. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned}
20 + \frac{19}{x} &= -\frac{3}{x^2} && \text{Original equation.}\\
x^2\left[20 + \frac{19}{x}\right] &= \left[-\frac{3}{x^2}\right]x^2 && \text{Multiply both sides by } x^2.\\
x^2\,[20] + x^2\left[\frac{19}{x}\right] &= \left[-\frac{3}{x^2}\right]x^2 && \text{Distribute } x^2.\\
20x^2 + 19x &= -3 && \text{Cancel and simplify.}
\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$20x^2 + 19x + 3 = 0 \qquad \text{Add 3 to both sides.}$$

Compare $20x^2 + 19x + 3$ with $ax^2 + bx + c$ and note that $ac = (20)(3)$ and $b = 19$. The integer pair 4 and 15 have product $ac = 60$ and sum $b = 19$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

$$\begin{aligned} 20x^2 + 4x + 15x + 3 &= 0 && 19x = 4x + 15x \\ 4x(5x+1) + 3(5x+1) &= 0 && \text{Factor by grouping.} \\ (4x+3)(5x+1) &= 0 && \text{Factor out } 5x+1. \end{aligned}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} 4x + 3 &= 0 &\quad \text{or} \quad&& 5x + 1 &= 0 \\ x &= -\frac{3}{4} &&& x &= -\frac{1}{5} \end{aligned}$$

Hence, the solutions are $x = -3/4$ and $x = -1/5$.

13. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} 8x &= 19 - \frac{11}{x} && \text{Original equation.} \\ x\,[8x] &= \left[19 - \frac{11}{x}\right] x && \text{Multiply both sides by } x. \\ x\,[8x] &= x\,[19] - x\left[\frac{11}{x}\right] && \text{Distribute } x. \\ 8x^2 &= 19x - 11 && \text{Cancel and simplify.} \end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$\begin{aligned} 8x^2 - 19x &= -11 && \text{Subtract } 19x \text{ from both sides.} \\ 8x^2 - 19x + 11 &= 0 && \text{Add 11 to both sides.} \end{aligned}$$

Compare $8x^2 - 19x + 11$ with $ax^2 + bx + c$ and note that $ac = (8)(11)$ and $b = -19$. The integer pair -8 and -11 have product $ac = 88$ and sum $b = -19$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

$$\begin{aligned} 8x^2 - 8x - 11x + 11 &= 0 && -19x = -8x - 11x \\ 8x(x-1) - 11(x-1) &= 0 && \text{Factor by grouping.} \\ (8x-11)(x-1) &= 0 && \text{Factor out } x-1. \end{aligned}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} 8x - 11 &= 0 \qquad \text{or} \qquad & x - 1 &= 0 \\ x &= \frac{11}{8} & x &= 1 \end{aligned}$$

Hence, the solutions are $x = 11/8$ and $x = 1$.

15. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} 40 + \frac{6}{x} &= \frac{1}{x^2} && \text{Original equation.} \\ x^2\left[40 + \frac{6}{x}\right] &= \left[\frac{1}{x^2}\right]x^2 && \text{Multiply both sides by } x^2. \\ x^2\,[40] + x^2\left[\frac{6}{x}\right] &= \left[\frac{1}{x^2}\right]x^2 && \text{Distribute } x^2. \\ 40x^2 + 6x &= 1 && \text{Cancel and simplify.} \end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$40x^2 + 6x - 1 = 0 \qquad \text{Subtract 1 from both sides.}$$

Compare $40x^2 + 6x - 1$ with $ax^2 + bx + c$ and note that $ac = (40)(-1)$ and $b = 6$. The integer pair -4 and 10 have product $ac = -40$ and sum $b = 6$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

$$\begin{aligned} 40x^2 - 4x + 10x - 1 &= 0 && 6x = -4x + 10x \\ 4x(10x - 1) + 1(10x - 1) &= 0 && \text{Factor by grouping.} \\ (4x + 1)(10x - 1) &= 0 && \text{Factor out } 10x - 1. \end{aligned}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} 4x + 1 &= 0 \qquad \text{or} \qquad & 10x - 1 &= 0 \\ x &= -\frac{1}{4} & x &= \frac{1}{10} \end{aligned}$$

Hence, the solutions are $x = -1/4$ and $x = 1/10$.

17. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned} 36x &= -13 - \frac{1}{x} && \text{Original equation.} \\ x\,[36x] &= \left[-13 - \frac{1}{x}\right] x && \text{Multiply both sides by } x. \\ x\,[36x] &= x\,[-13] - x\left[\frac{1}{x}\right] && \text{Distribute } x. \\ 36x^2 &= -13x - 1 && \text{Cancel and simplify.} \end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$\begin{aligned} 36x^2 + 13x &= -1 && \text{Add } 13x \text{ to both sides.} \\ 36x^2 + 13x + 1 &= 0 && \text{Add 1 to both sides.} \end{aligned}$$

Compare $36x^2 + 13x + 1$ with $ax^2 + bx + c$ and note that $ac = (36)(1)$ and $b = 13$. The integer pair 9 and 4 have product $ac = 36$ and sum $b = 13$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

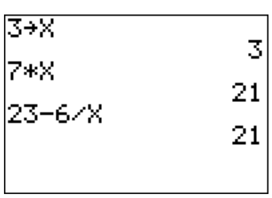
$$\begin{aligned} 36x^2 + 9x + 4x + 1 &= 0 && 13x = 9x + 4x \\ 9x(4x + 1) + 1(4x + 1) &= 0 && \text{Factor by grouping.} \\ (9x + 1)(4x + 1) &= 0 && \text{Factor out } 4x + 1. \end{aligned}$$

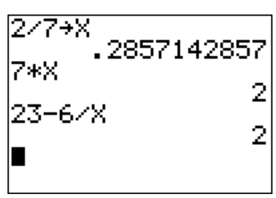
Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned} 9x + 1 &= 0 & \quad\text{or}\quad && 4x + 1 &= 0 \\ x &= -\frac{1}{9} & && x &= -\frac{1}{4} \end{aligned}$$

Hence, the solutions are $x = -1/9$ and $x = -1/4$.

Check: To check the solution $x = -1/9$, first enter **-1/9**, push the `STO▶` button, then the `X,T,θ,n` button and the ENTER key. Next, enter the left-hand side of the equation as **36*X** and press ENTER. Enter the right-hand side of the equation as **-13 - 1/X** and press ENTER. The results are the same (see the first image on the left below). This verifies that $-1/9$ is a solution of $36x = -13 - 1/x$. The calculator screen on the right shows a similar check of the solution $x = -1/4$.





19. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned}
14x &= 9 - \frac{1}{x} && \text{Original equation.}\\
x\,[14x] &= \left[9 - \frac{1}{x}\right] x && \text{Multiply both sides by } x.\\
x\,[14x] &= x\,[9] - x\left[\frac{1}{x}\right] && \text{Distribute } x.\\
14x^2 &= 9x - 1 && \text{Cancel and simplify.}
\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$\begin{aligned}
14x^2 - 9x &= -1 && \text{Subtract } 9x \text{ from both sides.}\\
14x^2 - 9x + 1 &= 0 && \text{Add 1 to both sides.}
\end{aligned}$$

Compare $14x^2 - 9x + 1$ with $ax^2 + bx + c$ and note that $ac = (14)(1)$ and $b = -9$. The integer pair -2 and -7 have product $ac = 14$ and sum $b = -9$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

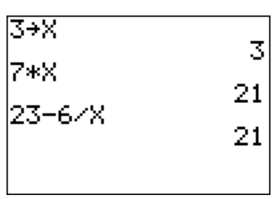
$$\begin{aligned}
14x^2 - 2x - 7x + 1 &= 0 && -9x = -2x - 7x\\
2x(7x - 1) - 1(7x - 1) &= 0 && \text{Factor by grouping.}\\
(2x - 1)(7x - 1) &= 0 && \text{Factor out } 7x - 1.
\end{aligned}$$

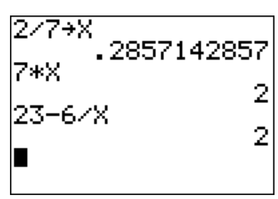
Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

$$\begin{aligned}
2x - 1 &= 0 &\quad \text{or} \quad&& 7x - 1 &= 0\\
x &= \frac{1}{2} &&& x &= \frac{1}{7}
\end{aligned}$$

Hence, the solutions are $x = 1/2$ and $x = 1/7$.

Check: To check the solution $x = 1/2$, first enter **1/2**, push the `STO▶` button, then the `X,T,θ,n` button and the ENTER key. Next, enter the left-hand side of the equation as **14*X** and press ENTER. Enter the right-hand side of the equation as **9 - 1/X** and press ENTER. The results are the same (see the first image on the left below). This verifies that $1/2$ is a solution of $14x = 9 - 1/x$. The calculator screen on the right shows a similar check of the solution $x = 1/7$.





21. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$
\begin{aligned}
1 - \frac{1}{x} &= \frac{12}{x^2} && \text{Original equation.}\\
x^2\left[1 - \frac{1}{x}\right] &= \left[\frac{12}{x^2}\right]x^2 && \text{Multiply both sides by } x^2.\\
x^2\left[1\right] - x^2\left[\frac{1}{x}\right] &= \left[\frac{12}{x^2}\right]x^2 && \text{Distribute } x^2.\\
x^2 - x &= 12 && \text{Cancel and simplify.}
\end{aligned}
$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero, then factor.

$$x^2 - x - 12 = 0 \qquad \text{Subtract 12 from both sides.}$$

Compare $x^2 - x - 12$ with $ax^2 + bx + c$ and note that $ac = (1)(-12)$ and $b = -1$. The integer pair 3 and -4 have product $ac = -12$ and sum $b = -1$. Hence, the trinomial factors as follows.

$$(x + 3)(x - 4) = 0 \qquad \text{Use } ac\text{-method to factor.}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

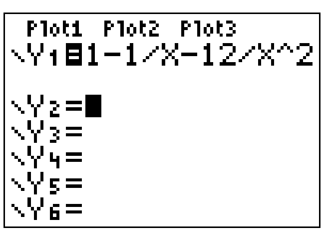
$$
\begin{aligned}
x + 3 &= 0 & \quad\text{or}\quad && x - 4 &= 0\\
x &= -3 & && x &= 4
\end{aligned}
$$

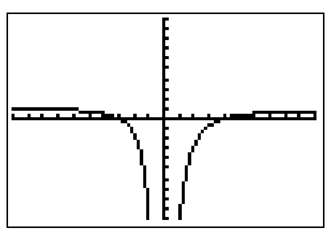
Hence, the solutions are $x = -3$ and $x = 4$.

Graphical solution: Make one side of the equation zero.

$$1 - \frac{1}{x} - \frac{12}{x^2} = 0$$

Load the left-hand side of the equation into **Y1** as **1-1/X-12/Xˆ2** (see image on the left), then select **6:ZStandard** from the ZOOM menu to produce the image at the right.

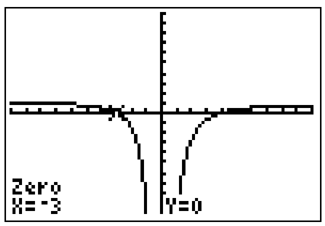


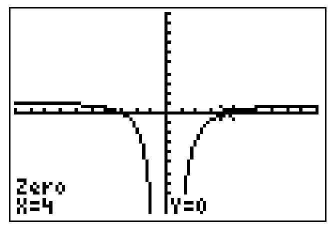


Next, the solutions of

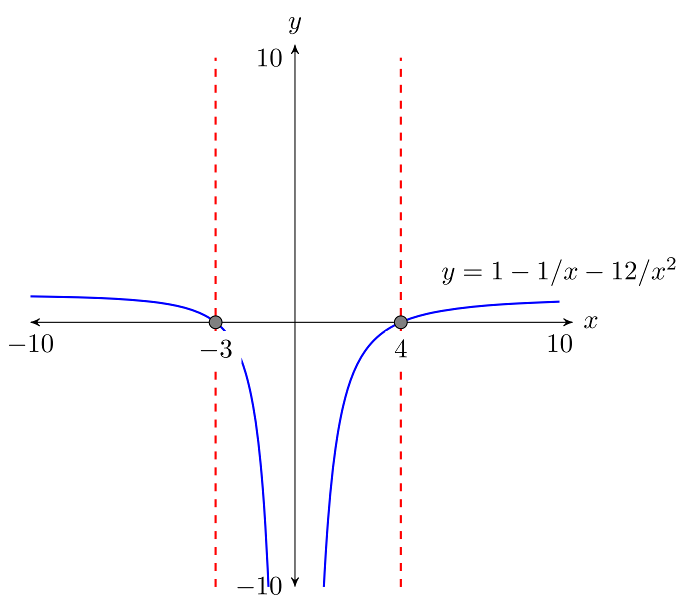
$$1 - \frac{1}{x} - \frac{12}{x^2} = 0$$

are found by noting where the graph of $y = 1 - 2/x - 12/x^2$ cross the x-axis. Select **2:zero** from the CALC menu. Use the arrow keys to move the cursor to the left of the first x-intercept, then press ENTER to set the "Left bound." Next, move the cursor to the right of the first x-intercept, then press ENTER to set the "Right bound." Finally, leave the cursor where it is and press ENTER to set your "Guess." The calculator responds with the result shown in the figure on the left. Repeat the zero-finding procedure to capture the coordinates of the second x-intercept (see the image on the right).





Reporting the solution on your homework.



Note that the calculator solutions, -3 and 4, match the algebraic solutions.

23. The least common denominator (LCD) is x^2, so first clear fractions by multiplying both sides of the equation by the LCD.

$$\begin{aligned}
2x &= 3 + \frac{44}{x} && \text{Original equation.}\\
x\,[2x] &= \left[3 + \frac{44}{x}\right] x && \text{Multiply both sides by } x.\\
x\,[2x] &= x\,[3] + x\left[\frac{44}{x}\right] && \text{Distribute } x.\\
2x^2 &= 3x + 44 && \text{Cancel and simplify.}
\end{aligned}$$

The resulting equation is nonlinear (x is raised to a power larger than 1). Make one side zero

$$\begin{aligned}
2x^2 - 3x &= 44 && \text{Subtract } 3x \text{ from both sides.}\\
2x^2 - 3x - 44 &= 0 && \text{Subtract 44 from both sides.}
\end{aligned}$$

Compare $2x^2 - 3x - 44$ with $ax^2 + bx + c$ and note that $ac = (2)(-44)$ and $b = -3$. The integer pair -11 and 8 have product $ac = -88$ and sum $b = -3$. Replace the middle term with a combination of like terms using this pair, then factor by grouping.

$$\begin{aligned}
2x^2 - 11x + 8x - 44 &= 0 && -3x = -11x + 8x\\
x(2x - 11) + 4(2x - 11) &= 0 && \text{Factor by grouping.}\\
(x + 4)(2x - 11) &= 0 && \text{Factor out } 2x - 11.
\end{aligned}$$

Use the zero product property to complete the solution. Either the first factor is zero or the second factor is zero.

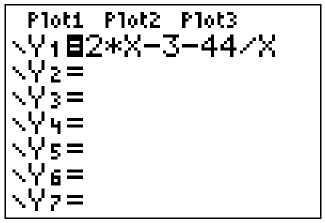
$$\begin{aligned}
x + 4 &= 0 &\quad \text{or} \quad&& 2x - 11 &= 0\\
x &= -4 &&& x &= \frac{11}{2}
\end{aligned}$$

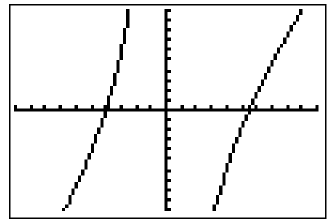
Hence, the solutions are $x = -4$ and $x = 11/2$.

Graphical solution: Make one side of the equation zero.

$$2x - 3 - \frac{44}{x} = 0$$

Load the left-hand side of the equation into **Y1** as **2*X-3-44/X** (see image on the left), then select **6:ZStandard** from the ZOOM menu to produce the image at the right.

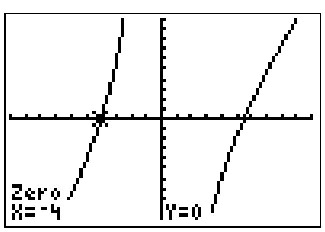


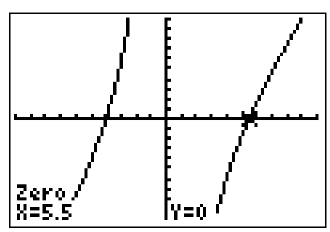


Next, the solutions of

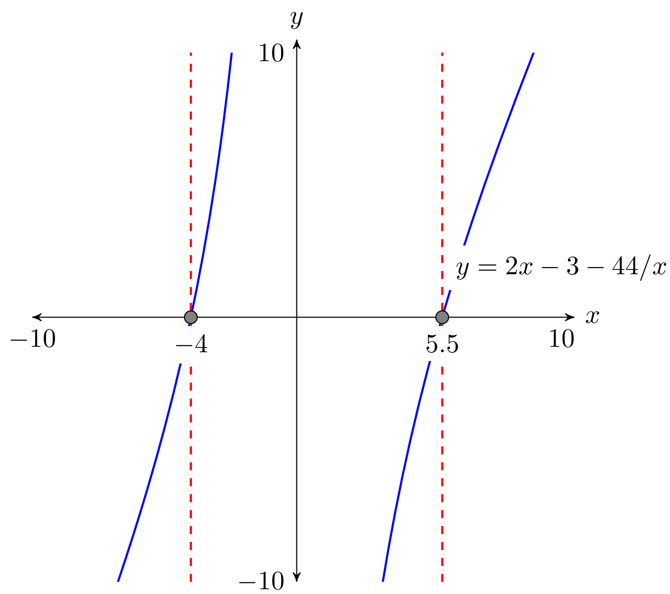
$$2x - 3 - \frac{44}{x} = 0$$

are found by noting where the graph of $y = 2x - 3 - 44/x$ cross the x-axis. Select **2:zero** from the CALC menu. Use the arrow keys to move the cursor to the left of the first x-intercept, then press ENTER to set the "Left bound." Next, move the cursor to the right of the first x-intercept, then press ENTER to set the "Right bound." Finally, leave the cursor where it is and press ENTER to set your "Guess." The calculator responds with the result shown in the figure on the left. Repeat the zero-finding procedure to capture the coordinates of the second x-intercept (see the image on the right).





Reporting the solution on your homework.



Note that the calculator solutions, -4 and 5.5, match the algebraic solutions.

25. In the solution, we address each step of the *Requirements for Word Problem Solutions*.

1. *Set up a variable dictionary.* Let x represent the unknown number.
2. *Set up an equation.* If the unknown number is x, then its reciprocal is $1/x$. Thus, the "sum of a number and its reciprocal is 5/2" becomes:

$$x + \frac{1}{x} = \frac{5}{2}$$

3. *Solve the equation.* Clear the fractions by multiplying both sides by $2x$, the least common denominator.

$$\begin{aligned} x + \frac{1}{x} &= \frac{5}{2} && \text{Model equation.} \\ 2x\left[x + \frac{1}{x}\right] &= \left[\frac{5}{2}\right]2x && \text{Multiply both sides by } 2x. \\ 2x\left[x\right] + 2x\left[\frac{1}{x}\right] &= \left[\frac{5}{2}\right]2x && \text{Distribute } 2x. \\ 2x^2 + 2 &= 5x && \text{Cancel and simplify.} \end{aligned}$$

The equation is nonlinear. Make one side zero.

$$2x^2 - 5x + 2 = 0 \qquad \text{Make one side zero.}$$

The integer pair -1 and -4 has product $ac = 4$ and sum $b = -5$. Break up the middle term in the last equation into a sum of like terms using this pair, then factor by grouping.

$$\begin{aligned}
2x^2 - x - 4x + 2 &= 0 && -x - 4x = -5x. \\
x(2x-1) - 2(2x-1) &= 0 && \text{Factor by grouping.} \\
(x-2)(2x-1) &= 0 && \text{Factor out } 2x-1.
\end{aligned}$$

We can now use the zero product property to write:

$$\begin{aligned}
x - 2 &= 0 \quad \text{or} \quad & 2x - 1 &= 0 \\
x &= 2 & x &= \frac{1}{2}
\end{aligned}$$

4. *Answer the question.* There are two possible numbers, 2 and $1/2$.

5. *Look back.* The sum of the unknown number and its reciprocal is supposed to equal $5/2$. The first answer 2 has reciprocal $1/2$. Their sum is:

$$\begin{aligned}
2 + \frac{1}{2} &= 2 \cdot \frac{2}{2} + \frac{1}{2} \cdot \frac{1}{1} \\
&= \frac{4}{2} + \frac{1}{2} \\
&= \frac{5}{2}
\end{aligned}$$

Thus, 2 is a valid solution. The second answer $1/2$ has reciprocal 2, so it is clear that their sum is also $5/2$. Hence, $1/2$ is also a valid solution.

27. In the solution, we address each step of the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let x represent the unknown number.

2. *Set up an equation.* If the unknown number is x, then its reciprocal is $1/x$. Thus, the "sum of a number and 8 times its reciprocal is $17/3$" becomes:

$$x + 8\left(\frac{1}{x}\right) = \frac{17}{3}$$

Or equivalently:

$$x + \frac{8}{x} = \frac{17}{3}$$

3. *Solve the equation.* Clear the fractions by multiplying both sides by $3x$, the least common denominator.

$$\begin{aligned}
x + \frac{8}{x} &= \frac{17}{3} && \text{Model equation.}\\
3x\left[x + \frac{8}{x}\right] &= \left[\frac{17}{3}\right]3x && \text{Multiply both sides by } 3x.\\
3x\,[x] + 3x\left[\frac{8}{x}\right] &= \left[\frac{17}{3}\right]3x && \text{Distribute } 3x.\\
3x^2 + 24 &= 17x && \text{Cancel and simplify.}
\end{aligned}$$

The equation is nonlinear. Make one side zero.

$$3x^2 - 17x + 24 = 0 \qquad \text{Make one side zero.}$$

The integer pair -8 and -9 has product $ac = 72$ and sum $b = -17$. Break up the middle term in the last equation into a sum of like terms using this pair, then factor by grouping.

$$\begin{aligned}
3x^2 - 8x - 9x + 24 &= 0 && -8x - 9x = -17x.\\
x(3x - 8) - 3(3x - 8) &= 0 && \text{Factor by grouping.}\\
(x - 3)(3x - 8) &= 0 && \text{Factor out } 3x - 8.
\end{aligned}$$

We can now use the zero product property to write:

$$\begin{aligned}
x - 3 &= 0 \qquad \text{or} \qquad & 3x - 8 &= 0\\
x &= 3 & x &= \frac{8}{3}
\end{aligned}$$

4. *Answer the question.* There are two possible numbers, 3 and 8/3.

5. *Look back.* The sum of the unknown number and 8 times its reciprocal is supposed to equal 17/3. The first answer 3 has reciprocal 1/3. Their sum of the first number and 8 times its reciprocal is:

$$\begin{aligned}
3 + 8 \cdot \frac{1}{3} &= 3 \cdot \frac{3}{3} + 8 \cdot \frac{1}{3} \cdot \frac{1}{1}\\
&= \frac{9}{3} + \frac{8}{3}\\
&= \frac{17}{3}
\end{aligned}$$

Thus, 3 is a valid solution. We'll leave it to readers to check that the second solution is also valid.

7.5 Direct and Inverse Variation

1. Given the fact the s is proportional to t, we know immediately that

$$s = kt,$$

where k is the proportionality constant. Because we are given that $s = 632$ when $t = 79$, we can substitute 632 for s and 79 for t to determine k.

$$\begin{aligned} s &= kt && \text{s is proportional to t.} \\ 632 &= k(79) && \text{Substitute 632 for s, 79 for t.} \\ \frac{632}{79} &= k && \text{Divide both sides by 79.} \\ k &= 8 && \text{Simplify.} \end{aligned}$$

Substitute 8 for k in $s = kt$, then substitute 50 for t to determine s when $t = 50$.

$$\begin{aligned} s &= 8t && \text{Substitue 8 for k.} \\ s &= 8(50) && \text{Substitute 50 for t.} \\ s &= 400 && \text{Multiply: } 8(50) = 400 \end{aligned}$$

Thus, $s = 400$ when $t = 50$.

3. Given the fact the s is proportional to the cube of t, we know immediately that

$$s = kt^3,$$

where k is the proportionality constant. Because we are given that $s = 1588867$ when $t = 61$, we can substitute 1588867 for s and 61 for t to determine k.

$$\begin{aligned} s &= kt^3 && \text{s is proportional to the cube of t.} \\ 1588867 &= k(61)^3 && \text{Substitute 1588867 for s, 61 for t.} \\ 1588867 &= k(226981) && \text{Simplify: } (61)^3 = 226981 \\ \frac{1588867}{226981} &= k && \text{Divide both sides by 226981.} \\ k &= 7 && \text{Simplify.} \end{aligned}$$

Substitute 7 for k in $s = kt^3$, then substitute 63 for t to determine s when $t = 63$.

$$\begin{aligned} s &= 7t^3 && \text{Substitue 7 for k.} \\ s &= 7(63)^3 && \text{Substitute 63 for t.} \\ s &= 7(250047) && \text{Exponent first: } (63)^3 = 250047 \\ s &= 1750329 && \text{Multiply: } 7(250047) = 1750329. \end{aligned}$$

Thus, $s = 1750329$ when $t = 63$.

5. Given the fact the q is proportional to the square of c, we know immediately that

$$q = kc^2,$$

where k is the proportionality constant. Because we are given that $q = 13448$ when $c = 82$, we can substitute 13448 for q and 82 for c to determine k.

$$\begin{aligned}
q &= kc^2 && q \text{ is proportional to the square of } c.\\
13448 &= k(82)^2 && \text{Substitute 13448 for } q\text{, 82 for } c.\\
13448 &= k(6724) && \text{Simplify: } (82)^2 = 6724\\
\frac{13448}{6724} &= k && \text{Divide both sides by 6724.}\\
k &= 2 && \text{Simplify.}
\end{aligned}$$

Substitute 2 for k in $q = kc^2$, then substitute 29 for c to determine q when $c = 29$.

$$\begin{aligned}
q &= 2c^2 && \text{Substitue 2 for } k.\\
q &= 2(29)^2 && \text{Substitute 29 for } c.\\
q &= 2(841) && \text{Exponent first: } (29)^2 = 841\\
q &= 1682 && \text{Multiply: } 2(841) = 1682.
\end{aligned}$$

Thus, $q = 1682$ when $c = 29$.

7. Given the fact the y is proportional to the square of x, we know immediately that

$$y = kx^2,$$

where k is the proportionality constant. Because we are given that $y = 14700$ when $x = 70$, we can substitute 14700 for y and 70 for x to determine k.

$$\begin{aligned}
y &= kx^2 && y \text{ is proportional to the square of } x.\\
14700 &= k(70)^2 && \text{Substitute 14700 for } y\text{, 70 for } x.\\
14700 &= k(4900) && \text{Simplify: } (70)^2 = 4900\\
\frac{14700}{4900} &= k && \text{Divide both sides by 4900.}\\
k &= 3 && \text{Simplify.}
\end{aligned}$$

Substitute 3 for k in $y = kx^2$, then substitute 45 for x to determine y when $x = 45$.

$$\begin{aligned}
y &= 3x^2 && \text{Substitue 3 for } k.\\
y &= 3(45)^2 && \text{Substitute 45 for } x.\\
y &= 3(2025) && \text{Exponent first: } (45)^2 = 2025\\
y &= 6075 && \text{Multiply: } 3(2025) = 6075.
\end{aligned}$$

Thus, $y = 6075$ when $x = 45$.

9. Given the fact the F is proportional to the cube of x, we know immediately that

$$F = kx^3,$$

where k is the proportionality constant. Because we are given that $F = 214375$ when $x = 35$, we can substitute 214375 for F and 35 for x to determine k.

$$\begin{aligned}
F &= kx^3 && \text{F is proportional to the cube of x.} \\
214375 &= k(35)^3 && \text{Substitute 214375 for F, 35 for x.} \\
214375 &= k(42875) && \text{Simplify: $(35)^3 = 42875$} \\
\frac{214375}{42875} &= k && \text{Divide both sides by 42875.} \\
k &= 5 && \text{Simplify.}
\end{aligned}$$

Substitute 5 for k in $F = kx^3$, then substitute 36 for x to determine F when $x = 36$.

$$\begin{aligned}
F &= 5x^3 && \text{Substitue 5 for k.} \\
F &= 5(36)^3 && \text{Substitute 36 for x.} \\
F &= 5(46656) && \text{Exponent first: $(36)^3 = 46656$} \\
F &= 233280 && \text{Multiply: $5(46656) = 233280$.}
\end{aligned}$$

Thus, $F = 233280$ when $x = 36$.

11. Given the fact the d is proportional to t, we know immediately that

$$d = kt,$$

where k is the proportionality constant. Because we are given that $d = 496$ when $t = 62$, we can substitute 496 for d and 62 for t to determine k.

$$\begin{aligned}
d &= kt && \text{d is proportional to t.} \\
496 &= k(62) && \text{Substitute 496 for d, 62 for t.} \\
\frac{496}{62} &= k && \text{Divide both sides by 62.} \\
k &= 8 && \text{Simplify.}
\end{aligned}$$

Substitute 8 for k in $d = kt$, then substitute 60 for t to determine d when $t = 60$.

$$\begin{aligned}
d &= 8t && \text{Substitue 8 for k.} \\
d &= 8(60) && \text{Substitute 60 for t.} \\
d &= 480 && \text{Multiply: $8(60) = 480$}
\end{aligned}$$

Thus, $d = 480$ when $t = 60$.

13. Given the fact the h is inversely proportional to x, we know immediately that

$$h = \frac{k}{x},$$

where k is the proportionality constant. Because we are given that $h = 16$ when $x = 29$, we can substitute 16 for h and 29 for x to determine k.

$$\begin{aligned}
h &= \frac{k}{x} && \text{h is inversely proportional to x.}\\
16 &= \frac{k}{29} && \text{Substitute 16 for h, 29 for x.}\\
29(16) &= \left[\frac{k}{29}\right]29 && \text{Multiply both sides by 29.}\\
464 &= k && \text{Cancel and simplify.}
\end{aligned}$$

Substitute 464 for k in $h = k/x$, then substitute 20 for x to determine h when $x = 20$.

$$\begin{aligned}
h &= \frac{464}{x} && \text{Substitue 464 for k.}\\
h &= \frac{464}{20} && \text{Substitute 20 for x.}\\
h &= \frac{116}{5} && \text{Reduce.}
\end{aligned}$$

Thus, $h = 116/5$ when $x = 20$.

15. Given the fact the q is inversely proportional to the square of c, we know immediately that

$$q = \frac{k}{c^2},$$

where k is the proportionality constant. Because we are given that $q = 11$ when $c = 9$, we can substitute 11 for q and 9 for c to determine k.

$$\begin{aligned}
q &= \frac{k}{c^2} && \text{q is inversely proportional to the square of c.}\\
11 &= \frac{k}{(9)^2} && \text{Substitute 11 for q, 9 for c.}\\
11 &= \frac{k}{81} && \text{Exponent first: $(9)^2 = 81$}\\
81(11) &= \left[\frac{k}{81}\right]81 && \text{Multiply both sides by 81.}\\
k &= 891 && \text{Cancel and simplify.}
\end{aligned}$$

Substitute 891 for k in $q = k/c^2$, then substitute 3 for c to determine q when $c = 3$.

$$\begin{aligned}
q &= \frac{891}{c^2} && \text{Substitue 891 for } k. \\
q &= \frac{891}{(3)^2} && \text{Substitute 3 for } c. \\
q &= \frac{891}{9} && \text{Exponent first: } (3)^2 = 9 \\
q &= 99 && \text{Reduce.}
\end{aligned}$$

Thus, $q = 99$ when $c = 3$.

17. Given the fact the F is inversely proportional to x, we know immediately that

$$F = \frac{k}{x},$$

where k is the proportionality constant. Because we are given that $F = 19$ when $x = 22$, we can substitute 19 for F and 22 for x to determine k.

$$\begin{aligned}
F &= \frac{k}{x} && F \text{ is inversely proportional to } x. \\
19 &= \frac{k}{22} && \text{Substitute 19 for } F\text{, 22 for } x. \\
22(19) &= \left[\frac{k}{22}\right] 22 && \text{Multiply both sides by 22.} \\
418 &= k && \text{Cancel and simplify.}
\end{aligned}$$

Substitute 418 for k in $F = k/x$, then substitute 16 for x to determine F when $x = 16$.

$$\begin{aligned}
F &= \frac{418}{x} && \text{Substitue 418 for } k. \\
F &= \frac{418}{16} && \text{Substitute 16 for } x. \\
F &= \frac{209}{8} && \text{Reduce.}
\end{aligned}$$

Thus, $F = 209/8$ when $x = 16$.

19. Given the fact the y is inversely proportional to the square of x, we know immediately that

$$y = \frac{k}{x^2},$$

where k is the proportionality constant. Because we are given that $y = 14$ when $x = 4$, we can substitute 14 for y and 4 for x to determine k.

$$\begin{aligned}
y &= \frac{k}{x^2} && \text{y is inversely proportional to the square of x.}\\
14 &= \frac{k}{(4)^2} && \text{Substitute 14 for y, 4 for x.}\\
14 &= \frac{k}{16} && \text{Exponent first: } (4)^2 = 16\\
16(14) &= \left[\frac{k}{16}\right]16 && \text{Multiply both sides by 16.}\\
k &= 224 && \text{Cancel and simplify.}
\end{aligned}$$

Substitute 224 for k in $y = k/x^2$, then substitute 10 for x to determine y when $x = 10$.

$$\begin{aligned}
y &= \frac{224}{x^2} && \text{Substitue 224 for k.}\\
y &= \frac{224}{(10)^2} && \text{Substitute 10 for x.}\\
y &= \frac{224}{100} && \text{Exponent first: } (10)^2 = 100\\
y &= \frac{56}{25} && \text{Reduce.}
\end{aligned}$$

Thus, $y = 56/25$ when $x = 10$.

21. Given the fact the d is inversely proportional to the cube of t, we know immediately that

$$d = \frac{k}{t^3},$$

where k is the proportionality constant. Because we are given that $d = 18$ when $t = 2$, we can substitute 18 for d and 2 for t to determine k.

$$\begin{aligned}
d &= \frac{k}{t^3} && \text{d is inversely proportional to the cube of t.}\\
18 &= \frac{k}{(2)^3} && \text{Substitute 18 for d, 2 for t.}\\
18 &= \frac{k}{8} && \text{Exponent first: } (2)^3 = 8\\
8(18) &= \left[\frac{k}{8}\right]8 && \text{Multiply both sides by 8.}\\
k &= 144 && \text{Cancel and simplify.}
\end{aligned}$$

Substitute 144 for k in $d = k/t^3$, then substitute 3 for t to determine d when $t = 3$.

$$\begin{aligned}
d &= \frac{144}{t^3} && \text{Substitue 144 for } k. \\
d &= \frac{144}{(3)^3} && \text{Substitute 3 for } t. \\
d &= \frac{144}{27} && \text{Exponent first: } (3)^3 = 27 \\
d &= \frac{16}{3} && \text{Reduce.}
\end{aligned}$$

Thus, $d = 16/3$ when $t = 3$.

23. Given the fact the q is inversely proportional to the cube of c, we know immediately that

$$q = \frac{k}{c^3},$$

where k is the proportionality constant. Because we are given that $q = 16$ when $c = 5$, we can substitute 16 for q and 5 for c to determine k.

$$\begin{aligned}
q &= \frac{k}{c^3} && q \text{ is inversely proportional to the cube of } c. \\
16 &= \frac{k}{(5)^3} && \text{Substitute 16 for } q, \text{ 5 for } c. \\
16 &= \frac{k}{125} && \text{Exponent first: } (5)^3 = 125 \\
125(16) &= \left[\frac{k}{125}\right]125 && \text{Multiply both sides by 125.} \\
k &= 2000 && \text{Cancel and simplify.}
\end{aligned}$$

Substitute 2000 for k in $q = k/c^3$, then substitute 6 for c to determine q when $c = 6$.

$$\begin{aligned}
q &= \frac{2000}{c^3} && \text{Substitue 2000 for } k. \\
q &= \frac{2000}{(6)^3} && \text{Substitute 6 for } c. \\
q &= \frac{2000}{216} && \text{Exponent first: } (6)^3 = 216 \\
q &= \frac{250}{27} && \text{Reduce.}
\end{aligned}$$

Thus, $q = 250/27$ when $c = 6$.

25. Let W represent the weight hung on the spring. Let x represent the distance the spring stretches. We're told that the distance x the spring stretches is proportional to the amount of weight W hung on the spring. Hence, we can write:

$$x = kW \qquad \text{x is proportional to W.}$$

We're told that a 2 pound weight stretches the spring 16 inches. Substitute 2 for W, 16 for x, then solve for k.

$$\begin{aligned} 16 &= k(2) && \text{Substitute 16 for x, 2 for W.} \\ \frac{16}{2} &= k && \text{Divide both sides by 2.} \\ k &= 8 && \text{Simplify.} \end{aligned}$$

Substitute 8 for k in $x = kW$ to produce:

$$x = 8W \qquad \text{Substitute 8 for k in $x = kW$.}$$

To determine the distance the spring will stretch when 5 pounds are hung on the spring, substitute 5 for W.

$$\begin{aligned} x &= 8(5) && \text{Substitute 5 for W.} \\ x &= 40 && \text{Simplify.} \end{aligned}$$

Thus, the spring will stretch 40 inches.

27. Given the fact that the intensity I of the light is inversely proportional to the square of the distance d from the light source, we know immediately that

$$I = \frac{k}{d^2},$$

where k is the proportionality constant. Because we are given that the intensity is $I = 20$ foot-candles at $d = 4$ feet from the light source, we can substitute 20 for I and 4 for d to determine k.

$$\begin{aligned} I &= \frac{k}{d^2} && \text{I is inversely proportional to d^2.} \\ 20 &= \frac{k}{(4)^2} && \text{Substitute 20 for I, 4 for d.} \\ 20 &= \frac{k}{16} && \text{Exponent first: $(4)^2 = 16$} \\ 320 &= k && \text{Multiply both sides by 25.} \end{aligned}$$

Substitute 320 for k in $I = k/d^2$, then substitute 18 for d to determine I when $d = 18$.

$$\begin{aligned} I &= \frac{320}{d^2} && \text{Substitue 320 for } k. \\ I &= \frac{320}{(18)^2} && \text{Substitute 18 for } d. \\ I &= \frac{320}{324} && \text{Simplify.} \\ I &= 1.0 && \text{Divide. Round to nearest tenth.} \end{aligned}$$

Thus, the intensity of the light 18 feet from the light source is 1.0 foot-candles.

29. Let p represent the price per person and let N be the number of people who sign up for the camping experience. Because we are told that the price per person is inversely proportional to the number of people who sign up for the camping experience, we can write:

$$p = \frac{k}{N},$$

where k is the proportionality constant. Because we are given that the price per person is \$204 when 18 people sign up, we can substitute 204 for p and 18 for N to determine k.

$$\begin{aligned} p &= \frac{k}{N} && p \text{ is inversely proportional to } N. \\ 204 &= \frac{k}{18} && \text{Substitute 204 for } p\text{, 18 for } N. \\ 3672 &= k && \text{Multiply both sides by 18.} \end{aligned}$$

Substitute 3672 for k in $p = k/N$, then substitute 35 for N to determine p when $N = 35$.

$$\begin{aligned} p &= \frac{3672}{N} && \text{Substitute 3672 for } k. \\ p &= \frac{3672}{35} && \text{Substitute 35 for } N. \\ p &= 105 && \text{Round to the nearest dollar.} \end{aligned}$$

Thus, the price per person is \$105 if 35 people sign up for the camping experience.

Chapter 8

Quadratic Functions

8.1 Introduction to Radical Notation

1. We are looking for a number whose square is -400.

- However, every time you square a real number, the result is never negative. Hence, -400 has no *real* square roots.

3. We are looking for a number whose square is -25.

- However, every time you square a real number, the result is never negative. Hence, -25 has no *real* square roots.

5. We are looking for a number whose square is 49.

- Because $(-7)^2 = 49$, the negative square root of 49 is -7.
- Because $(7)^2 = 49$, the nonnegative square root of 49 is 7.

Hence, 49 has two real square roots, -7 and 7.

7. We are looking for a number whose square is 324.

- Because $(-18)^2 = 324$, the negative square root of 324 is -18.
- Because $(18)^2 = 324$, the nonnegative square root of 324 is 18.

Hence, 324 has two real square roots, -18 and 18.

9. We are looking for a number whose square is -225.

- However, every time you square a real number, the result is never negative. Hence, -225 has no *real* square roots.

11. Every time you square a real number, the result is never negative. Hence, the equation $x^2 = -225$ has no *real* solutions.

13. There are two numbers whose square equals 361, namely -19 and 19.

$$\begin{aligned} x^2 &= 361 && \text{Original equation.} \\ x &= \pm 19 && \text{Two answers: } (-19)^2 = 361 \text{ and } (19)^2 = 361. \end{aligned}$$

Thus, the real solutions of $x^2 = 361$ are $x = -19$ or $x = 19$. Writing $x = \pm 19$ ("x equals plus or minus 19) is a shortcut for writing $x = -19$ or $x = 19$.

15. Every time you square a real number, the result is never negative. Hence, the equation $x^2 = -400$ has no *real* solutions.

17. There are two numbers whose square equals 169, namely -13 and 13.

$$\begin{aligned} x^2 &= 169 && \text{Original equation.} \\ x &= \pm 13 && \text{Two answers: } (-13)^2 = 169 \text{ and } (13)^2 = 169. \end{aligned}$$

Thus, the real solutions of $x^2 = 169$ are $x = -13$ or $x = 13$. Writing $x = \pm 13$ ("x equals plus or minus 13) is a shortcut for writing $x = -13$ or $x = 13$.

19. There are two numbers whose square equals 625, namely -25 and 25.

$$\begin{aligned} x^2 &= 625 && \text{Original equation.} \\ x &= \pm 25 && \text{Two answers: } (-25)^2 = 625 \text{ and } (25)^2 = 625. \end{aligned}$$

Thus, the real solutions of $x^2 = 625$ are $x = -25$ or $x = 25$. Writing $x = \pm 25$ ("x equals plus or minus 25) is a shortcut for writing $x = -25$ or $x = 25$.

21. The expression $\sqrt{64}$ calls for the nonnegative square root of 64. Because $(8)^2 = 64$ and 8 is nonnegative,

$$\sqrt{64} = 8.$$

23. The expression $-\sqrt{-256}$ calls for the negative square root of -256. Because you cannot square a real number and get -256, the expression $-\sqrt{-256}$ is not a real number.

25. The expression $-\sqrt{361}$ calls for the negative square root of 361. Because $(-19)^2 = 361$ and -19 is negative,

$$-\sqrt{361} = -19.$$

27. The expression $-\sqrt{100}$ calls for the negative square root of 100. Because $(-10)^2 = 100$ and -10 is negative,

$$-\sqrt{100} = -10.$$

29. The expression $\sqrt{441}$ calls for the nonnegative square root of 441. Because $(21)^2 = 441$ and 21 is nonnegative,

$$\sqrt{441} = 21.$$

31. If $a > 0$, then $-\sqrt{a}$ is the negative solution of $x^2 = a$. Hence, when we substitute $-\sqrt{a}$ into the equation $x^2 = a$, we must get a true statement:

$$(-\sqrt{a})^2 = a.$$

Thus:

$$(-\sqrt{17})^2 = 17.$$

33. If $a > 0$, then $\sqrt{a}$ is the nonnegative solution of $x^2 = a$. Hence, when we substitute $\sqrt{a}$ into the equation $x^2 = a$, we must get a true statement:

$$(\sqrt{a})^2 = a.$$

Thus:

$$(\sqrt{59})^2 = 59.$$

35. If $a > 0$, then $-\sqrt{a}$ is the negative solution of $x^2 = a$. Hence, when we substitute $-\sqrt{a}$ into the equation $x^2 = a$, we must get a true statement:

$$(-\sqrt{a})^2 = a.$$

Thus:

$$(-\sqrt{29})^2 = 29.$$

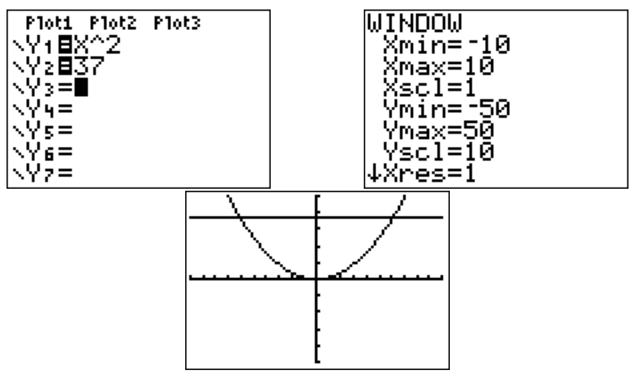
37. If $a > 0$, then $\sqrt{a}$ is the nonnegative solution of $x^2 = a$. Hence, when we substitute $\sqrt{a}$ into the equation $x^2 = a$, we must get a true statement:

$$(\sqrt{a})^2 = a.$$

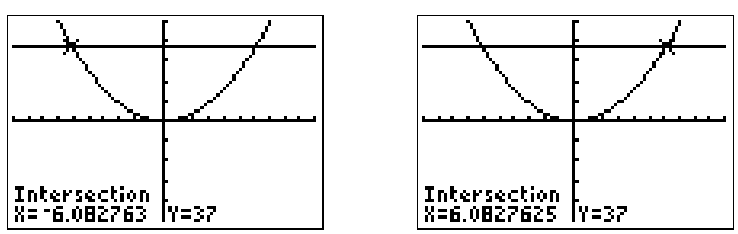
Thus:

$$(\sqrt{79})^2 = 79.$$

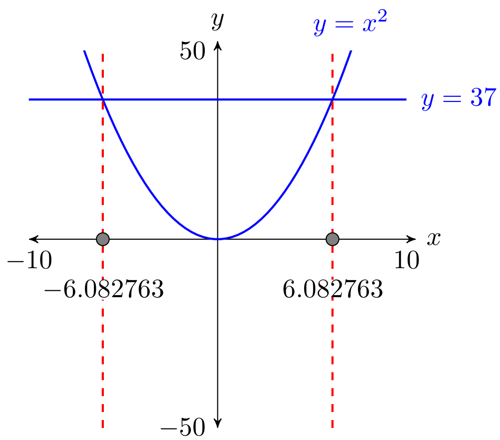
39. Enter each side of the equation $x^2 = 37$ in the **Y=** menu, then adjust the WINDOW parameters so the intersection points are visible in the viewing window.



Use the **5:intersect** utility on the CALC menu to find the points of intersection.

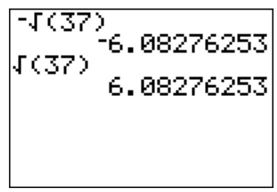


Reporting the solution on your homework: Duplicate the image in your calculator's viewing window on your homework page.



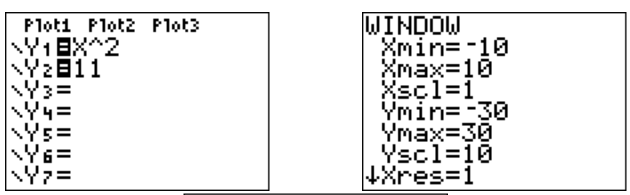
Next, solve the equation algebraically, then enter the results in your calculator to see if they match those found above.

$$x^2 = 37$$
$$x = \pm\sqrt{37}$$

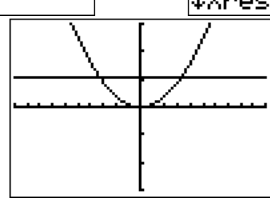

-√(37)
-6.08276253
√(37)
6.08276253

Note how these solutions match those found using the **5:intersect** utility on the calculator.

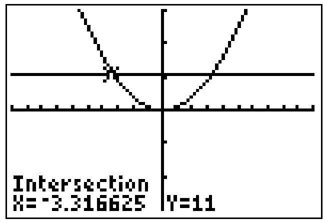
41. Enter each side of the equation $x^2 = 11$ in the **Y=** menu, then adjust the WINDOW parameters so the intersection points are visible in the viewing window.

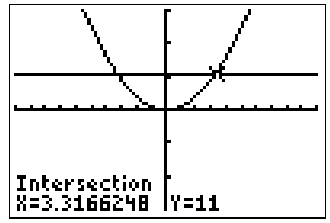

Plot1 Plot2 Plot3
\Y1=X^2
\Y2=11
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=

WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-30
Ymax=30
Yscl=10
↓Xres=1

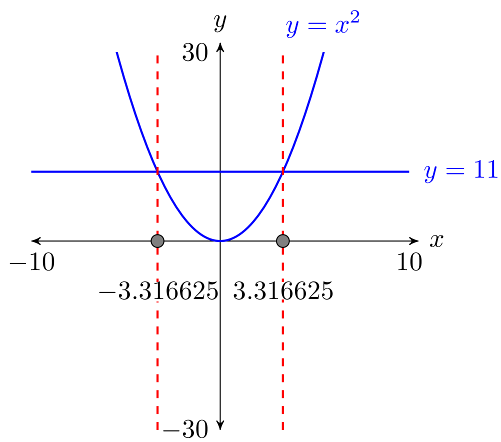


Use the **5:intersect** utility on the CALC menu to find the points of intersection.



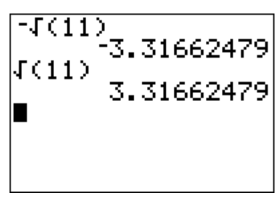


Reporting the solution on your homework: Duplicate the image in your calculator's viewing window on your homework page.



Next, solve the equation algebraically, then enter the results in your calculator to see if they match those found above.

$$x^2 = 11$$
$$x = \pm\sqrt{11}$$



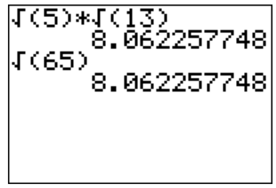
Note how these solutions match those found using the **5:intersect** utility on the calculator.

8.2 Simplifying Radical Expressions

1. Use the property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ to multiply the radicals.

$$\begin{aligned}\sqrt{5}\sqrt{13} &= \sqrt{(5)(13)}\\ &= \sqrt{65}\end{aligned}$$

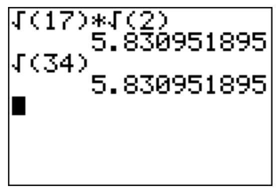
Check: Use the graphing calculator to check the result.



3. Use the property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ to multiply the radicals.

$$\begin{aligned}\sqrt{17}\sqrt{2} &= \sqrt{(17)(2)}\\ &= \sqrt{34}\end{aligned}$$

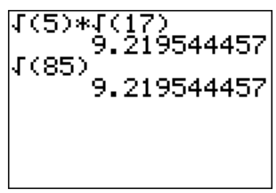
Check: Use the graphing calculator to check the result.



5. Use the property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ to multiply the radicals.

$$\begin{aligned}\sqrt{5}\sqrt{17} &= \sqrt{(5)(17)}\\ &= \sqrt{85}\end{aligned}$$

Check: Use the graphing calculator to check the result.



7. From $\sqrt{56}$, we can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned}\sqrt{56} &= \sqrt{4}\sqrt{14} && \text{Factor out a perfect square.}\\ &= 2\sqrt{14} && \text{Simplify: } \sqrt{4} = 2.\end{aligned}$$

9. From $\sqrt{99}$, we can factor out a perfect square, in this case $\sqrt{9}$.

$$\begin{aligned}\sqrt{99} &= \sqrt{9}\sqrt{11} && \text{Factor out a perfect square.}\\ &= 3\sqrt{11} && \text{Simplify: } \sqrt{9} = 3.\end{aligned}$$

11. From $\sqrt{150}$, we can factor out a perfect square, in this case $\sqrt{25}$.

$$\begin{aligned} \sqrt{150} &= \sqrt{25}\sqrt{6} && \text{Factor out a perfect square.} \\ &= 5\sqrt{6} && \text{Simplify: } \sqrt{25} = 5. \end{aligned}$$

13. From $\sqrt{40}$, we can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned} \sqrt{40} &= \sqrt{4}\sqrt{10} && \text{Factor out a perfect square.} \\ &= 2\sqrt{10} && \text{Simplify: } \sqrt{4} = 2. \end{aligned}$$

15. From $\sqrt{28}$, we can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned} \sqrt{28} &= \sqrt{4}\sqrt{7} && \text{Factor out a perfect square.} \\ &= 2\sqrt{7} && \text{Simplify: } \sqrt{4} = 2. \end{aligned}$$

17. From $\sqrt{153}$, we can factor out a perfect square, in this case $\sqrt{9}$.

$$\begin{aligned} \sqrt{153} &= \sqrt{9}\sqrt{17} && \text{Factor out a perfect square.} \\ &= 3\sqrt{17} && \text{Simplify: } \sqrt{9} = 3. \end{aligned}$$

19. From $\sqrt{50}$, we can factor out a perfect square, in this case $\sqrt{25}$.

$$\begin{aligned} \sqrt{50} &= \sqrt{25}\sqrt{2} && \text{Factor out a perfect square.} \\ &= 5\sqrt{2} && \text{Simplify: } \sqrt{25} = 5. \end{aligned}$$

21. From $\sqrt{18}$, we can factor out a perfect square, in this case $\sqrt{9}$.

$$\begin{aligned} \sqrt{18} &= \sqrt{9}\sqrt{2} && \text{Factor out a perfect square.} \\ &= 3\sqrt{2} && \text{Simplify: } \sqrt{9} = 3. \end{aligned}$$

23. From $\sqrt{44}$, we can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned} \sqrt{44} &= \sqrt{4}\sqrt{11} && \text{Factor out a perfect square.} \\ &= 2\sqrt{11} && \text{Simplify: } \sqrt{4} = 2. \end{aligned}$$

25. From $\sqrt{104}$, we can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned} \sqrt{104} &= \sqrt{4}\sqrt{26} && \text{Factor out a perfect square.} \\ &= 2\sqrt{26} && \text{Simplify: } \sqrt{4} = 2. \end{aligned}$$

27. First, write out the Pythagorean Theorem, then substitute the given values in the appropriate places.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem.} \\ (14)^2 + b^2 &= (16)^2 && \text{Substitute: 14 for } a\text{, 16 for } c. \\ 196 + b^2 &= 256 && \text{Square: } (14)^2 = 196,\ (16)^2 = 256. \\ b^2 &= 60 && \text{Subtract 196 from both sides.} \end{aligned}$$

The equation $b^2 = 60$ has two real solutions, $b = -\sqrt{60}$ and $b = \sqrt{60}$. However, in this situation, b represents the length of one leg of the right triangle and must be a positive number. Hence:

$$b = \sqrt{60} \qquad \text{Nonnegative square root.}$$

However, this answer is not in simple radical form. In this case, we can factor out the perfect square $\sqrt{4}$.

$$\begin{aligned} b &= \sqrt{4}\sqrt{15} && \sqrt{60} = \sqrt{4}\sqrt{15}. \\ b &= 2\sqrt{15} && \sqrt{4} = 2. \end{aligned}$$

Thus, the length of the missing leg is $b = 2\sqrt{15}$.

29. First, write out the Pythagorean Theorem, then substitute the given values in the appropriate places.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem.} \\ (3)^2 + b^2 &= (25)^2 && \text{Substitute: 3 for } a\text{, 25 for } c. \\ 9 + b^2 &= 625 && \text{Square: } (3)^2 = 9,\ (25)^2 = 625. \\ b^2 &= 616 && \text{Subtract 9 from both sides.} \end{aligned}$$

The equation $b^2 = 616$ has two real solutions, $b = -\sqrt{616}$ and $b = \sqrt{616}$. However, in this situation, b represents the length of one leg of the right triangle and must be a positive number. Hence:

$$b = \sqrt{616} \qquad \text{Nonnegative square root.}$$

However, this answer is not in simple radical form. In this case, we can factor out the perfect square $\sqrt{4}$.

$$\begin{aligned} b &= \sqrt{4}\sqrt{154} && \sqrt{616} = \sqrt{4}\sqrt{154}. \\ b &= 2\sqrt{154} && \sqrt{4} = 2. \end{aligned}$$

Thus, the length of the missing leg is $b = 2\sqrt{154}$.

31. First, write out the Pythagorean Theorem, then substitute the given values in the appropriate places.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem.} \\ (2)^2 + (12)^2 &= c^2 && \text{Substitute: 2 for } a\text{, 12 for } b. \\ 4 + 144 &= c^2 && \text{Square: } (2)^2 = 4,\ (12)^2 = 144. \\ 148 &= c^2 && \text{Add: } 4 + 144 = 148. \end{aligned}$$

The equation $c^2 = 148$ has two real solutions, $c = -\sqrt{148}$ and $c = \sqrt{148}$. However, in this situation, c represents the length of the hypotenuse and must be a positive number. Hence:

$$c = \sqrt{148} \qquad \text{Nonnegative square root.}$$

However, this answer is not in simple radical form. In this case, we can factor out the perfect square $\sqrt{4}$.

$$\begin{aligned} c &= \sqrt{4}\sqrt{37} && \sqrt{148} = \sqrt{4}\sqrt{37}. \\ c &= 2\sqrt{37} && \sqrt{4} = 2. \end{aligned}$$

Thus, the length of the hypotenuse is $c = 2\sqrt{37}$.

33. First, write out the Pythagorean Theorem, then substitute the given values in the appropriate places.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem.} \\ (10)^2 + (14)^2 &= c^2 && \text{Substitute: 10 for } a\text{, 14 for } b. \\ 100 + 196 &= c^2 && \text{Square: } (10)^2 = 100,\ (14)^2 = 196. \\ 296 &= c^2 && \text{Add: } 100 + 196 = 296. \end{aligned}$$

The equation $c^2 = 296$ has two real solutions, $c = -\sqrt{296}$ and $c = \sqrt{296}$. However, in this situation, c represents the length of the hypotenuse and must be a positive number. Hence:

$$c = \sqrt{296} \qquad \text{Nonnegative square root.}$$

However, this answer is not in simple radical form. In this case, we can factor out the perfect square $\sqrt{4}$.

$$\begin{aligned} c &= \sqrt{4}\sqrt{74} && \sqrt{296} = \sqrt{4}\sqrt{74}. \\ c &= 2\sqrt{74} && \sqrt{4} = 2. \end{aligned}$$

Thus, the length of the hypotenuse is $c = 2\sqrt{74}$.

35. To find the area of the shaded region, subtract the area of the triangle from the area of the semicircle.

$$\text{Area of shaded region} = \text{Area of semicircle} - \text{Area of triangle}$$

To find the area of the semicircle, we'll need to find the radius of the circle. Note that the hypotenuse of the right triangle $\triangle ABC$ is the diameter of the circle. We can use the Pythagorean Theorem to find its length.

$$\begin{aligned} (AB)^2 &= (AC)^2 + (BC)^2 \\ (AB)^2 &= (4)^2 + (3)^2 \\ (AB)^2 &= 16 + 9 \\ (AB)^2 &= 25 \end{aligned}$$

Hence, the diameter of the semicircle is $AB = 5$. The radius of the semicircle is $1/2$ of the diameter. Hence, the radius of the semicircle is $r = 5/2$. The area of a full circle is given by the formula $A = \pi r^2$, so the area of the semicircle is found by:

$$\begin{aligned} \text{Area of semicircle} &= \frac{1}{2}\pi r^2 \\ &= \frac{1}{2}\pi\left(\frac{5}{2}\right)^2 \\ &= \frac{25}{8}\pi \end{aligned}$$

To find the area of $\triangle ABC$, we take half of the base times the height.

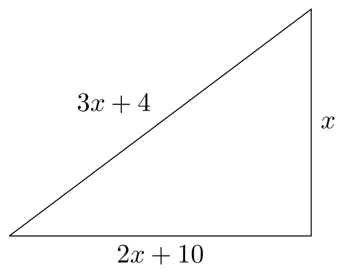
$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}(3)(4) \\ &= 6 \end{aligned}$$

We can now find the area of the shaded region.

$$\begin{aligned} \text{Area of shaded region} &= \text{Area of semicircle} - \text{Area of triangle} \\ &= \frac{25}{8}\pi - 6 \end{aligned}$$

37. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* Let x represent the length of the shorter leg of the right triangle. Because the longer leg is 10 feet longer than twice the length of the shorter leg, its length is $2x + 10$. The length of the hypotenuse is 4 feet longer than three times the length of its shorter leg, so its length is $3x + 4$. A sketch will help us maintain focus.



2. *Set up an equation.* By the Pythagorean Theorem, the sum of the squares of the legs must equal the square of the hypotenuse.

$$x^2 + (2x + 10)^2 = (3x + 4)^2$$

3. *Solve the equation.* Use the shortcut $(a + b)^2 = a^2 + 2ab + b^2$ to expand.

$$\begin{aligned} x^2 + (2x + 10)^2 &= (3x + 4)^2 \\ x^2 + 4x^2 + 40x + 100 &= 9x^2 + 24x + 16 \end{aligned}$$

Simplify the left-hand side.

$$5x^2 + 40x + 100 = 9x^2 + 24x + 16$$

The equation is nonlinear, so make one side zero by subtracting $5x^2$, $40x$, and 100 from both sides of the equation.

$$0 = 4x^2 - 16x - 84$$

Note that each coefficient is divisible by 4. Divide both sides of the equation by 4.

$$0 = x^2 - 4x - 21$$

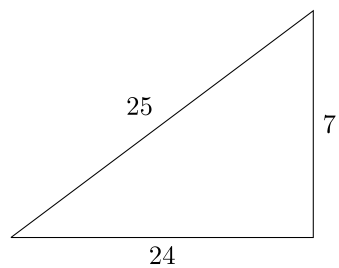
Note that the integer pair 3 and -7 has a product equal to $ac = (1)(-21) = -21$ and sum equal to $b = -4$. Because the leading coefficient is a 1, we can "drop this pair in place" to factor.

$$0 = (x - 7)(x + 3)$$

Hence, $x = 7$ and $x = -3$.

4. *Answer the question.* Because x represents the length of the shortest side of the right triangle, and length is a positive quantity, we discard the answer $x = -3$. Next, if the length of the shorter leg is $x = 7$ feet, then the length of the longer leg is $2x + 10 = 2(7) + 10$, or 24 feet. Finally, the length of the hypotenuse is $3x + 4 = 3(7) + 4$, or 25 feet.

5. *Look back.* Note that the length of the longer leg is 24 feet, which is 10 feet longer than twice 7 feet, the length of the shorter leg. The length of the hypotenuse is 25, which is 4 feet longer than three times 7 feet, the length of the shorter leg. But do the numbers satisfy the Pythagorean Theorem?



That is, is it true that

$$7^2 + 24^2 \stackrel{?}{=} 25^2$$

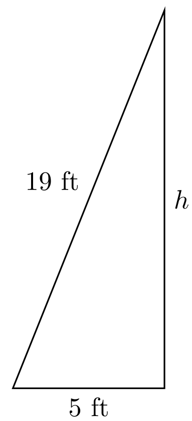
Simplify each square.

$$49 + 576 \stackrel{?}{=} 625$$

Note that this last statement is true, so our solution checks.

39. As always, we obey the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* We'll create a well-marked diagram for this purpose, letting h represent the distance between the base of the garage wall and the upper tip of the ladder.



2. *Set up an equation.* Using the Pythagorean Theorem, we can write:

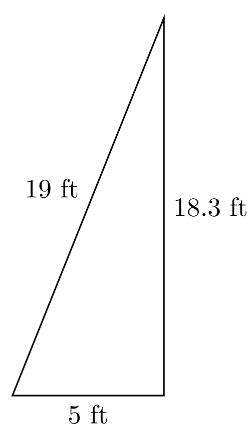
$$5^2 + h^2 = 19^2 \qquad \text{Pythagorean Theorem.}$$
$$25 + h^2 = 361 \qquad \text{Square: } 5^2 = 25 \text{ and } 19^2 = 361.$$

3. *Solve the equation.*

$$h^2 = 336 \qquad \text{Subtract 25 from both sides.}$$
$$h = \sqrt{336} \qquad h \text{ will be the nonnegative square root.}$$

4. *Answer the question.* As we're asked for a decimal approximation, we won't worry about simple radical form in this situation. The ladder reaches $\sqrt{336}$ feet up the wall. Using a calculator, this is about 18.3 feet, rounded to the nearest tenth of a foot.

5. *Look back.* Understand that when we use 18.3 ft, an approximation, our solution will only check approximately.



Using the Pythagorean Theorem:

$$\begin{aligned} 5^2 + (18.3)^2 &= 19^2 \\ 25 + 334.89 &= 361 \\ 359.89 &= 361 \end{aligned}$$

The approximation is not perfect, but it seems close enough!

8.3 Completing the Square

1. There are two square roots of 84, one positive and one negative.

$$\begin{aligned} x &= 84 && \text{Original equation.} \\ x &= \pm\sqrt{84} && \text{Two square roots of 84.} \end{aligned}$$

However, these answers are not in simple radical form. In this case, we can factor out a perfect square, namely $\sqrt{4}$, then simplify.

$$\begin{aligned} x &= \pm\sqrt{4}\sqrt{21} && \sqrt{84} = \sqrt{4}\sqrt{21} \\ x &= \pm 2\sqrt{21} && \text{Simplify: } \sqrt{4} = 2 \end{aligned}$$

Therefore, the solutions of $x^2 = 84$ are $x = \pm 2\sqrt{21}$.

3. There are two square roots of 68, one positive and one negative.

$$\begin{aligned} x &= 68 && \text{Original equation.} \\ x &= \pm\sqrt{68} && \text{Two square roots of 68.} \end{aligned}$$

However, these answers are not in simple radical form. In this case, we can factor out a perfect square, namely $\sqrt{4}$, then simplify.

$$\begin{aligned} x &= \pm\sqrt{4}\sqrt{17} && \sqrt{68} = \sqrt{4}\sqrt{17} \\ x &= \pm 2\sqrt{17} && \text{Simplify: } \sqrt{4} = 2 \end{aligned}$$

Therefore, the solutions of $x^2 = 68$ are $x = \pm 2\sqrt{17}$.

5. Consider again the equation

$$x^2 = -16$$

You cannot square a real number and get a negative result. Hence, this equation has no real solutions.

7. There are two square roots of 124, one positive and one negative.

$$\begin{aligned} x &= 124 && \text{Original equation.} \\ x &= \pm\sqrt{124} && \text{Two square roots of 124.} \end{aligned}$$

However, these answers are not in simple radical form. In this case, we can factor out a perfect square, namely $\sqrt{4}$, then simplify.

$$\begin{aligned} x &= \pm\sqrt{4}\sqrt{31} && \sqrt{124} = \sqrt{4}\sqrt{31} \\ x &= \pm 2\sqrt{31} && \text{Simplify: } \sqrt{4} = 2 \end{aligned}$$

Therefore, the solutions of $x^2 = 124$ are $x = \pm 2\sqrt{31}$.

9. Much like the solutions of $x^2 = 36$ are $x = \pm 6$, we use a similar approach on $(x + 19)^2 = 36$ to obtain:

$$\begin{aligned} (x + 19)^2 &= 36 && \text{Original equation.} \\ x + 19 &= \pm 6 && \text{There are two square roots.} \end{aligned}$$

To complete the solution, subtract 19 to both sides of the equation.

$$x = -19 \pm 6 \qquad \text{Subtract 19 from both sides.}$$

Note that this means that there are two answers, namely:

$$\begin{aligned} x &= -19 - 6 && \text{or} && x = -19 + 6 \\ x &= -25 && && x = -13 \end{aligned}$$

11. Much like the solutions of $x^2 = 100$ are $x = \pm 10$, we use a similar approach on $(x + 14)^2 = 100$ to obtain:

$$\begin{aligned} (x + 14)^2 &= 100 && \text{Original equation.} \\ x + 14 &= \pm 10 && \text{There are two square roots.} \end{aligned}$$

To complete the solution, subtract 14 to both sides of the equation.

$$x = -14 \pm 10 \qquad \text{Subtract 14 from both sides.}$$

Note that this means that there are two answers, namely:

$$\begin{aligned} x &= -14 - 10 && \text{or} && x = -14 + 10 \\ x &= -24 && && x = -4 \end{aligned}$$

13. Using the shortcut $(a+b)^2 = a^2 + 2ab + b^2$, we square the binomial as follows:

$$\begin{aligned}(x+23)^2 &= x^2 + 2(x)(23) + (23)^2 \\ &= x^2 + 46x + 529\end{aligned}$$

15. Using the shortcut $(a+b)^2 = a^2 + 2ab + b^2$, we square the binomial as follows:

$$\begin{aligned}(x+11)^2 &= x^2 + 2(x)(11) + (11)^2 \\ &= x^2 + 22x + 121\end{aligned}$$

17. Using the shortcut $(a-b)^2 = a^2 - 2ab + b^2$, we square the binomial as follows:

$$\begin{aligned}(x-25)^2 &= x^2 - 2(x)(25) + (25)^2 \\ &= x^2 - 50x + 625\end{aligned}$$

19. Using the shortcut $a^2 + 2ab + b^2 = (a+b)^2$, we factor the trinomial by taking the square roots of the first and last terms, then writing:

$$x^2 + 24x + 144 = (x+12)^2$$

Note that $2(x)(12) = 24x$, so the middle term checks.

21. Using the shortcut $a^2 - 2ab + b^2 = (a-b)^2$, we factor the trinomial by taking the square roots of the first and last terms, then writing:

$$x^2 - 34x + 289 = (x-17)^2$$

Note that $2(x)(17) = 34x$, so the middle term checks.

23. Using the shortcut $a^2 - 2ab + b^2 = (a-b)^2$, we factor the trinomial by taking the square roots of the first and last terms, then writing:

$$x^2 - 20x + 100 = (x-10)^2$$

Note that $2(x)(10) = 20x$, so the middle term checks.

25. Compare $x^2 - 20x$ with $x^2 + bx$ and note that $b = -20$.

1. Take one-half of -20: -10
2. Square the result of step 1: $(-10)^2 = 100$
3. Add the result of step 2 to $x^2 - 20x$: $x^2 - 20x + 100$

Check: Note that the first and last terms of $x^2 - 20x + 100$ are perfect squares. Take the square roots of the first and last terms and factor as follows:

$$x^2 - 20x + 100 = (x - 10)^2$$

Note that $2(x)(-10) = -20x$, so the middle term checks.

27. Compare $x^2 - 6x$ with $x^2 + bx$ and note that $b = -6$.

1. Take one-half of -6: -3
2. Square the result of step 1: $(-3)^2 = 9$
3. Add the result of step 2 to $x^2 - 6x$: $x^2 - 6x + 9$

Check: Note that the first and last terms of $x^2 - 6x + 9$ are perfect squares. Take the square roots of the first and last terms and factor as follows:

$$x^2 - 6x + 9 = (x - 3)^2$$

Note that $2(x)(-3) = -6x$, so the middle term checks.

29. Compare $x^2 + 20x$ with $x^2 + bx$ and note that $b = 20$.

1. Take one-half of 20: 10
2. Square the result of step 1: $(10)^2 = 100$
3. Add the result of step 2 to $x^2 + 20x$: $x^2 + 20x + 100$

Check: Note that the first and last terms of $x^2 + 20x + 100$ are perfect squares. Take the square roots of the first and last terms and factor as follows:

$$x^2 + 20x + 100 = (x + 10)^2$$

Note that $2(x)(10) = 20x$, so the middle term checks.

31. Compare $x^2 + 7x$ with $x^2 + bx$ and note that $b = 7$.

1. Take one-half of 7: $7/2$
2. Square the result of step 1: $(7/2)^2 = 49/4$
3. Add the result of step 2 to $x^2 + 7x$: $x^2 + 7x + 49/4$

Check: Note that the first and last terms of $x^2+7x+49/4$ are perfect squares. Take the square roots of the first and last terms and factor as follows:

$$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

Note that $2(x)(7/2) = 7x$, so the middle term checks.

33. Compare $x^2 + 15x$ with $x^2 + bx$ and note that $b = 15$.

1. Take one-half of 15: $15/2$
2. Square the result of step 1: $(15/2)^2 = 225/4$
3. Add the result of step 2 to $x^2 + 15x$: $x^2 + 15x + 225/4$

Check: Note that the first and last terms of $x^2 + 15x + 225/4$ are perfect squares. Take the square roots of the first and last terms and factor as follows:

$$x^2 + 15x + \frac{225}{4} = \left(x + \frac{15}{2}\right)^2$$

Note that $2(x)(15/2) = 15x$, so the middle term checks.

35. Compare $x^2 - 5x$ with $x^2 + bx$ and note that $b = -5$.

1. Take one-half of -5: $-5/2$
2. Square the result of step 1: $(-5/2)^2 = 25/4$
3. Add the result of step 2 to $x^2 - 5x$: $x^2 - 5x + 25/4$

Check: Note that the first and last terms of $x^2-5x+25/4$ are perfect squares. Take the square roots of the first and last terms and factor as follows:

$$x^2 - 5x + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2$$

Note that $2(x)(-5/2) = -5x$, so the middle term checks.

37. Note that the equation $x^2 = 18x - 18$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 - 18x + 18 = 0$$

We would then calculate $ac = (1)(18)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 18$ and whose sum is $b = -18$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 - 18x = -18$$

On the left, take one-half of the coefficient of x: $(1/2)(-18) = -9$. Square the result: $(-9)^2 = 81$. Add this result to both sides of the equation.

$$\begin{aligned} x^2 - 18x + 81 &= -18 + 81 \\ x^2 - 18x + 81 &= 63 \end{aligned}$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x - 9)^2 = 63$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x - 9 = \pm\sqrt{63}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{9}$.

$$\begin{aligned} x - 9 &= \pm\sqrt{9}\sqrt{7} \\ x - 9 &= \pm 3\sqrt{7} \end{aligned}$$

Finally, add 9 to both sides of the equation.

$$x = 9 \pm 3\sqrt{7}$$

Thus, the equation $x^2 = 18x - 18$ has two answers, $x = 9 - 3\sqrt{7}$ and $x = 9 + 3\sqrt{7}$.

39. Note that the equation $x^2 = 16x - 16$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 - 16x + 16 = 0$$

We would then calculate $ac = (1)(16)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 16$ and whose sum is $b = -16$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 - 16x = -16$$

On the left, take one-half of the coefficient of x: $(1/2)(-16) = -8$. Square the result: $(-8)^2 = 64$. Add this result to both sides of the equation.

$$\begin{aligned} x^2 - 16x + 64 &= -16 + 64 \\ x^2 - 16x + 64 &= 48 \end{aligned}$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x - 8)^2 = 48$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x - 8 = \pm\sqrt{48}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{16}$.

$$\begin{aligned} x - 8 &= \pm\sqrt{16}\sqrt{3} \\ x - 8 &= \pm 4\sqrt{3} \end{aligned}$$

Finally, add 8 to both sides of the equation.

$$x = 8 \pm 4\sqrt{3}$$

Thus, the equation $x^2 = 16x - 16$ has two answers, $x = 8 - 4\sqrt{3}$ and $x = 8 + 4\sqrt{3}$.

41. Note that the equation $x^2 = -16x - 4$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 + 16x + 4 = 0$$

We would then calculate $ac = (1)(4)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 4$ and whose sum is $b = 16$. Hence, this trinomial will not factor using the ac-method. Therefore,

we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 + 16x = -4$$

On the left, take one-half of the coefficient of x: $(1/2)(16) = 8$. Square the result: $(8)^2 = 64$. Add this result to both sides of the equation.

$$\begin{aligned} x^2 + 16x + 64 &= -4 + 64 \\ x^2 + 16x + 64 &= 60 \end{aligned}$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x + 8)^2 = 60$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x + 8 = \pm\sqrt{60}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned} x + 8 &= \pm\sqrt{4}\sqrt{15} \\ x + 8 &= \pm 2\sqrt{15} \end{aligned}$$

Finally, subtract 8 from both sides of the equation.

$$x = -8 \pm 2\sqrt{15}$$

Thus, the equation $x^2 = -16x - 4$ has two answers, $x = -8 - 2\sqrt{15}$ and $x = -8 + 2\sqrt{15}$.

43. Note that the equation $x^2 = 18x - 9$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 - 18x + 9 = 0$$

We would then calculate $ac = (1)(9)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 9$ and whose sum is $b = -18$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 - 18x = -9$$

On the left, take one-half of the coefficient of x: $(1/2)(-18) = -9$. Square the result: $(-9)^2 = 81$. Add this result to both sides of the equation.

$$\begin{aligned} x^2 - 18x + 81 &= -9 + 81 \\ x^2 - 18x + 81 &= 72 \end{aligned}$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x - 9)^2 = 72$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x - 9 = \pm\sqrt{72}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{36}$.

$$\begin{aligned} x - 9 &= \pm\sqrt{36}\sqrt{2} \\ x - 9 &= \pm 6\sqrt{2} \end{aligned}$$

Finally, add 9 to both sides of the equation.

$$x = 9 \pm 6\sqrt{2}$$

Thus, the equation $x^2 = 18x-9$ has two answers, $x = 9-6\sqrt{2}$ and $x = 9+6\sqrt{2}$.

45. Note that the equation $x^2 = 16x - 8$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 - 16x + 8 = 0$$

We would then calculate $ac = (1)(8)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 8$ and whose sum is $b = -16$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 - 16x = -8$$

On the left, take one-half of the coefficient of x: $(1/2)(-16) = -8$. Square the result: $(-8)^2 = 64$. Add this result to both sides of the equation.

$$\begin{aligned} x^2 - 16x + 64 &= -8 + 64 \\ x^2 - 16x + 64 &= 56 \end{aligned}$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x-8)^2 = 56$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x - 8 = \pm\sqrt{56}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{4}$.

$$\begin{aligned} x - 8 &= \pm\sqrt{4}\sqrt{14} \\ x - 8 &= \pm 2\sqrt{14} \end{aligned}$$

Finally, add 8 to both sides of the equation.

$$x = 8 \pm 2\sqrt{14}$$

Thus, the equation $x^2 = 16x - 8$ has two answers, $x = 8 - 2\sqrt{14}$ and $x = 8 + 2\sqrt{14}$.

47. Note that the equation $x^2 = -18x - 18$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 + 18x + 18 = 0$$

We would then calculate $ac = (1)(18)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 18$ and whose sum is $b = 18$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 + 18x = -18$$

On the left, take one-half of the coefficient of x: $(1/2)(18) = 9$. Square the result: $(9)^2 = 81$. Add this result to both sides of the equation.

$$\begin{aligned} x^2 + 18x + 81 &= -18 + 81 \\ x^2 + 18x + 81 &= 63 \end{aligned}$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x+9)^2 = 63$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x + 9 = \pm\sqrt{63}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{9}$.

$$x + 9 = \pm\sqrt{9}\sqrt{7}$$
$$x + 9 = \pm 3\sqrt{7}$$

Finally, subtract 9 from both sides of the equation.

$$x = -9 \pm 3\sqrt{7}$$

Thus, the equation $x^2 = -18x - 18$ has two answers, $x = -9 - 3\sqrt{7}$ and $x = -9 + 3\sqrt{7}$.

49. Note that the equation $x^2 = -16x - 20$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 + 16x + 20 = 0$$

We would then calculate $ac = (1)(20)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 20$ and whose sum is $b = 16$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 + 16x = -20$$

On the left, take one-half of the coefficient of x: $(1/2)(16) = 8$. Square the result: $(8)^2 = 64$. Add this result to both sides of the equation.

$$x^2 + 16x + 64 = -20 + 64$$
$$x^2 + 16x + 64 = 44$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x + 8)^2 = 44$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x + 8 = \pm\sqrt{44}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{4}$.

$$x + 8 = \pm\sqrt{4}\sqrt{11}$$
$$x + 8 = \pm 2\sqrt{11}$$

Finally, subtract 8 from both sides of the equation.

$$x = -8 \pm 2\sqrt{11}$$

Thus, the equation $x^2 = -16x - 20$ has two answers, $x = -8 - 2\sqrt{11}$ and $x = -8 + 2\sqrt{11}$.

51. Note that the equation $x^2 = -18x - 1$ is nonlinear (there is a power of x greater than one). Normal procedure would be to first make one side zero.

$$x^2 + 18x + 1 = 0$$

We would then calculate $ac = (1)(1)$. However, after some exploration, we discover that there is no integer pair whose product is $ac = 1$ and whose sum is $b = 18$. Hence, this trinomial will not factor using the ac-method. Therefore, we'll use the technique of *completing the square* to solve the equation. First, move the constant term to the right-hand side of the equation.

$$x^2 + 18x = -1$$

On the left, take one-half of the coefficient of x: $(1/2)(18) = 9$. Square the result: $(9)^2 = 81$. Add this result to both sides of the equation.

$$x^2 + 18x + 81 = -1 + 81$$
$$x^2 + 18x + 81 = 80$$

We can now factor the left-hand side as a perfect square trinomial.

$$(x + 9)^2 = 80$$

Now, as in Examples **??**, **??**, and **??**, we can take the square root of both sides of the equation. Remember, there are two square roots.

$$x + 9 = \pm\sqrt{80}$$

The right hand side is not in simple radical form. We can factor out a perfect square, in this case $\sqrt{16}$.

$$x + 9 = \pm\sqrt{16}\sqrt{5}$$
$$x + 9 = \pm 4\sqrt{5}$$

Finally, subtract 9 from both sides of the equation.

$$x = -9 \pm 4\sqrt{5}$$

Thus, the equation $x^2 = -18x - 1$ has two answers, $x = -9 - 4\sqrt{5}$ and $x = -9 + 4\sqrt{5}$.

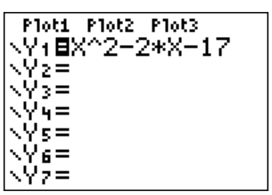
53. First, move the constant -17 to the right-hand side of of the equation.

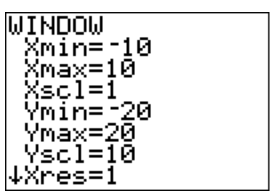
$$\begin{aligned} x^2 - 2x - 17 &= 0 && \text{Original equation.} \\ x^2 - 2x &= 17 && \text{Add 17 to both sides.} \end{aligned}$$

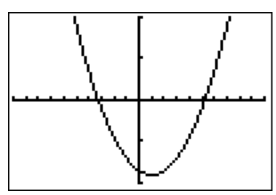
Take half of the coefficient of x: $(1/2)(-2) = -1$. Square, $(-1)^2 = 1$, then add this result to both sides of the equation.

$$\begin{aligned} x^2 - 2x + 1 &= 17 + 1 && \text{Add 1 to both sides.} \\ (x-1)^2 &= 18 && \text{Factor left-hand side.} \\ x - 1 &= \pm\sqrt{18} && \text{There are two square roots.} \\ x - 1 &= \pm\sqrt{9}\sqrt{2} && \text{Factor out a perfect square.} \\ x - 1 &= \pm 3\sqrt{2} && \text{Simplify: } \sqrt{9} = 3. \\ x &= 1 \pm 3\sqrt{2} && \text{Add 1 to both sides.} \end{aligned}$$

Graphical solution: Enter the equation $y = x^2 - 2x - 17$ in **Y1** of the Y= menu (see the first image below). After some experimentation, we settled on the WINDOW parameters shown in the middle image below. Once you've entered these WINDOW parameters, push the GRAPH button to produce the the graph of $y = x^2 - 2x - 17$, as shown in the rightmost image below.

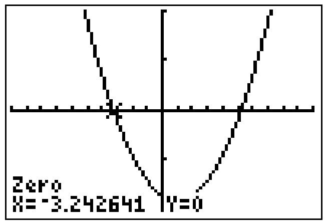


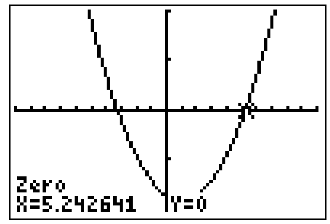




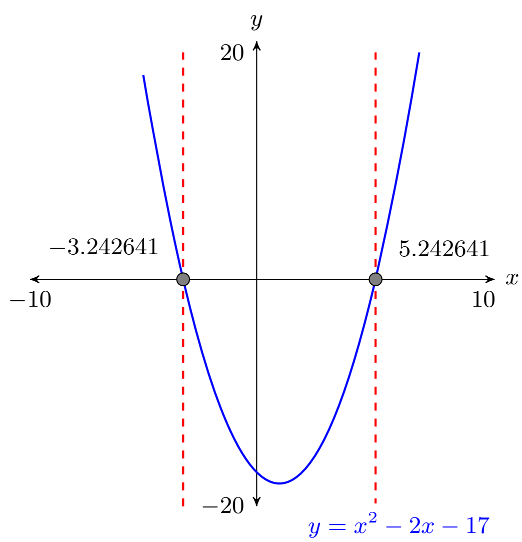
We're looking for solutions of $x^2 - 2x - 17 = 0$, so we need to locate where the graph of $y = x^2 - 2x - 17$ intercepts the x-axis. That is, we need to find the zeros of $y = x^2 - 2x - 17$. Select **2:zero** from the CALC menu, move the cursor slightly to the left of the first x-intercept and press ENTER in response to "Left bound." Move the cursor slightly to the right of the first x-intercept and press ENTER in response to "Right bound." Leave the cursor where it sits

and press ENTER in response to "Guess." The calculator responds by finding the x-coordinate of the x-intercept, as shown in the first image below. Repeat the process to find the second x-intercept of $y = x^2 - 2x - 17$ shown in the second image below.

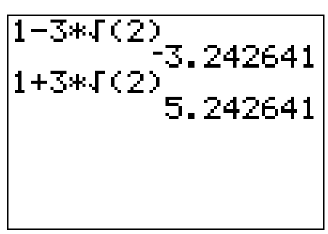




Reporting the solution on your homework: Duplicate the image in your calculator's viewing window on your homework page. Use a ruler to draw all lines, but freehand any curves.



Comparing exact and calculator approximations. How well do the graphing calculator solutions compare with the exact solutions, $x = 1 - 3\sqrt{2}$ and $x = 1 + 3\sqrt{2}$? After entering each in the calculator, the comparison is excellent!



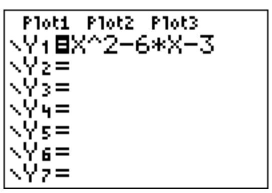
55. First, move the constant -3 to the right-hand side of of the equation.

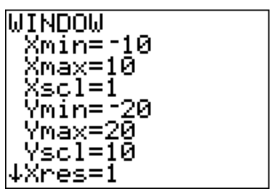
$$\begin{aligned} x^2 - 6x - 3 &= 0 && \text{Original equation.} \\ x^2 - 6x &= 3 && \text{Add 3 to both sides.} \end{aligned}$$

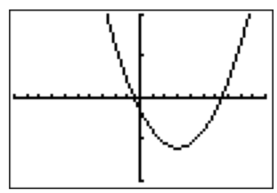
Take half of the coefficient of x: $(1/2)(-6) = -3$. Square, $(-3)^2 = 9$, then add this result to both sides of the equation.

$$\begin{aligned} x^2 - 6x + 9 &= 3 + 9 && \text{Add 9 to both sides.} \\ (x-3)^2 &= 12 && \text{Factor left-hand side.} \\ x - 3 &= \pm\sqrt{12} && \text{There are two square roots.} \\ x - 3 &= \pm\sqrt{4}\sqrt{3} && \text{Factor out a perfect square.} \\ x - 3 &= \pm 2\sqrt{3} && \text{Simplify: } \sqrt{4} = 2. \\ x &= 3 \pm 2\sqrt{3} && \text{Add 3 to both sides.} \end{aligned}$$

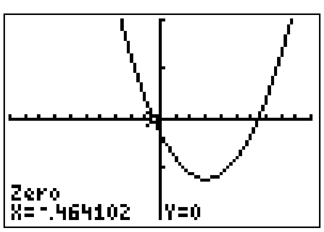
Graphical solution: Enter the equation $y = x^2 - 6x - 3$ in **Y1** of the Y= menu (see the first image below). After some experimentation, we settled on the WINDOW parameters shown in the middle image below. Once you've entered these WINDOW parameters, push the GRAPH button to produce the the graph of $y = x^2 - 6x - 3$, as shown in the rightmost image below.

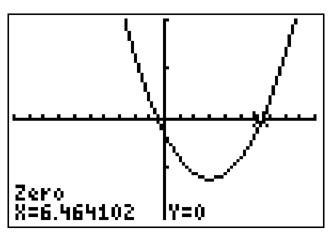




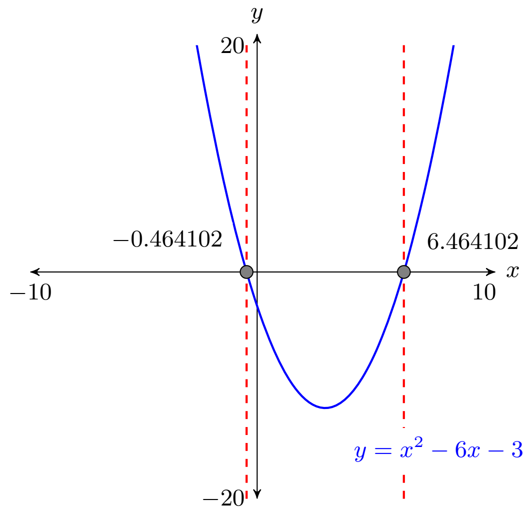


We're looking for solutions of $x^2 - 6x - 3 = 0$, so we need to locate where the graph of $y = x^2 - 6x - 3$ intercepts the x-axis. That is, we need to find the zeros of $y = x^2 - 6x - 3$. Select **2:zero** from the CALC menu, move the cursor slightly to the left of the first x-intercept and press ENTER in response to "Left bound." Move the cursor slightly to the right of the first x-intercept and press ENTER in response to "Right bound." Leave the cursor where it sits and press ENTER in response to "Guess." The calculator responds by finding the x-coordinate of the x-intercept, as shown in the first image below. Repeat the process to find the second x-intercept of $y = x^2 - 6x - 3$ shown in the second image below.

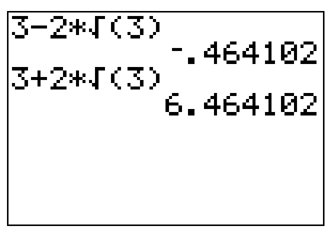




Reporting the solution on your homework: Duplicate the image in your calculator's viewing window on your homework page. Use a ruler to draw all lines, but freehand any curves.



Comparing exact and calculator approximations. How well do the graphing calculator solutions compare with the exact solutions, $x = 3 - 2\sqrt{3}$ and $x = 3 + 2\sqrt{3}$? After entering each in the calculator, the comparison is excellent!



8.4 The Quadratic Formula

1. The integer pair 4, −7 has product $ac = -28$ and sum $b = -3$. Hence, this trinomial factors.

$$x^2 - 3x - 28 = 0$$
$$(x + 4)(x - 7) = 0$$

Now we can use the zero product property to write:

$$x + 4 = 0 \quad \text{or} \quad x - 7 = 0$$
$$x = -4 \qquad\qquad x = 7$$

Thus, the solutions are $x = -4$ or $x = 7$.

Quadratic formula: Compare $ax^2 + bx + c = 0$ and $x^2 - 3x - 28 = 0$ and note that $a = 1$, $b = -3$, and $c = -28$. Next, replace each occurrence of a, b, and c in the quadratic formula with open parentheses.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ The quadratic formula.

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ Replace a, b, and c with open parentheses.

Now we can substitute: 1 for a, -3 for b, and -28 for c.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-28)}}{2(1)}$$ Substitute: 1 for a, -3 for b, and -28 for c

$$x = \frac{3 \pm \sqrt{9 + 112}}{2}$$ Simplify. Exponents, then multiplication.

$$x = \frac{3 \pm \sqrt{121}}{2}$$ Simplify: $9 + 112 = 121$.

$$x = \frac{3 \pm 11}{2}$$ Simplify: $\sqrt{121} = 11$.

Note that because of the "plus or minus" symbol, we have two answers.

$$x = \frac{3 - 11}{2} \quad \text{or} \quad x = \frac{3 + 11}{2}$$
$$x = \frac{-8}{2} \qquad\qquad x = \frac{14}{2}$$
$$x = -4 \qquad\qquad x = 7$$

Note that these answers match the answers found using the ac-test to factor the trinomial.

3. The integer pair -3, -5 has product $ac = 15$ and sum $b = -8$. Hence, this trinomial factors.

$$x^2 - 8x + 15 = 0$$
$$(x - 3)(x - 5) = 0$$

Now we can use the zero product property to write:

$$\begin{aligned} x - 3 &= 0 \quad &\text{or} \quad x - 5 &= 0 \\ x &= 3 & x &= 5 \end{aligned}$$

Thus, the solutions are $x = 3$ or $x = 5$.

Quadratic formula: Compare $ax^2 + bx + c = 0$ and $x^2 - 8x + 15 = 0$ and note that $a = 1$, $b = -8$, and $c = 15$. Next, replace each occurrence of a, b, and c in the quadratic formula with open parentheses.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ The quadratic formula.

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ Replace a, b, and c with open parentheses.

Now we can substitute: 1 for a, -8 for b, and 15 for c.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$ Substitute: 1 for a, -8 for b, and 15 for c

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$ Simplify. Exponents, then multiplication.

$$x = \frac{8 \pm \sqrt{4}}{2}$$ Simplify: $64 - 60 = 4$.

$$x = \frac{8 \pm 2}{2}$$ Simplify: $\sqrt{4} = 2$.

Note that because of the "plus or minus" symbol, we have two answers.

$$\begin{aligned} x &= \frac{8-2}{2} \quad &\text{or} \quad x &= \frac{8+2}{2} \\ x &= \frac{6}{2} & x &= \frac{10}{2} \\ x &= 3 & x &= 5 \end{aligned}$$

Note that these answers match the answers found using the ac-test to factor the trinomial.

5. The integer pair 6, -8 has product $ac = -48$ and sum $b = -2$. Hence, this trinomial factors.

$$\begin{aligned} x^2 - 2x - 48 &= 0 \\ (x + 6)(x - 8) &= 0 \end{aligned}$$

Now we can use the zero product property to write:

$$\begin{aligned} x + 6 &= 0 \quad &\text{or}\quad x - 8 &= 0 \\ x &= -6 & x &= 8 \end{aligned}$$

Thus, the solutions are $x = -6$ or $x = 8$.

Quadratic formula: Compare $ax^2 + bx + c = 0$ and $x^2 - 2x - 48 = 0$ and note that $a = 1$, $b = -2$, and $c = -48$. Next, replace each occurrence of a, b, and c in the quadratic formula with open parentheses.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ The quadratic formula.

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ Replace a, b, and c with open parentheses.

Now we can substitute: 1 for a, -2 for b, and -48 for c.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-48)}}{2(1)}$$ Substitute: 1 for a, -2 for b, and -48 for c

$$x = \frac{2 \pm \sqrt{4 + 192}}{2}$$ Simplify. Exponents, then multiplication.

$$x = \frac{2 \pm \sqrt{196}}{2}$$ Simplify: $4 + 192 = 196$.

$$x = \frac{2 \pm 14}{2}$$ Simplify: $\sqrt{196} = 14$.

Note that because of the "plus or minus" symbol, we have two answers.

$$\begin{aligned} x &= \frac{2 - 14}{2} \quad &\text{or}\quad x &= \frac{2 + 14}{2} \\ x &= \frac{-12}{2} & x &= \frac{16}{2} \\ x &= -6 & x &= 8 \end{aligned}$$

Note that these answers match the answers found using the ac-test to factor the trinomial.

7. The integer pair 6, -5 has product $ac = -30$ and sum $b = 1$. Hence, this trinomial factors.

$$\begin{aligned} x^2 + x - 30 &= 0 \\ (x + 6)(x - 5) &= 0 \end{aligned}$$

Now we can use the zero product property to write:

$$\begin{aligned} x + 6 &= 0 \quad &\text{or} \quad x - 5 &= 0 \\ x &= -6 & x &= 5 \end{aligned}$$

Thus, the solutions are $x = -6$ or $x = 5$.

Quadratic formula: Compare $ax^2 + bx + c = 0$ and $x^2 + x - 30 = 0$ and note that $a = 1$, $b = 1$, and $c = -30$. Next, replace each occurrence of a, b, and c in the quadratic formula with open parentheses.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{The quadratic formula.}$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{Replace } a,\ b, \text{ and } c \text{ with open parentheses.}$$

Now we can substitute: 1 for a, 1 for b, and -30 for c.

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-30)}}{2(1)} \qquad \text{Substitute: 1 for } a, \text{ 1 for } b, \text{ and } -30 \text{ for } c$$

$$x = \frac{-1 \pm \sqrt{1 + 120}}{2} \qquad \text{Simplify. Exponents, then multiplication.}$$

$$x = \frac{-1 \pm \sqrt{121}}{2} \qquad \text{Simplify: } 1 + 120 = 121.$$

$$x = \frac{-1 \pm 11}{2} \qquad \text{Simplify: } \sqrt{121} = 11.$$

Note that because of the "plus or minus" symbol, we have two answers.

$$\begin{aligned} x &= \frac{-1 - 11}{2} \quad &\text{or} \quad x &= \frac{-1 + 11}{2} \\ x &= \frac{-12}{2} & x &= \frac{10}{2} \\ x &= -6 & x &= 5 \end{aligned}$$

Note that these answers match the answers found using the ac-test to factor the trinomial.

9. Compare $x^2 - 7x - 5 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 1$, $b = -7$, and $c = -5$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 1 for a, -7 for b, and -5 for c.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-5)}}{2(1)}$$ Substitute: 1 for a, -7 for b, and -5 for c

$$x = \frac{7 \pm \sqrt{49 + 20}}{2}$$ Exponent, then multiplication.

$$x = \frac{7 \pm \sqrt{69}}{2}$$ Simplify.

11. Compare $2x^2 + x - 4 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 2$, $b = 1$, and $c = -4$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula. Replace a, b,

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ and c with open parentheses.

Substitute 2 for a, 1 for b, and -4 for c.

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(2)(-4)}}{2(2)}$$ Substitute: 2 for a, 1 for b, and -4 for c

$$x = \frac{-1 \pm \sqrt{1 + 32}}{4}$$ Exponent, then multiplication.

$$x = \frac{-1 \pm \sqrt{33}}{4}$$ Simplify.

13. Compare $x^2 - 7x - 4 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 1$, $b = -7$, and $c = -4$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula. Replace a, b,

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ and c with open parentheses.

Substitute 1 for a, -7 for b, and -4 for c.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-4)}}{2(1)}$$ Substitute: 1 for a, -7 for b, and -4 for c

$$x = \frac{7 \pm \sqrt{49 + 16}}{2}$$ Exponent, then multiplication.

$$x = \frac{7 \pm \sqrt{65}}{2}$$ Simplify.

15. Compare $4x^2 - x - 2 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 4$, $b = -1$, and $c = -2$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$
$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 4 for a, -1 for b, and -2 for c.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-2)}}{2(4)} \qquad \text{Substitute: 4 for } a,\ -1 \text{ for } b, \text{ and } -2 \text{ for } c$$
$$x = \frac{1 \pm \sqrt{1 + 32}}{8} \qquad \text{Exponent, then multiplication.}$$
$$x = \frac{1 \pm \sqrt{33}}{8} \qquad \text{Simplify.}$$

17. Compare $x^2 - x - 11 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 1$, $b = -1$, and $c = -11$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$
$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 1 for a, -1 for b, and -11 for c.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-11)}}{2(1)} \qquad \text{Substitute: 1 for } a,\ -1 \text{ for } b, \text{ and } -11 \text{ for } c$$
$$x = \frac{1 \pm \sqrt{1 + 44}}{2} \qquad \text{Exponent, then multiplication.}$$
$$x = \frac{1 \pm \sqrt{45}}{2} \qquad \text{Simplify.}$$

In this case, note that we can factor out a perfect square, namely $\sqrt{9}$.

$$x = \frac{1 \pm \sqrt{9}\sqrt{5}}{2} \qquad \sqrt{45} = \sqrt{9}\sqrt{5}.$$
$$x = \frac{1 \pm 3\sqrt{5}}{2} \qquad \text{Simplify: } \sqrt{9} = 3$$

19. Compare $x^2-9x+9=0$ with $ax^2+bx+c=0$ and note that $a=1$, $b=-9$, and $c=9$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2-4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 1 for a, -9 for b, and 9 for c.

$$x = \frac{-(-9) \pm \sqrt{(-9)^2-4(1)(9)}}{2(1)} \qquad \text{Substitute: 1 for } a, \ -9 \text{ for } b, \text{ and 9 for } c$$

$$x = \frac{9 \pm \sqrt{81-36}}{2} \qquad \text{Exponent, then multiplication.}$$

$$x = \frac{9 \pm \sqrt{45}}{2} \qquad \text{Simplify.}$$

In this case, note that we can factor out a perfect square, namely $\sqrt{9}$.

$$x = \frac{9 \pm \sqrt{9}\sqrt{5}}{2} \qquad \sqrt{45} = \sqrt{9}\sqrt{5}.$$

$$x = \frac{9 \pm 3\sqrt{5}}{2} \qquad \text{Simplify: } \sqrt{9}=3$$

21. Compare $x^2-3x-9=0$ with $ax^2+bx+c=0$ and note that $a=1$, $b=-3$, and $c=-9$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2-4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 1 for a, -3 for b, and -9 for c.

$$x = \frac{-(-3) \pm \sqrt{(-3)^2-4(1)(-9)}}{2(1)} \qquad \text{Substitute: 1 for } a, \ -3 \text{ for } b, \text{ and } -9 \text{ for } c$$

$$x = \frac{3 \pm \sqrt{9+36}}{2} \qquad \text{Exponent, then multiplication.}$$

$$x = \frac{3 \pm \sqrt{45}}{2} \qquad \text{Simplify.}$$

In this case, note that we can factor out a perfect square, namely $\sqrt{9}$.

$$x = \frac{3 \pm \sqrt{9}\sqrt{5}}{2} \qquad \sqrt{45} = \sqrt{9}\sqrt{5}.$$

$$x = \frac{3 \pm 3\sqrt{5}}{2} \qquad \text{Simplify: } \sqrt{9} = 3$$

23. Compare $x^2 - 7x - 19 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 1$, $b = -7$, and $c = -19$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 1 for a, -7 for b, and -19 for c.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-19)}}{2(1)} \qquad \text{Substitute: 1 for } a,\ -7 \text{ for } b, \text{ and } -19 \text{ for } c$$

$$x = \frac{7 \pm \sqrt{49 + 76}}{2} \qquad \text{Exponent, then multiplication.}$$

$$x = \frac{7 \pm \sqrt{125}}{2} \qquad \text{Simplify.}$$

In this case, note that we can factor out a perfect square, namely $\sqrt{25}$.

$$x = \frac{7 \pm \sqrt{25}\sqrt{5}}{2} \qquad \sqrt{125} = \sqrt{25}\sqrt{5}.$$

$$x = \frac{7 \pm 5\sqrt{5}}{2} \qquad \text{Simplify: } \sqrt{25} = 5$$

25. Compare $12x^2 + 10x - 1 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 12$, $b = 10$, and $c = -1$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 12 for a, 10 for b, and -1 for c.

$$x = \frac{-(10) \pm \sqrt{(10)^2 - 4(12)(-1)}}{2(12)} \qquad \text{Substitute: 12 for } a, \text{ 10 for } b, \text{ and } -1 \text{ for } c$$

$$x = \frac{-10 \pm \sqrt{100 + 48}}{24} \qquad \text{Exponent, then multiplication.}$$

$$x = \frac{-10 \pm \sqrt{148}}{24} \qquad \text{Simplify.}$$

In this case, note that we can factor out a perfect square, namely $\sqrt{4}$.

$$x = \frac{-10 \pm \sqrt{4}\sqrt{37}}{24} \qquad \sqrt{148} = \sqrt{4}\sqrt{37}.$$

$$x = \frac{-10 \pm 2\sqrt{37}}{24} \qquad \text{Simplify: } \sqrt{4} = 2$$

Finally, notice that both numerator and denominator are divisible by 2.

$$x = \frac{\dfrac{-10 \pm 2\sqrt{37}}{2}}{\dfrac{24}{2}} \qquad \text{Divide numerator and denominator by 2.}$$

$$x = \frac{\dfrac{-10}{2} \pm \dfrac{2\sqrt{37}}{2}}{\dfrac{24}{2}} \qquad \text{Distribute the 2.}$$

$$x = \frac{-5 \pm \sqrt{37}}{12} \qquad \text{Simplify.}$$

27. Compare $7x^2 - 10x + 1 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 7$, $b = -10$, and $c = 1$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a, b,$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 7 for a, -10 for b, and 1 for c.

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(7)(1)}}{2(7)}$$ Substitute: 7 for a, -10 for b, and 1 for c

$$x = \frac{10 \pm \sqrt{100 - 28}}{14}$$ Exponent, then multiplication.

$$x = \frac{10 \pm \sqrt{72}}{14}$$ Simplify.

In this case, note that we can factor out a perfect square, namely $\sqrt{36}$.

$$x = \frac{10 \pm \sqrt{36}\sqrt{2}}{14}$$ $\sqrt{72} = \sqrt{36}\sqrt{2}$.

$$x = \frac{10 \pm 6\sqrt{2}}{14}$$ Simplify: $\sqrt{36} = 6$

Finally, notice that both numerator and denominator are divisible by 2.

$$x = \frac{\dfrac{10 \pm 6\sqrt{2}}{2}}{\dfrac{14}{2}}$$ Divide numerator and denominator by 2.

$$x = \frac{\dfrac{10}{2} \pm \dfrac{6\sqrt{2}}{2}}{\dfrac{14}{2}}$$ Distribute the 2.

$$x = \frac{5 \pm 3\sqrt{2}}{7}$$ Simplify.

29. Compare $2x^2 - 12x + 3 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 2$, $b = -12$, and $c = 3$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula. Replace a, b,

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ and c with open parentheses.

Substitute 2 for a, -12 for b, and 3 for c.

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(2)(3)}}{2(2)}$$ Substitute: 2 for a, -12 for b, and 3 for c

$$x = \frac{12 \pm \sqrt{144 - 24}}{4}$$ Exponent, then multiplication.

$$x = \frac{12 \pm \sqrt{120}}{4}$$ Simplify.

In this case, note that we can factor out a perfect square, namely $\sqrt{4}$.

$$x = \frac{12 \pm \sqrt{4}\sqrt{30}}{4}$$ $\sqrt{120} = \sqrt{4}\sqrt{30}$.

$$x = \frac{12 \pm 2\sqrt{30}}{4}$$ Simplify: $\sqrt{4} = 2$

Finally, notice that both numerator and denominator are divisible by 2.

$$x = \frac{\dfrac{12 \pm 2\sqrt{30}}{2}}{\dfrac{4}{2}}$$ Divide numerator and denominator by 2.

$$x = \frac{\dfrac{12}{2} \pm \dfrac{2\sqrt{30}}{2}}{\dfrac{4}{2}}$$ Distribute the 2.

$$x = \frac{6 \pm \sqrt{30}}{2}$$ Simplify.

31. Compare $13x^2 - 2x - 2 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 13$, $b = -2$, and $c = -2$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Quadratic formula. Replace a, b,

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$ and c with open parentheses.

Substitute 13 for a, -2 for b, and -2 for c.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(13)(-2)}}{2(13)} \qquad \text{Substitute: 13 for } a, \ -2 \text{ for } b, \text{ and } -2 \text{ for } c$$

$$x = \frac{2 \pm \sqrt{4 + 104}}{26} \qquad \text{Exponent, then multiplication.}$$

$$x = \frac{2 \pm \sqrt{108}}{26} \qquad \text{Simplify.}$$

In this case, note that we can factor out a perfect square, namely $\sqrt{36}$.

$$x = \frac{2 \pm \sqrt{36}\sqrt{3}}{26} \qquad \sqrt{108} = \sqrt{36}\sqrt{3}.$$

$$x = \frac{2 \pm 6\sqrt{3}}{26} \qquad \text{Simplify: } \sqrt{36} = 6$$

Finally, notice that both numerator and denominator are divisible by 2.

$$x = \frac{\dfrac{2 \pm 6\sqrt{3}}{2}}{\dfrac{26}{2}} \qquad \text{Divide numerator and denominator by 2.}$$

$$x = \frac{\dfrac{2}{2} \pm \dfrac{6\sqrt{3}}{2}}{\dfrac{26}{2}} \qquad \text{Distribute the 2.}$$

$$x = \frac{1 \pm 3\sqrt{3}}{13} \qquad \text{Simplify.}$$

33. When the object returns to ground level, its height y above ground level is $y = 0$ feet. To find the time when this occurs, substitute $y = 0$ in the formula $y = 240 + 160t - 16t^2$ and solve for t.

$$y = 240 + 160t - 16t^2 \qquad \text{Original equation.}$$

$$0 = 240 + 160t - 16t^2 \qquad \text{Set } y = 0.$$

Each of the coefficients is divisible by 16.

$$0 = t^2 - 10t - 15 \qquad \text{Divide both sides by } -16.$$

Compare $t^2 - 10t - 15 = 0$ with $at^2 + bt + c = 0$ and note that $a = 1$, $b = -10$, and $c = -15$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution. Note that we are solving for t this time, not x.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{Quadratic formula. Replace } a,\ b,$$

$$t = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \qquad \text{and } c \text{ with open parentheses.}$$

Substitute 1 for a, -10 for b, and -15 for c.

$$t = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-15)}}{2(1)} \qquad \text{Substitute: 1 for } a,\ -10 \text{ for } b, \text{ and } -15 \text{ for } c$$

$$t = \frac{10 \pm \sqrt{100 + 60}}{2} \qquad \text{Exponent, then multiplication.}$$

$$t = \frac{10 \pm \sqrt{160}}{2} \qquad \text{Simplify.}$$

The answer is not in simple form, as we can factor out $\sqrt{16}$.

$$t = \frac{10 \pm \sqrt{16}\sqrt{10}}{2} \qquad \sqrt{160} = \sqrt{16}\sqrt{10}.$$

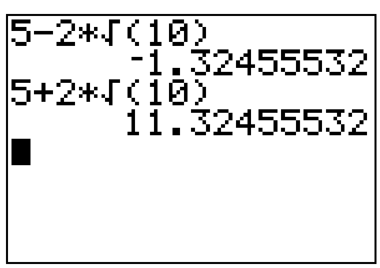
$$t = \frac{10 \pm 4\sqrt{10}}{2} \qquad \text{Simplify: } \sqrt{16} = 4$$

Use the distributive property to divide both terms in the numerator by 2.

$$t = \frac{10}{2} \pm \frac{4\sqrt{10}}{2} \qquad \text{Divide both terms by 2.}$$

$$t = 5 \pm 2\sqrt{10} \qquad \text{Simplify.}$$

Thus, we have two solutions, $t = 5 - 2\sqrt{10}$ and $t = 5 + 2\sqrt{10}$. Use your calculator to find decimal approximations.



Thus:

$$t \approx -1.32455532 \qquad \text{or} \qquad t \approx 11.3245532$$

Rounding to the nearest tenth, $t \approx -1.3, 11.3$. The negative time is irrelevant, so to the nearest tenth of a second, it takes the object approximately 11.3 seconds to return to ground level.

35. The manufacturer breaks even if his costs equal his incoming revenue.

$$R = C$$

Replace R with $6000x - 5x^2$ and C with $500000 + 5.25x$.

$$6000x - 5x^2 = 500000 + 5.25x$$

The equation is nonlinear, so make one side equal to zero by moving all terms to one side of the equation.

$$\begin{aligned} 0 &= 5x^2 + 5.25x - 6000x + 500000 \\ 0 &= 5x^2 - 5994.75x + 500000 \end{aligned}$$

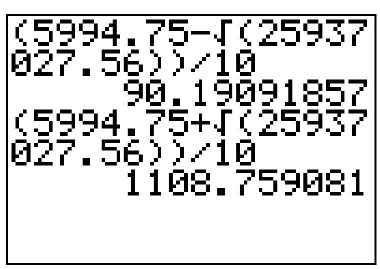
Compare $5x^2 - 5994.75x + 500000 = 0$ with $ax^2 + bx + c = 0$ and note that $a = 5$, $b = -5994.75$, and $c = 500000$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \end{aligned}$$

Substitute 5 for a, -5994.75 for b, and 500000 for c, then simplify.

$$\begin{aligned} x &= \frac{-(-5994.75) \pm \sqrt{(-5994.75)^2 - 4(5)(500000)}}{2(5)} \\ x &= \frac{5994.75 \pm \sqrt{35937037.56 - 10000000}}{10} \\ x &= \frac{5994.75 \pm \sqrt{25937027.56}}{10} \end{aligned}$$

Enter each answer into your calculator.



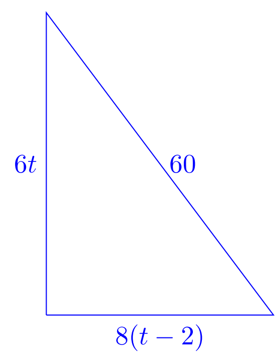
Thus, the solutions are

$$x \approx 90.19091857 \qquad \text{or} \qquad x \approx 1108.759081$$

The first answer rounds to 90 widgets, the second answer to 1109 widgets.

37. At the moment they are 60 miles apart, let t represent the time that Mike has been riding since noon. Because Todd started at 2 pm, he has been riding for two hours less than Mike. So, let $t-2$ represent the number of hours that Todd has been riding at the moment they are 60 miles apart.

Now, if Mike has been riding at a constant rate of 6 miles per hour for t hours, then he has travelled a distance of $6t$ miles. Because Todd has been riding at a constant rate of 8 miles per hour for $t-2$ hours, he has travelled a distance of $8(t-2)$ miles.



The distance and direction traveled by Mike and Todd are marked in the figure above. Note that we have a right triangle, so the sides of the triangle must satisfy the Pythagorean Theorem. That is,

$$(6t)^2 + [8(t-2)]^2 = 60^2. \qquad \text{Use the Pythagorean Theorem.}$$

Distribute the 8.

$$(6t)^2 + (8t-16)^2 = 60^2 \qquad \text{Distribute the 8.}$$

Square each term. Use $(a-b)^2 = a^2 - 2ab + b^2$ to expand $(8t-16)^2$.

$$\begin{aligned} 36t^2 + 64t^2 - 256t + 256 &= 3600 && \text{Square each term.} \\ 100t^2 - 256t + 256 &= 3600 && \text{Simplify: } 36t^2 + 64t^2 = 100t^2 \end{aligned}$$

The resulting equation is nonlinear. Make one side equal to zero.

$$\begin{aligned} 100t^2 - 256t - 3344 &= 0 && \text{Subtract 3600 from both sides.} \\ 25t^2 - 64t - 836 &= 0 && \text{Divide both sides by 4.} \end{aligned}$$

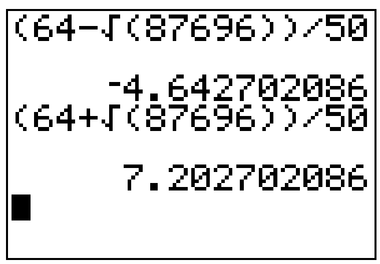
Compare $25t^2 - 64t - 836 = 0$ with $at^2 + bt + c = 0$ and note that $a = 25$, $b = -64$, and $c = -836$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution. Note that we are solving for t this time, not x.

$$
\begin{aligned}
t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{Quadratic formula. Replace } a,\ b, \\
t &= \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} && \text{and } c \text{ with open parentheses.}
\end{aligned}
$$

Substitute 25 for a, -64 for b, and -836 for c.

$$
\begin{aligned}
t &= \frac{-(-64) \pm \sqrt{(-64)^2 - 4(25)(-836)}}{2(25)} && \text{Substitute: 25 for } a, \\
& && -64 \text{ for } b, \text{ and } -836 \text{ for } c \\
t &= \frac{64 \pm \sqrt{4096 + 83600}}{50} && \text{Exponent, then multiplication.} \\
t &= \frac{64 \pm \sqrt{87696}}{50} && \text{Simplify.}
\end{aligned}
$$

Now, as the request is for an approximate time, we won't bother with simple form and reduction, but proceed immediately to the calculator to approximate this last result.



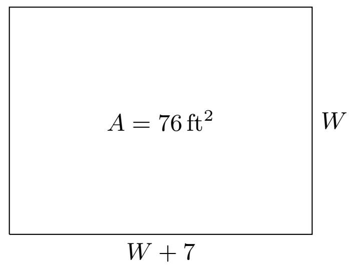
The negative time is irrelevant and discarded. Thus, Mike has been riding for approximately 7.202702086 hours. To change 0.202702086 hours to minutes,

$$0.202702086\,\text{hr} = 0.202702086\,\text{hr} \times \frac{60\,\text{min}}{\text{hr}} = 12.16212516\,\text{min}$$

Rounding to the nearest minute, Mike has been riding for approximately 7 hours and 12 minutes. Because Mike started riding at noon, the time at which he and Todd are 60 miles apart is approximately 7:12 pm.

39. We follow the *Requirements for Word Problem Solutions.*

1. *Set up a variable dictionary.* A carefully labeled figure will help us maintain our focus. We'll let W represent the uniform width of the field. Because the length is 7 feet longer than its width, the length is $W + 7$.



2. *Set up an equation.* The area of the rectangular field is found by multiplying the width and the length. Thus,

$$\begin{aligned} \text{Area} &= \text{Width} \times \text{Length} \\ 76 &= W(W+7) \end{aligned}$$

3. *Solve the equation.* We start by expanding the right-hand side of the equation.

$$76 = W^2 + 7W$$

The resulting equation is nonlinear. Make one side zero.

$$0 = W^2 + 7W - 76$$

Compare $W^2 + 7W - 76 = 0$ with $aW^2 + bW + c = 0$ and note that $a = 1$, $b = 7$, and $c = -76$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution. Note that we are solving for W this time, not x.

$$W = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$W = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

Quadratic formula. Replace a, b, and c with open parentheses.

Substitute 1 for a, 7 for b, and -76 for c.

$$W = \frac{-(7) \pm \sqrt{(7)^2 - 4(1)(-76)}}{2(1)}$$

Substitute: 1 for a, 7 for b, and -76 for c

$$W = \frac{-7 \pm \sqrt{49 + 304}}{2}$$

Exponent, then multiplication.

$$W = \frac{-7 \pm \sqrt{353}}{2}$$

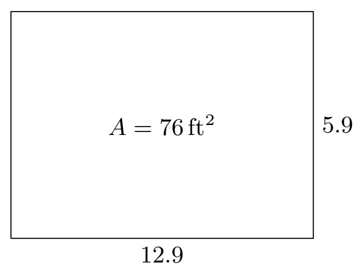
Simplify.

Now, as the request is for an approximate time, we won't bother with simple form and reduction, but proceed immediately to the calculator to approximate this last result. Enter the expression **(-7-sqrt(353))(2)** and press the ENTER key. Enter the expression **(-7+sqrt(353))(2)** and press the ENTER key. The results are

$$W \approx -12.894147114 \qquad \text{or} \qquad W \approx 5.894147114.$$

4. *Answer the question.* The negative width is irrelevant and discarded. Thus, rounding to the nearest tenth of a foot, the width of the field is 5.9 feet. Because the length is 7 feet longer than the width, the length of the field is 12.9 feet.

5. *Look back.* Because we rounded to the nearest tenth of a foot, we don't expect the answers to check exactly.



Note that $(5.9)(12.9) = 76.11$, which is very close to the given area.

41. Substitute the concentration $C = 330$ into the given equation.

$$\begin{aligned} C &= 0.01125t^2 + 0.925t + 318 \\ 330 &= 0.01125t^2 + 0.925t + 318 \end{aligned}$$

The equation is nonlinear, so make one side equal to zero by subtracting 330 from both sides of the equation.

$$0 = 0.01125t^2 + 0.925t - 12$$

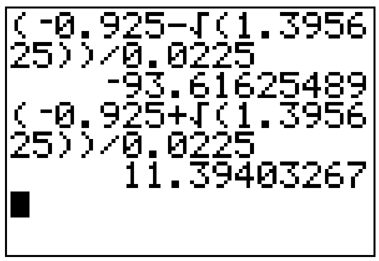
Compare $0.01125t^2 + 0.925t - 12 = 0$ with $at^2 + bt + c = 0$ and note that $a = 0.01125$, $b = 0.925$, and $c = -12$. Replace each occurrence of a, b, and c with open parentheses to prepare the quadratic formula for substitution. Note that we are solving for t this time, not x.

$$\begin{aligned} W &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ W &= \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)} \end{aligned}$$

Substitute 0.01125 for a, 0.925 for b, and -12 for c, then simplify.

$$
\begin{aligned}
W &= \frac{-(0.925) \pm \sqrt{(0.925)^2 - 4(0.01125)(-12)}}{2(0.01125)} \\
W &= \frac{-0.925 \pm \sqrt{0.855625 + 0.54}}{0.0225} \\
W &= \frac{-0.925 \pm \sqrt{1.395625}}{0.0225}
\end{aligned}
$$

Enter each answer into your calculator.



Thus, the solutions are

$$t \approx -93.61625489 \qquad \text{or} \qquad t \approx 11.39493267.$$

The negative answer is irrelevant and is discarded. Remember, t represents the number of years after the year 1962. Rounding the second answer to the nearest year, the concentration reached a level of 330 parts per million 11 years after the year 1962, that is, in the year 1973. *It is interesting to compare this answer with the actual data:* `ftp://ftp.cmdl.noaa.gov/ccg/co2/trends/co2_annmean_mlo.txt`.